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ANALYSIS OF MATHEMATICAL COMMUNICATION SKILLS IN TRIGONOMETRY TOPIC BASED ON HABITS OF MIND

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ABSTRACT

Mathematical communication is an important skill that must be developed in trigonometry learning but is often not developed optimally. Students tend to rely on procedural formulas without being able to explain, represent, and systematically justify their reasons. Habits of mind are one of the internal factors indicated to influence students' mathematical communication skills. The purpose of the study to describe students' mathematical communication skills in trigonometry based on habits of mind. This research uses descriptive approach and qualitative data collection techniques such as mathematical communication skill test and in-depth interviews. The subjects of the study were 22 students of class X E3 at MA Negeri Sukoharjo selected purposively based on their habits of mind categories which are limited, developer and expert, from the results of using the Habits of Mind questionnaire. The result showed that: (1) student with the limited habits of mind category had weak mathematical communication skills in all indicators namely drawing, mathematical expression, and writing; (2) student with the developer habits of mind category showed better mathematical communication skills, but still showed inconsistencies and incompleteness in written representations and narratives; (3) Student in the expert category have excellent mathematical communication skills across all indicators, can communicate mathematical ideas completely, accurately, and systematically. The findings of this study indicate that habits of mind have an important role in developing students' mathematical communication skills in trigonometry topic. Therefore, teachers are encouraged to design learning activities that encourage habits of mind to improve students' ability to communicate mathematical ideas effectively.

Keywords: Mathematical Communication Skills, Habits of Mind, Trigonometry.

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PRELIMINARY

Mathematics plays a strategic role in shaping the scientific thinking required in various fields of science and its application in everyday life. Therefore, this view emphasizes that mathematics learning should not only aim to master concepts and procedures, but also develop students' mathematical thinking and communication skills (Maulyda, 2019)

According to The National Council of Teachers of Mathematics (NCTM) (2014), five skills as process standards in mathematics learning that student must possess are the skill to communicate, skill to solve problems, skill to represent, skill to connect, and skill to reason. These five skills are needed by students so that they have the skill to obtain, utilize, and process information to survive in a dynamic and competitive situation. Mathematical communication skills are important skills because they play a role in expressing ideas and concept both verbally and in writing as in effort to solve mathematical problem.

The Ontario Ministry of Education (2005) (in Hendriana et al., 2017) categorizes mathematical communication skills into three main aspects: (1) written text, providing answers using one's own language, creating models of situations or problems using spoken, written, concrete, graphic and algebraic methods, explaining and asking questions about mathematics that has been studied, listening, discussing and writing about mathematics, making conjectures, constructing arguments and generalizations; (2) drawing, reflecting real objects, pictures, and diagrams into mathematical ideas and; (3) mathematical expression, express mathematical concepts by stating everyday events in mathematical language or symbols. These three aspects reflect students' skills to convey mathematical ideas in writing, visually, and symbolically.

Mathematical communication skills are important because mathematics serves not only as a thinking tool but also as a language for conveying ideas and solutions to problems (Robiah et al., 2019). With good communication skills, students are expected to be able to understand concepts more deeply and solve mathematical problems meaningfully

Various studies have shown that mathematical communication skills support student achievement in understanding mathematical concepts and solving mathematical problems. Mathematical communication enables students to share ideas, clarify their understanding, and express their knowledge. The acquisition of proficient mathematical communication skills will facilitate a more profound comprehension of mathematical concepts (Nurun et al., 2024). Good mathematical communication skills tend to enable students to explain solution steps in an organized manner, use mathematical notation correctly, and draw logical and comprehensive conclusions (Hendriana et al., 2017).

Conversely, low mathematical communication skills have an impact on student's difficulty in understanding mathematical concepts optimally even though they able to remember certain formulas or methods. However, the reality is quite worrying. Various studies show that students' mathematical communication skill tend to be low when studying abstract mathematics that requires a high level of presentation.

Trigonometry is a mathematics topic considered difficult by students because it requires a deep conceptual understanding and the skill to relate mathematical representations and rules to each other (Nurfauziah & Sari, 2018). The characteristics of trigonometry topic, which requires visualization, representation, and mathematical modeling skills, make this topic relevant for assessing students' mathematical communication skills, Students' difficulties in trigonometry are often related to their skill to represent problem situations in appropriate mathematical images, symbols, and explanations. Trigonometry studies trigonometric ratios by relating the angles and sides of triangles and applying trigonometric functions. Learning trigonometry requires strong visual intelligence and mathematical communication skills (Sánchez et al., 2023). Previous research conducted by Utami et al. (2021) showed that students' mathematical communication skills in solving trigonometry problems were still at a low to intermediate level, particularly in the indicators of explaining solution steps and interpreting mathematical symbols. This is in line with the research findings of Faradillah (2022), which stated that even though students master basic calculations, they still difficulty experience expressing mathematical ideas in writing in trigonometry problems.

Furthermore, Sari et al. (2025) found in their research that some students were able to create visual or symbolic representations but were unable to connect these representations to logical mathematical explanations. This suggests that mathematical communication skills are not the most important skill in solving trigonometry problems, but rather formula mastery. Therefore, it can be argued that mathematical communication skills are a crucial factor to consider in trigonometry learning.

In line with the findings of previous research, the results of interviews with mathematics teachers at MA Negeri Sukoharjo also showed that students still experience difficulties in solving mathematical problems, especially in describing problem situations, translating contextual problems into mathematical models, and compiling written answers and conclusions. This can be seen from the results of students' answers when given an assessment on trigonometry Topic, as in Figure 1.

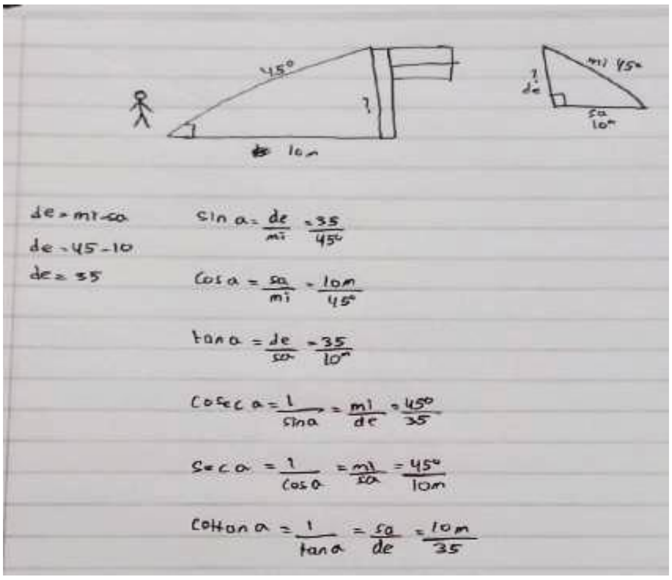

Figure 1. The Student's Answer sheet

Figure 1 is the answer to a question that instructs us to determine the height of a flagpole given the angle of elevation and the observer's height. Student was unable to describe the situation in the story problem and was unable to express it in mathematical symbols, thus failing to find the answer. Student was also unable to express the conclusion of the solution in words. This is an indication that students are not yet accustomed to communicating mathematical ideas systematically so that their conceptual understanding has not been internalized well and completely.

It appears that students able to represent problems in the form of triangles and use trigonometric ratios, so it can be said that students' representation skills have emerged but are not communicated well. Furthermore, mathematical connection skills are also seen to be present because students able to link concepts, namely trigonometry with the Pythagorean theorem. Next, reasoning skills appear to be quite running, seen in the logical sequence of steps but are not expressed in writing. In problem-solving skills, the strategies used by students are also appropriate and contextual, so it can be said that students are able to solve problems but the results and processes are not communicated optimally. Based on the initial analysis of several of these skills, it can be said that the most common obstacles are in mathematical communication skills in the form of drawing, written text, and mathematical expressions, which are not yet optimally visible.

One internal factor thought to influence students' mathematical communication skills is habits of mind. In line with Astuti & Yuliani (2024) stated that there is an influence of habits of mind on students' mathematical communication abilities of 84.8%. Habits of mind refer to an individual's tendency to consistently use certain thought patterns when faced with problems that cannot be solved immediately (Costa & Kallick, 2008).

Habits of mind play a crucial role in mathematics learning, shaping students' understanding of problems, selecting problem solving strategies, and communicating the process and results of problem solving. Hendriana et al. (2017) stated that students with good thinking habits tend to be able to explain mathematical ideas coherently and logically. Habits of mind and students' mathematical communication skills show a positive relationship (Ilmi et al., 2022). Habits of mind are defined as the intelligent thinking habits individuals possess when facing problems for which solutions cannot be readily obtained (Costa & Kallick, 2008). Habits of mind reflect patterns of intellectual behavior that develop through continuous learning and experience. In the context of mathematics education, habits of mind play a role in shaping how students think, make decisions, and solve mathematical problems Cuaco (1997) (in Hendriana et al., 2017).

Costa & Kallick (2012) describe habits of mind into sixteen indicators as follows: (1) Persistence or never giving up; (2) Managing your conscience; (3) Listening to other people's opinions with empathy; (4) Thinking flexibly; (5) Thinking metacognitively; (6) Trying to work carefully and precisely; (7) Asking and presenting problems effectively; (8) Utilizing past experiences to form new knowledge; (9) Thinking and communicating clearly and precisely; (10) Utilizing the senses in collecting and processing data; (11) Creating, imagining, and innovating; (12) Being enthusiastic in responding; (13) Dare to take responsibility and face risks; (14) Humorous; (15) Thinking interdependently; (16) Continuous learning.

Costa and Kallick proposed sixteen indicators of habits of mind, including persistence or never giving up, flexible thinking, metacognitive thinking, striving for thoroughness and accuracy, and thinking and communicating clearly and accurately. Several researchers subsequently adapted the framework into levels or stages of development for assessment purposes, these categories range from the initial level with very limited emergence of indicators to the advanced level which shows consistency in almost all habits of mind indicators (Dwirahayu et al., 2018)

Rahmah et al. (2022) in their research divided the stages of habits of mind based on Costa & Kallick (2012) into 4 categories, namely; (1) Beginner (no concept), the habits of mind possessed by students are very minimal, indicated by providing a simple response. (2) Limited, at this stage the habits of mind possessed by students have begun to show their characteristics but the responses given are still limited. (3) Developing, at this stage the habits of mind possessed by students have been developed with good responses. (4) Expert (proficient), this stage shows that students have almost 16 indicators of habits of mind and the responses they give are good.

Based on the adaptation of Costa and Kallick's habits of mind indicators, the abilities of students in this study were classified into four stages of development, namely:
(1) the beginner stage, characterized by the emergence of very minimal indicators; (2) the limited stage, when students began to show the characteristics of habits of mind but were not yet consistent; (3) the development stage, when the habits of mind indicators began to develop with more appropriate responses; and (4) the advanced stage, which showed the emergence of almost all habits of mind indicators consistently.

Differences in the level of habits of mind are thought to influence students' skill to understand problems, communicate ideas, and develop mathematical solutions. Students with more developed habits of mind tend to show better mathematical communication skills than students with low habits of mind (Ilmi et al., 2022)

Various previous studies have examined students' mathematical communication skills and habits of mind separately and then linked them to learning outcomes or problem-solving abilities. However, most of these studies have focused on quantitatively measuring ability levels or the relationships between variables, thus failing to provide an in-depth understanding of how students' mathematical communication processes are shaped by their habits of mind. However, mathematical communication skills are not solely determined by whether an answer is correct or incorrect, but also by how students describe, write, and express mathematical ideas coherently and meaningfully.

Furthermore, there is limited research examining mathematical communication skills from the perspective of habits of mind. This study employed a descriptive qualitative approach that deeply analyzed each indicator of mathematical communication: drawing, writing, and mathematical expression. Then, trigonometry remains a challenging topic for students because it requires several skills, such as visual representation, modeling, and explaining symbols. Therefore, students' habits of mind are suspected to play a role in determining the quality of communication. Therefore, research is needed that specifically analyzes students' mathematical communication skills in trigonometry material from the perspective of the habits of mind category, so that it is hoped that it can provide a complete picture of the characteristics of students' mathematical communication based on their thinking habits.

Therefore, based on the foregoing description, this study aims to describe students' mathematical communication skills in solving trigonometry problems, assessed by habits of mind categories: beginner, limited, developing, and expert. This research is expected to provide a more comprehensive picture of the characteristics of students' mathematical communication within each habit of mind category, enabling teachers to better understand their individual abilities, which can ultimately inform their considerations in designing more effective learning that focuses on developing students' thinking processes.

METHODS

This study used a qualitative method and a descriptive approach, aiming to provide an in-depth description of students' mathematical communication skills in trigonometry viewed from perspective of habits of mind. The qualitative approach was used because this study focuses on the meaning, processes, and characteristics experienced by the research subjects naturally (Creswell & Creswell, 2017; Moleong, 2021).

The research design, a case study, allowed researchers to explore students' mathematical communication skills more deeply, holistically, and comprehensively based on their habits of mind (Anggito & Setiawan, 2018). This study was conducted at MA Negeri Sukoharjo, with 22 students in grade X E3 selected through purposive sampling. Purposive sampling was used to select subjects who could provide relevant and in-depth information in line with the research objectives (Sugiyono, 2019).

The instruments used included a habits of mind questionnaire, a mathematical communication skills test, and an interview guide. The use of more than one data collection technique aims to obtain rich and complementary data (Creswell & Creswell, 2017). The Habits of Mind questionnaire was used to group students according to their Habits of Mind categories. This questionnaire adapts the statement items developed by Sumarmo (in Hendriana et al., 2017) based on 16 indicators according to Costa & Kallick.

There are 32 statements with 16 positive statements and 16 negative statements, then they were tested and 27 valid statements were obtained. There are 27 statements consisting of 13 positive statements and 14 negative statements. This questionnaire uses a Likert scale with four answer choices: strongly disagree (SD), disagree (D), agree (A), and strongly agree (SA). The scoring guidelines for each questionnaire question based on the choices are presented in Table 1.

Table 1. Habits of Mind Questionnaire Scoring Guidelines

Statement	SA	A	D	SD
Positive	4	3	2	1
Negative	1	2	3	4

The Habits of Mind criteria interval is determined using an interval formula. Determination of the questionnaire score category is done using the class interval technique, namely dividing the difference between the maximum and minimum scores by the number of categories specified (Sudjana, 2017). The maximum score for each statement is 4, with 27 statements the maximum score obtained is 108 and the minimum score is 27 with a range of 81 and 4 categories of habits of mind, resulting in an interval of 20,25. From these calculations, the intervals and categories are then determined as in Table 2.

Table 2. Intervals and Categories of Habits of Mind Scores

No	Interval	Categories
1	27 – 47	Beginner
2	48 – 68	Limited
2	69 – 88	Developer
3	89 – 108	Expert

Tests were used to identify students' mathematical communication skills based on indicators according to the Ontario Ministry of Education (in Hendriana et al., 2017), namely drawing, mathematical expression, and writing. This test is in the form of descriptive questions consisting of three mathematical communication skills questions, which are given for 60 minutes. Test question grid is presented in the Table 3.

Table 3. Test Question Grid

<table>
<tr><th>No</th><th>Indicators of Competence Achievement</th><th>Mathematical Communication Indicators</th><th>Cognitive Dimension</th></tr>
<tr><td>1</td><td>Using the concept of trigonometric ratios of special angles in solving problems</td><td rowspan="3">The ability to express and write down mathematical ideas through writing or describing information in the form of pictures, tables, graphs (Drawing)
The ability to analyze and evaluate mathematical ideas in written form (Writing)
The ability to use mathematical terms, notation, and structures to present ideas (Mathematical Expression)</td><td>C3 (Apply)</td></tr>
<tr><td>2</td><td>Analyze a trigonometric ratio value to find a solution to a problem.</td><td>C4 (Analyze)</td></tr>
<tr><td>3</td><td>Using the concept of trigonometric comparisons (sine, cosine, tangent, cosecant, secant, and cotangent) in right triangles to solve problems in everyday life.</td><td>C5 (Evaluate)</td></tr>
</table>

The scoring guidelines for the mathematical communication test are in Table 4.

Table 4. Mathematical Communication Skill Test Scoring Guidelines

Indicator	Student Answer	Score
The ability to express and write down mathematical ideas through writing or depicting information in the form of pictures, tables, graphs. (Drawing)	Describe the information correctly and completely	4
	Describe the information correctly and incomplete	3
	Describe the state of the information in writing, some of which is correct	2
	Describes the state of information in a way that is wrong. However, it is wrong	1
	Does not describe the state of the information	0
The ability to analyze and produce mathematical ideas in written form (Writing)	Write the answer by providing logical, correctly and completely arguments	4
	Write the answer by providing logical, correct arguments. This is incomplete	3
	Write an answer by providing logical arguments and some of which are correct	2
	Writing an answer by providing illogical arguments	1
	Does not make an answer	0
The ability to use mathematical terms, notation and structures to present ideas. (Mathematical Expression)	Write down the terms, mathematical notation, and structures correctly and completely	4
	Writing the terms, mathematical notation, and structure correctly. This is incomplete	3
	Writing terms, mathematical notation, and structures with a small amount of correctness	2
	Writing down terms, mathematical notation, and structures even if they are wrong	1
	No written answer	0
Maximum Score		12

Based on the results of the students' mathematical communication skill test, the criteria were then determined. The determination of the mathematical communication skill criteria was carried out using a normative approach based on the average value and standard deviation. According to Azwar (2022), score grouping can be done by comparing individual scores to the group average with a reference of one standard deviation. Guidelines for determining the criteria are as shown in Table 5.

Table 5. Table of Criteria for Mathematical Communication Skill

No	Interval	Criteria
1	$X < \mu - 1.\sigma$	Low
2	$\mu - 1.\sigma \leq X < \mu + 1.\sigma$	Medium
3	$X \geq \mu + 1.\sigma$	High

Source: Azwar (2022)

After carrying out the test and being assessed based on the mathematical communication skill indicators, the average obtained was 72 and the standard deviation was 19. From these results, the scores for the mathematical communication skill criteria were obtained as in Table 6.

Table 6. Mathematical Communication Skill Criteria Score

No	Interval	Criteria
1	$X < 53$	Low
2	$53 \leq X < 91$	Medium
3	$X \geq 91$	High

Interviews were used to strengthen and deepen the data obtained from the test results. The data obtained were then analyzed using qualitative data analysis techniques including the stages of data reduction, data presentation, and drawing conclusions. Data validity was through technical triangulation, namely comparing test results and interview results to obtain consistent and reliable data.

RESULT AND DISCUSSION

Based on the results of the habits of mind questionnaire completed by 22 research subjects, it was found that there were no students in the beginner category. A total of 2 students (9.1%) were in the limited category, 17 students (77.3%) were in the developer category, and 3 students (13.6%) were in the expert category. From the distribution of the habits of mind categories, it could be seen that the majority of students with habits of mind were in the developer category, although there were still some students with habits of mind in the limited category.

Based on the results of the mathematical communication skill test of 22 research subjects, there were 4 students (18.2%) in the low criteria, 12 students (54.5%) in the medium criteria, and 6 students (27.3%) in the high criteria. The findings of this study indicate that students' majority have medium mathematical communication abilities, this indicates that mathematical communication abilities have not been achieved optimally in trigonometry topic.

Limited Category

Students with this limited category of habits of mind are represented by subject T-16. Subject T-16's written answers are shown in Figure 2.

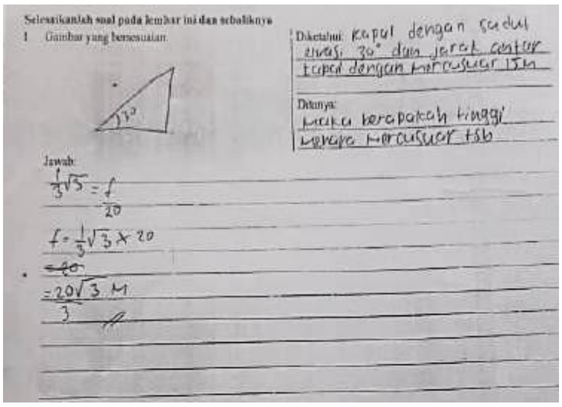

Figure 2. T-16's Answer Sheet

Based on Figure 2, it could be seen that subject T-16 was unable to draw a triangle representing the problem in the drawing indicator. The drawing was incomplete and its accuracy was not verified. In the mathematical expression indicator, students tended to be able to select the trigonometric ratios but made errors in notation, value substitution, and calculations. In the writing indicator, students did not provide a written narrative or justification, so the problem solution consisted only of a series of symbols without a logical explanation.

Furthermore, T-16 code represents students with a limited category with absence number 16 and R2 is the code for researchers when interviewing T-16 subjects. During the interview session, some information was obtained regarding subject T-16's communication skills, as seen in the following interview excerpt.

R2-04 : Do you think your picture is correct or is it still wrong?
T-1604 : This is just nonsense, ma'am, I do not know ma'am. I want to see it but I am confused.
R2-05 : So, you are not sure about that?
T-1605 : Not sure ma'am
R2-06 : From your drawing, did you immediately continue working on it?
T-1606 : Yes ma'am, I have difficulty seeing things. So, I just write.

From the interview, it was found that the drawings were made carelessly and were only seen as a formality, not as a communication or problem-solving tool.

R2-25	:	Are you sure your solution steps are correct?
T-1625	:	Wrong ma'am
R2-26	:	If it was wrong, where is the mistake?
T-1626	:	My mistake, ah I just wrote it, ma'am, I did not know where it's going
R2-31	:	Number 3 what trigonometric ratio do you use?
T-1631	:	This is what I remember ma'am
R2-32	:	Why do you use this method?
T-1632	:	I remember when Mrs. Nur Indah was teaching, I thought, "Just try this. Before working on this problem, I studied a little, so when I worked on it, I just wrote down what I remembered, I did not count.

From the interview, information was obtained that the selection of trigonometric ratios was based on the memory of the T-16 subjects when taught by the subject teacher, not because of their understanding of the concept, so that the selection of the formula was wrong, causing the wrong results and ultimately not being able to provide the right conclusions.

Triangulation results showed consistency between students' written test answers and the information obtained from interviews, thus confirming the validity of the data obtained. The triangulation results suggest that students' weaknesses in mathematical communication skills are related to their impulsive, inaccurate, and unreflective thinking habits.

Developer Category

Students with developer habits of mind are represented by subject P-21. Subject P-21's written answers are shown in Figure 3.

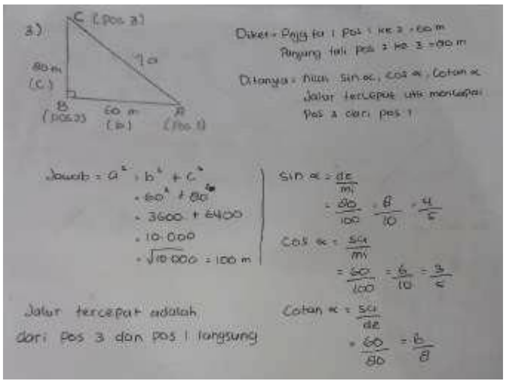

Figure 3. P-21's Answer Sheet

Based on Figure 3, it could be seen that in the drawing indicator, subject P-21 was able to draw a triangle that represents the problem. The drawing was quite complete, but forgot to label the angles, so the information produced by the drawing was incomplete. In the mathematical expression indicator, students were able to select trigonometric ratios, substitute values, and perform calculations correctly. In the writing indicator, students wrote down what was known, asked, and answered in a coherent manner, but did not provide logical justification for arriving at conclusions.

Furthermore, P-21 code represents students with developer category with absence number 21 and R4 is the code for researchers when interviewing P-21 subjects. During the interview session, some information was obtained regarding subject P-21's communication skills, as seen in the following interview excerpt

R4-14	:	Is your drawing for number 3 complete? If not, what is missing?
P-2114	:	Corner mam
R4-15	:	Why didn't you think of that at the time?
P-2115	:	Kinda forgot. Suddenly not using corners
R4-16	:	But now you just realized, huh?
P-2116	:	Yes ma'am, after seeing the other question pictures
R4-17	:	So, where should α be?
P-2117	:	α is between the ropes of post 1 and post 2 and post 1 and post 3
R4-18	:	So, what do you think about this picture you made?
P-2118	:	It's correct but incomplete, ma'am

From the interview, information was obtained that the image created was correct but incomplete, however, the image could be used as a basis for mathematical calculations and problem solving.

R4-36	:	What is the unknown number 3 in the dance sketch?
P-2136	:	Length of rope from post 1 to post 3 ma'am
R4-37	:	So, what do you use to find the length of the rope?
P-2137	:	Using the surrounding sides. What was it called? Oh yeah, the Pythagorean theorem.
R4-38	:	Is the solution written correctly or incorrect? This is a square, while the other side has been rooted.
P-2138	:	There should not to be any squares, ma'am.
R4-2139	:	Now for sin α in front per hypotenuse, cos α in side per hypotenuse with hypotenuse 100. Are you sure about this answer?
P-2139	:	Sure ma'am
R4-40	:	Next, why can you say the fastest route is from post 3 and post 1 directly?
P-2140	:	I checked that Post 1 to Post 2 was still 60 meters, then after that it was another 80 meters there
R4-41	:	Why don't you write down the steps on the answer sheet? Why jump straight to a conclusion without explaining first?
P-2141	:	I previously scribbled on the calculation sheet, ma'am.

The interview revealed that subject P-21 was able to use trigonometric ratios and symbolic notation quite accurately, although some calculation errors and data inconsistencies were still found. Furthermore, subject P-21 was able to write down information and justify the steps taken to solve the problem.

In the writing indicator, students began to write down known and requested information, but the narrative explanation of the solution steps was not yet fully coherent and in-depth. These findings indicate that students in the developer category have developed fairlygood thinking habits, but are not yet fully reflective and consistent in communicating their mathematical ideas.

Expert Category

Students with Expert category habits of mind are represented by subject A-17. Subject A-17's written answer can be seen in Figure 4.

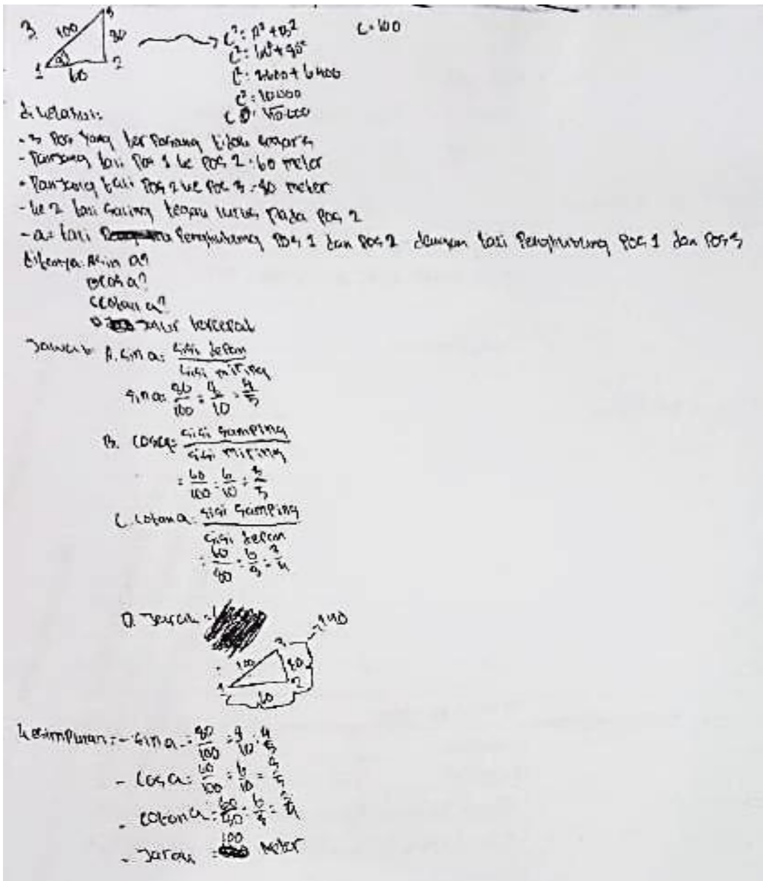

Figure 4. A-17's Answer Sheet

Based on Figure 4, it could be seen that in the drawing indicator, subject A-17 is able to draw completely and accurately. The created drawing can represent the story situation and be used as a basis for problem solving. In addition, subject A-17 is also able to create a drawing to evaluate the answer to the question in the problem. In the mathematical expression indicator, students able to choose the trigonometric ratio, substitute values, and perform calculations correctly and precisely. In the writing indicator, students write what is known, is asked and answered in a coherent manner, providing logical justification to reach a correct conclusion.

Furthermore, A-17 code represents students with a Expert category with absence number 17 and R6 is the code for researchers when interviewing A-17 subjects. During the interview session, some information was obtained regarding subject A-17's communication skills, as seen in the following interview excerpt

R6-09 : Now the third picture, how can you explain this third picture?
A-1709 : I remember there were soldiers who were training and the rope from post 1 to post 2 was 60 m long, then from post 2 to post 3 it was 80 m and it was not perpendicular
R6-10 : Then what post is the perpendicular image in?
A-1710 : At post 2 ma'am
R6-11 : Where is the known angle located?
A-1711 : Angle α is at the rope post between post 1 to post 2 and post 1 to post 3
R6-12 : Do you think this picture is enough as a basis for solving the problem or not?
A-1712 : Yes, ma'am
R6-13 : So, do you think your picture is correct?
A-1713 : Yes ma'am, that was right
R6-48 : So your fourth picture is to check which route is the fastest, right? So you check them one by one?
A-1748 : Yes ma'am, if from post 1 to post 2 60 then post 2 to post 3 80 and then add it up it turns out to be 140, if from post 1 straight to post 3 only the sloping side is 100

The interview revealed that subject A-17 was able to create precise and detailed drawings, which were very strong enough to serve as the basis for calculations in problem-solving. Furthermore, subject A-17 also created drawings to check and evaluate answers to problems.

R6-35 : Moving on to number 3, okay? You have written down everything you know completely and correctly. What was the question?
A-1735 : Sin, cos and cotan α mam, also the fastest path that can be chosen
R6-36 : So what are the steps?
A-1736 : Find the hypotenuse first
R6-37 : What do you use to find the hypotenuse first?
A-1737 : Pythagoras mam
R6-38 : How much did you get?
A-1738 : 100
R6-39 : Where did you get the 100 from?
A-1739 : $\sqrt{100000}$ mam
R6-40 : Try to explain
A-1740 : From 60^2 the result is 3600 plus 80^2 the result is 6400 the result is taken as a root and we get 100 mam
A-1741 : The front is 80 and the slope is 100
R6-42 : how much do you simplify it to?
A-1742 : $\frac{8}{10}$ become $\frac{4}{5}$

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The interview revealed that subject A-17 was able to select solution steps, write notations, perform substitutions, and solve problems accurately. Subject A-17 was able to explain the steps in detail and systematically. He also demonstrated a habit of double-checking the answers obtained. This indicates that well-developed habits of mind contribute positively to the quality of students' mathematical communication.

Based on the research findings, students' mathematical communication skills in trigonometry differed according to their habits of mind with details:

1. Subjects with limited habits of mind have limitations in visual communication caused by impulsiveness and lack of visualization analysis, images do not function as a communication tool but only as a formal requirement for problem solving, subjects make mistakes in using impulsive notations and formulas. Subjects use mathematical expressions only through memorization rather than reasoning, they are only able to copy memories without validating them with problem data. Subjects view solving as a mechanical activity and not a process of communicating ideas, have a priority to complete their writing rather than explaining logical procedures. Subjects fail to write and use data consistently which reflects a thought process without unfocused purpose;
2. Subjects with developed habits of mind able to understand geometric concepts well, as seen from their ability to make drawings quite accurately and are aware of their weaknesses when reflected on, able to model problems and make good mathematical calculations, and have a clear solution structure, but there are shortcomings in the written results due to inaccuracy, not lack of understanding;
3. Subjects with expert habits of mind able to draw, perform calculations accurately, have a deep understanding of calculation operations and attention to detail used for mathematical calculations. They have a problem-solving frame work, this shows a conceptual understanding that supports the use of notation rather than just memorization. This indicates that habits of mind are not merely a function of students' thinking processes but also impact the quality of their mathematical communication.

Student with limited habits of mind tended to have weak and inconsistent communication skills, drew triangles simply as drawings, not as tools for problem solving. This finding aligns with research by Ilmi et al. (2022), which found that students with limited habits of mind tended to use visual representations procedurally without conceptual meaning. Furthermore, Fitriani & Soebagyo (2022) also found in her research that students with weak visual communication skills were associated with an skill to connect contextual problems with appropriate mathematical models.

The weakest indicator demonstrated by student with limited habits of mind was writing. Problem solutions were written in symbols and short calculations without explanations or justification for the steps. This finding aligns with research by Hendriana and Sumarmo (2021), which stated that reflective thinking habits and the skill to consciously explain mathematical reason influence students' written communication skills. With these habits, mathematical writing appears conceptual and communicative.

Student with developing habits of mind demonstrated better communication skills than students with limited habits of mind. In the drawing indicator, students able to draw triangles that corresponded to the problem context but tended to be incomplete in their labeling. This finding aligns with research by Thasiro & Ruzai (2023), which stated that middle-level students are often able to visualize problems, although they are not yet fully and accurately presenting mathematical information.

The mathematical expression indicators of students with developer habits of mind appeared better, as evidenced by their relatively accurate use of trigonometric ratios and symbolic notation. However, calculation errors and inconsistencies in data usage were still found. This indicates that students' habits of mind have developed, but not yet fully maximized, in terms of accuracy and self-evaluation. Aligns with research by Purnomo et al. (2024) which stated that reflective practice is necessary for students with moderate habits of mind to improve the accuracy of symbolic communication. Developer student began to write down known and questionable information, but their solution narratives were not yet coherent and in depth. Student tended to jump directly to calculations without explaining the rationale for their chosen strategies, because they are accustomed to pursuing the correct result, rather than the thought process and explanation of the strategy. This finding indicates that mathematical communication skills have developed, but written argumentation still needs to be strengthened. This finding aligns with research by Astuti & Yuliani (2024), which stated that improving mathematical communication skills depend not only on conceptual understanding but also on the habits of explaining thought processes in writing.

Conversely, student with expert habits of mind demonstrated good mathematical communication skills across all three indicators. The drawings made by students in this category appear complete and accurate in depicting the situation and are used as a basis for selecting formulas and mathematical calculations. Students use mathematical expressions appropriately and write solutions accurately, logically, and systematically. In addition, student in this category demonstrate a habit of double checking their answers to obtain the correct answer. Aligns with research by Juandi et al. (2025) which shows that student with more advanced thinking habits tend to have systematic and reflective mathematical communication, especially in topics that require visualization such as trigonometry.

The impact of this research on trigonometry learning is that it shifts the focus of instruction from procedural calculations to reflective reasoning. Trigonometry requires an understanding of the relationships between angles and ratios. By understanding students' habits of mind, teachers can design trigonometry instruction that enhances conceptual understanding, mathematical communication, and metacognitive awareness, ultimately improving the quality of students' problem-solving, not just the accuracy of their calculations.

CONCLUSION

The result of this study demonstrates that students' mathematical communication skills in trigonometry are aligned and relate to their level of habits of mind. The diversity of habits of mind categories demonstrates differences in skill across three indicators of mathematical communication skill are drawing, mathematical expression, and writing.

Students with limited habits of mind categories appear weak in all indicators of mathematical communication skills, as evidenced by crude drawings, errors in mathematical expressions, and minimal written explanations. Students with developing habits of mind categories have better communication skills, although they still exhibit inconsistencies and incompleteness in their written representations and narratives. Meanwhile, students with expert habits of mind categories demonstrate excellent mastery of all three habits of mind indicators, able to communicate mathematical ideas and concepts more completely, accurately, and systematically.

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