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STUDENTS' GENERALIZATION ON FUNCTIONAL THINKING: A STUDY OF THREE-DIMENSIONAL COGNITIVE STYLE

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ABSTRACT

This study is a descriptive qualitative study that aims to explore junior high school students' generalization on functional thinking viewed from students' differences on cognitive style, verbalizer, spatial visualizer and object visualizer. The subjects of this study were three male students, ages 14-15 years old, on East Java Indonesia. The Object-Spatial Imagery and Verbal Questionnaire (OSIVQ) were used to get the data of students' cognitive style. This study used in-depth interviews using a pattern task and an interview guide. Time triangulation used for internal validity. The process of analyzing the data consists of data condensation, presentation of data and drawing conclusions. This study found that students with cognitive styles differences have different strategies in making generalization on functional thinking. The strategies used are counting from a drawing, whole-object strategies, and functional strategies. This study further examined how these three students came up with the strategies and what the mathematics expression used of nth configuration. The results of this research show the importance of identifying students' cognitive styles before studying mathematics so that teachers can provide appropriate treatment and scaffolding so that students can achieve their learning goals optimally.
Keywords : Generalization, Functional Thinking, Three-Dimensional Cognitive Style, Verbalizer, Spatial Visualizer, Object Visualizer.

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PRELIMINARY

Algebraic thinking can be broken down into four main areas: generalized arithmetic, functional thinking, modeling languages, and algebraic proof (Kaput & Blanton, 2005; Smith, 2008). A path analysis shows that there is a hierarchy in algebraic thinking, with functional thinking as the main branch. This means students are considered to be able to complete the task if they can complete functional thinking tasks, present them graphically, predict unknown terms based on data and identify patterns (Pitta-Pantazi, Chimoni, & Christou, 2020). Building upon the notion that functional thinking is a cornerstone of mathematical development (Syawahid, Purwanto, Sukoriyanto, & Sulandra, 2020), Khikmiyah et al. (2024) stated that it holds the greatest significance in fostering algebraic thinking compared to other cognitive skills. This emphasis is further proposed by Wilkie

(2016), who highlights the fundamental role of functional thinking as a prerequisite for success in advanced algebraic and calculus studies at the secondary and tertiary levels.

Functional thinking entails a mode of representational thought that emphasizes the relationship between multiple changing quantities, specifically the cognitive processes that lead from specific instances of relationships to broader generalizations encompassing various occurrences (Smith, 2008). A key aspect of functional thinking involves generalizing functions themselves (Kaput, Carraher, & Blanton, 2008). Functional thinking as representational thinking which is focused on the relationship between two or more different quantities (Smith, 2008). In order to provoke students' functional thinking, we could give these following tasks: 1) symbolizing numbers with variables or operating variables, 2) presenting data graphically, 3) finding functional relationships, 4) predicting unknown numbers based on data, 5) identify and explain arithmetic sequences, 6) identify and explain geometric sequences. (Kaput & Blanton, 2005)

Functional thinking hinges on the ability to generalize patterns. There are three types of pattern generalization, 1) immediate generalization tasks, contain calculating the value of a step based on the preceding one, 2) Near generalization tasks involve identifying the value of a step that closely aligns with provided values of preceding steps; and (3) Far generalization tasks require determining the value of a step that significantly deviates from the provided figures of preceding steps (El Mouhayar & Jurdak, 2016). Research suggests that generalizing patterns mathematically is often a challenging concept for students (Firdaus, Juniati, & Wijayanti, 2019). While the majority students succeed in immediate and near generalization tasks, they often faced difficulties in justifying rules for far generalization.

Students' ability to visualize the problems significantly impacts their chosen strategies for solving them. Differences in students' cognitive styles cause differences in procedural errors in solving mathematical problems (Muhassanah, N. 2023). Furthermore, research suggests that an individual's cognitive style influences how they approach visualization tasks (Lannin, Barker, & Townsend, 2006). This paper investigates how students with different cognitive styles; verbalizer, visualizer spatial, and visualizer object, develop strategies for pattern generalization. By examining the generalization process and the strategies employed, we gain insight into the thought processes that lead students to accurate solutions, offering a more systematic and meaningful view of their reasoning. This research provides valuable insights for both teachers and researchers. It allows educators to understand students' perspectives on generalization strategies, informing the development of appropriate learning methods. In addition, teachers can identify areas where students might struggle, such as object configuration or number pattern generalization, and provide targeted scaffolding to support their learning.

The concept of cognitive style describes how an individual consistently organize and process information (Ausburn & Ausburn, 1978; Messick, 1984). Paivio (1971) suggested a foundational theory, proposed that our cognitive system is divided into two components: verbal and visual. The verbal system deals with linguistic information, while the visual system processes and stores information in the form of images. These systems can work independently but can also integrate information, encoding it simultaneously using both verbal and spatial codes. Furthermore, (Paivio, 1971) was the first to propose an individual differences questionnaire that aims to determine the extent to which a person uses his habits to think using images and words (verbal). Kozhevnikov, M. et al. (Kozhevnikov, Kosslyn, & Shephard, 2005) found that visualizers can be divided into two groups: spatial visualizers and object visualizers. Specifically, verbalizers perform an average level on imagery tasks. A group of visualizers scored poorly on the spatial imagery task but excellent on the contrast of the object imagery task, and another group in reverse.

Blazenkova, O & Kozhevnikov, M (2009) suggest a three-dimensional cognitive style that distinguishes verbal, object imagery, and spatial imagery based on cognitive science theories. Data on neuropsychology suggest that there are two different image subsystems, an object imagery and a spatial imagery system, that encode and process visual information differently. The first imagery processes the visual appearance of objects, color, texture information, and scenes of their shape. Meanwhile, spatial imagery processes object location, spatial relationship, movement, transformations, and other spatial attributes of processing.

Previous studies empirically found that students use different strategies and reasoning ways to generalize patterns (El Mouhayar & Jurdak, 2016; Rivera, 2010; Rivera & Becker, 2008). Other study demonstrated that male subjects used a counting strategy, different-rate adjustment, and whole object-no adjustments (Firdaus et al., 2019). The female subject also used a counting strategy and was more explicit in generalization. El Mouhayar & Jurdak (2016) found out that students frequently use the recursive and functional strategy to generalize patterns. In this study we used a list of strategies with the definitions were used as the basis for analysis, 1) counting form a drawing means counting the components of a specific geometric figure within a pattern, 2) recursive means pointing the common difference of consecutive terms and continuously adding the constant from on configuration to the next to extend the pattern 3) chunking; taking the common difference between two terms in a sequence, multiplying it by the number of steps, and then adding this product to the initial term in the sequence., 4) functional; Establishing connections between components of the pattern and the numerical order of the steps within the figure, 5) whole-object; Determining the value of a term by utilizing multiples of a preceding term (El Mouhayar & Jurdak, 2014)

Some studies reported that there are many factors influence students' strategies in pattern generalization, including input values, mathematical structures of the task, a visual image of the situation, students' experience in prior strategies, and their social interaction. (Lannin et al., 2006; Yeap & Kaur, 2008) indicated that students' differences in cognitive style will affect students' strategies because the pattern is given in a visual image. Therefore, the aim of this study is to explore the variation in students' strategy in generalization of functional thinking based on their differences in cognitive style by addressing the following questions:

1. What strategy is used by student with verbalizer cognitive style in generalization on functional thinking?
2. What strategy is used by student with spatial visualizer cognitive style in generalization on functional thinking?
3. What strategy is used by student with object visualizer cognitive style in generalization on functional thinking?

METHODS

This study explores the strategies students use to generalize functional thinking processes. Data was collected form the students answers to functional thinking problems and through in-depth interviews. Employing descriptive qualitative approach with a case study method (Creswell, 2012), we facilitated an in-depth exploration of a specific subject area. This approach involved extended contact with the participants in a natural setting (Miles, n.d., 2014). Thi study was conducted during the 2021/2022 academic year. We asked a group of 30 eighth graders from SMP Negeri 1 Gresik, a junior high school in Gresik regency, East Java, Indonesia. These students filled out a cognitive style questionnaire and participated in an an algebraic thinking test.

Students were categorized into eight categories based on their answers to a cognitive style questionnaire assessing three dimensions (Blazhenkova & Kozhevnikov, 2009): verbalizer cognitive style (high/low), object visualizer cognitive style (high/low), and visualizer cognitive style (high/low). The categories are as follow; 1) TvTsTo: High verbal, High Spatial and High Object, 2)TvTsRo: High verbal, High Spatial and Low Object, 3) TvRsTo: High verbal, Low Spatial and High Object, 4) TvRsRo: High verbal, Low Spatial and Low Object, 5) RvTsTo: Low verbal, High Spatial and High Object, 6) RvTsRo: Low verbal, High Spatial and Low Object, 7) RvRsTo: Low verbal, Low Spatial and High Object, 8) RvRsRo: Low verbal, Low Spatial and Low Object. High or low scores in each category are based on comparing the average value of all students in each type of cognitive style with the acquisition of students' scores on each type of cognitive style. If the acquisition value of a student is greater than the average value, it is categorized as high. In contrast, if the acquisition value of a student is less than the average, it is categorized as low. Table 1 shows the distribution of students across these categories.

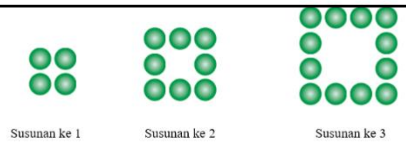
Table 1. Number of Students in each Cognitive Style (CS) Category

CS Category	Number of Students	Percentage (%)
TvTsTo	10	33,33
TvTsRo	4	13,33
TvRsTo	5	16,67
TvRsRo	1	3,33
RvTsTo	3	10
RvTsRo	2	6,67
RvRsTo	3	10
RvRsRo	2	6,67
Total	30	100

Students' performance in algebraic thinking was also assessed. Scores were categorized as high (≥75), medium (60-74), and low (<60). However, for further analysis, the focus was narrowed down to students with specific cognitive style profiles (TvRsRo, RvTsRo, and RvRsTo) who also demonstrated a consistent level of performance in algebraic thinking (either high or low) and shared the same gender. From the results of the cognitive style questionnaire and the algebraic thinking assessment, three students were chosen for in-depth interviews: FHS (assumed to be the verbalizer student), RMZ (assumed to be the spatial visualizer student), and RSS (assumed to be the Object Visualizer student). Other consideration is that they are male and their mathematics teacher recommended them due to their strong communication skills. We anticipated that during the task, these students would collaborate and share their thought processes openly, facilitating a rich exploration for the researcher.

This study employed a self-report instrument called the Object-Spatial Imagery and Verbal Questionnaire (OSIVQ) developed by Blazhenkova & Kozhevnikov (2009). The researcher got the license from the authors to adapt OSIVQ for use in this study. The original questionnaire consists of 45 items with five response options: strongly agree, agree, hesitate, disagree, and strongly disagree. Four of them were negatively structured, meaning a lower score reflected a high ability. The OSIVQ was translated into Indonesian by the researcher, ensuring the terminology used was appropriate for the target age group (eight grader) while maintaining the original meaning of the questions. In addition, the results of this adaptation also got expert judgments to further ensure the content validity. Finally, the questionnaire was assessed through a pilot test with five students at the same level from different classes to test its readability.

Beyond the researcher acting as the primary data collection tool, the study utilized two additional instruments: pattern tasks and semi-structured interviews. Pattern tasks were used to investigate the strategies used by the subject in functional thinking processes that involve generalization. The task is presented below:



Pay attention to the configuration of those colored balls:
1. Redraw the ball configuration up to 5th and determine the number of balls needed to configure? Explain your answer
2. How many of those colored balls are needed for the fiftieth configuration? Explain your answer
3. How many of those colored balls are needed for the nth configuration? Explain your answer

Figure 1. Pattern Task Given to the Students

We used an interview guide as a guideline so that the researcher got the main objective of the research without ignoring deep attentiveness, empathetic understanding, or bracketing preconceptions about the topics under discussion. To enhance the overall validity of the interview data, two experts reviewed and validated both the interview guide and the pattern tasks used in the study. Time triangulation was also applied for internal validity.

The process of analyzing the data involve of: 1) Data condensation, the initial stage involves selecting, focusing, simplifying, abstracting, and/or transforming data from various sources such as written field notes, interview transcripts, documents, and other empirical sources. The process may involve writing a summary, coding, developing themes, creating categories, and writing analytical memos to aid in further analysis. 2) Data Presentation, the data is organized and presented in a way that facilitates drawing meaningful conclusions. At this stage, the reduced data is presented by compiling complex information into a more straightforward and systematic way to understand its meaning. In this study, students' strategy in generalization is described narratively. 3) Drawing conclusions and verification, the final stage involves interpreting the presented data and drawing conclusions. (Miles, n.d., 2014). Conclusion drawing aims to make sense of the data and its implications. In comparison, verification activities include testing the truth, robustness, and compatibility of meanings by looking back at the existing data or by asking for the opinion of a competent expert.

RESULT AND DISCUSSION

The purpose of this study was to describe the strategies used by students in making generalization on functional thinking based on their differences in cognitive style, which are verbalizer, spatial visualizer and object visualizer. Functional thinking refers to the cognitive processes students use to identify and represent the relationships between multiple changing quantities. These relationships can be expressed mathematically through equations, tables, graphs, diagrams, or written descriptions. Generalization actions consist of recursive generalization and explicit generalization (Wilkie & Clarke, 2016).The following is a description of the students' strategies in generalization on functional thinking process in solving a pattern task.

Subject 1: Students with Verbalizer Cognitive Style (FHS)

Immediate and Near Generalization

At understanding the problem stage, FHS identify changes in two successive objects in the configuration given, namely 1st to 3rd configuration. FHS said "at the 1st arrangement there are 4 balls, 8 balls in the 2nd configuration and 12 balls in the 3rd configuration. So, for the next configuration always add by 4 balls". FHS wrote the information given using his own language "It is known that 4 ballas are added to the next arrangement" as seen at the figure 2.

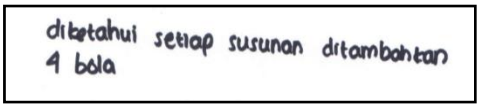


Figure 2. FHS wrote the information given and generalization on it

At the stage of devising a plan, FHS also stated a sentence with the same meaning, only he expanded it to the 4th and 5th configuration. FHS then answer that there are 16 balls at 4th configuration and 20 balls at 5th configuration as shown at figure 3.

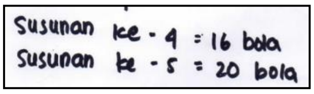


Figure 3. FHS answer for 4th and 5th configuration

A transcript from the dialogue between the researcher (R) with FHS mentions in immediate and near generalization presented at table

Table 2. The dialogue between the researcher and FHS

Code		Transcript
FHS	:	at the 1st arrangement there are 4 balls, 8 balls in the 2nd configuration and 12 balls in the 3rd configuration. So, for the next configuration always add by 4 balls
R	:	… How do you know that the number on the 4th arrangement should be 16?
FHS	:	We know that at the 1st arrangement there are 4 balls, 8 balls in the 2nd configuration and 12 balls in the 3rd configuration. Hence, it must be 16 balls in the 4th configuration
R	:	How do you count that?
FHS	:	I only add the next configuration by 4 so that there're 20 balls in the 5th configuration

Describe the relationship between two groups of quantities was appeared at the stage carrying out the plan. To determine the number of balls in the 50th configuration, FHS used the value of input "4" as the multiplier factor. FHS explained that $50 \times 4 = 200$, multiplication of the multiple factors with the order of the configuration. Asking for the number of balls in other order, he wrote for a hundred and a thousand. Figure 4 shows FHS answer for the second problem.

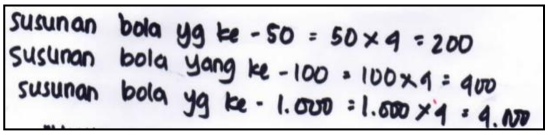


Figure 4. FHS answer for 50th configuration.

Far Generalization

To determine the general rule for nth configuration FHS wrote that formula used should be $n \times 4$ since it works for those problems before. Although the mathematics expression is not appropriate, FHS explain that $n \times 4$ will be correct for any order of the configuration. FHS Mathematics expression shows at figure 5.

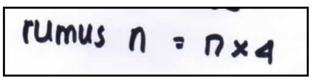


Figure 5. FHS formula to find the number of colored balls needed for the nth configuration

This study identified an interesting strategy employed by FHS, a student with a visualizer cognitive style. FHS successfully solved all the given problems by leveraging a pattern recognition approach. As illustrated in Figure 2, FHS observed that the number of balls increased by 4 in each consecutive arrangement. Notably, FHS recognized the constant value of 4 and used it as a multiplier factor. Furthermore, FHS exhibited consistent application of this strategy across various tasks, demonstrating both immediate and near to far generalization. This whole-object strategy involved multiplying the configuration number by the value of the first term. This finding highlights the potential of visualizer learners to identify and exploit patterns effectively to solve problems.

Subject 2: Students with Spatial Visualizer Cognitive Style (RMZ)

Immediate and Near Generalization

At understanding the problem stage, RMZ counted the number of the object in each configuration one by one. He drew the object on the configuration with the same color but different with other configurations. RMZ made mistake when drawing the 2nd configuration but later he made the correct one. RMZ wrote the number of the balls beside his drawing. Figure 6 shows RMZ drawing for the given problem.

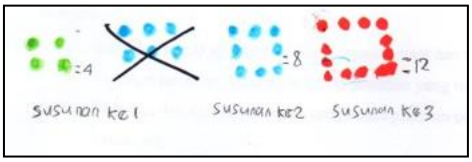


Figure 6. RMZ drawing the balls configuration given in the problem

At devising a plan stage, RMZ said to determine the number of balls in the 5th and 50th configurations, RMZ would make pictures for the configurations asked in the problem. When drawing the 4th configuration, RMZ stated that there were 16 balls and he explained that to draw the 4th configuration he added 1 ball on each side. The relationship between successive objects in the configuration is expressed by RMZ in the 1st to 4th arrangement. An excerpt from the dialogue with RMZ which mentions the recursive generalization aspect of the indicator generalizing the relationship between quantities in TPMM1a is presented in table 3

Table 3. The dialogue between the researcher and RMZ on immediate generalization

Code		Transcript
R	:	How many balls are at the 4th arrangement?

RMZ	:	16 balls
R	:	How do you determine the number of those balls?
RMZ	:	I add a ball on each place
R	:	What do you mean each place?
RMZ	:	These one, on the side that I make a circle on it (Every side of the configuration)

In order to find the number on the 5th configuration, RMZ drew with red color for the 4th and purple for 5th. Consistent with previous, he also wrote the number of the balls beside the drawing. RMZ made a sign, looks like a curve, to show how he counting the number of the object on each configuration.

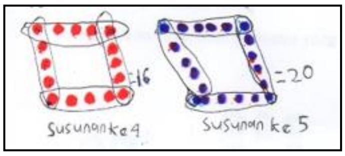


Figure 7. RMZ drawing for 4th and 5th configuration

RMZ explain that the number of the object in 5th configuration is 20 since for the horizontal part, he had five balls each. For the vertical part he also had five balls each. Since the balls at the corner counted twice so that he subtracts the number with 4.

In order to determine the number of balls at 50th configuration, RMZ made a model of the situation. It looks like a square with the number 48 at the center of the side. He used the same strategy for solving the second problem. However, it seems that he confused because he wrote 48 and didn't add a number of the ball in the corner. Figure 8 shows RMZ answer for the second.

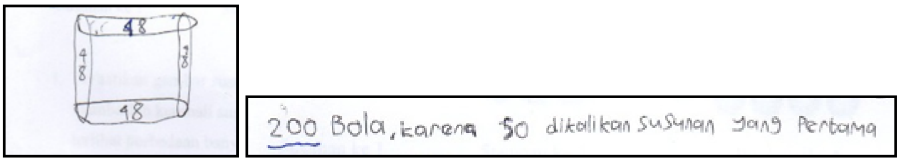


Figure 8. RMZ answer for 50th configuration

An excerpt from the dialogue with RMZ which mentions the explicit generalization aspect of the indicator generalizing the relationship between quantities in TPMM1a is presented in table 4.

Table 4. The dialogue between the researcher and RMZ on near generalization

Code		Transcript
RMZ	:	For every configuration, we multiply the order of the configuration and the number of the object on the first configuration
R	:	Explain more

RMZ	:	The number of the object on the first configuration is 4, for the 2nd configuration we multiply 2 with 4, for the 3rd configuration we multiply 3 with 4, for the 4th configuration we multiply 4 with 4 so that for the 5th
R	:	So, what is the relationship between the order of the configuration and the number of the object on the configuration?
RMZ	:	We can determine the number of the object on the configuration by multiply the order by 4.
R	:	Can you determine the number of the object on the 100th order?
RMZ	:	Yes, of course. We multiply 100 by 4 and we get 400

Far Generalization

RMZ, exhibiting a spatial visualizer cognitive style, encountered challenges in determining the number of objects for the nth configuration (far generalization). This suggests limitations in his ability to translate the visual pattern into a generalized rule applicable to unseen configurations. The concept of "n" likely represented an abstract variable, which may not have readily corresponded to his visual understanding of the problem.

RMZ couldn't come up with the answer for the nth configuration becauses meaning of "n" is didn't make a sense for him.

RMZ, a student exhibiting a spatial visualizer cognitive style, demonstrated success in solving problems with immediate and near generalization, but faced challenges with far generalization. His approach revealed an interesting shift in strategy. At the immediate generalization level, RMZ employed a counting strategy. He segmented the configuration into four distinct parts, meticulously drawing each section with dots and using curves to visually separate them. This meticulous approach suggests a focus on the concrete details of the visual representation. For near generalization, RMZ transitioned to a more abstract counting strategy. While he still utilized curves to define sections, he replaced the dots with numerals, indicating a shift towards a symbolic representation. Notably, RMZ successfully applied the "whole-object strategy" at this stage, multiplying the order by the value of the first term to determine the total number of objects in the 50th configuration. This finding suggests his ability to identify and apply a generalizable rule, albeit within a familiar range. However, RMZ's strategy did not extend to far generalization tasks. This highlights the potential limitations of a purely visualizer approach when faced with significantly different configurations.

Subject 3: Students with Object Visualizer Cognitive Style (RSS)

Immediate and Near Generalization

At understanding the problem stage, RSS explain the relationship between two successive objects in the configuration given, the 1st to 3rd configuration. At devising a plan stage, RSS said that to ensure that he gave the right answer then RSS do these steps, 1) counted the number of balls on each side, 2) ensured that the number of balls on each side was the same, 3) at the next configuration 1 ball was added on each side. RSS states "At the 1st configuration there are 2 balls (on each side), 3 balls are at 2nd configuration and 4 balls at 3rd configuration and so on.

A transcript from the dialogue between the researcher (R) with RSS mentions recursive generalization presented in table 5.

Table 5. The dialogue between the researcher and RSS

Code		Transcript
R	:	… How do you ensure that your drawing is correct?
RSS	:	I'll count the number of balls on each side
R	:	How do you determine the number of balls on each side?
RSS	:	The number must be the same
R	:	How many balls on each side for the 4th configuration and so forth?
RSS	:	We should add one ball on each
R	:	What do you mean add one ball?
RSS	:	There're 2 balls (on each side) at the 1st configuration, 3 balls (on each side) at the 2nd configuration, 4 balls (on each side) at the 3rd configuration and so forth

Reveals the relationship between two groups of quantities appears at the stage of carrying out the plan. RSS redrew the 1st to 2nd configuration of objects with small dots with numbers on them. This number shows the number of objects that compose a configuration. At the third configuration, RSS crossed out the picture because he thought it was wrong. RSS stated that "Each side should have 4 balls. Here I made 5 dots". RSS realizes that on each side, the number of constituent objects must be the same. RSS drew the 3rd and 4th configuration again and this time the drawing was correct. When asked not to count the points that had been created one by one, RSS then created curves on the image of known object configurations. On the 1st configuration, RSS divides the configuration into 2 parts, in the 2nd and 3rd configurations RSS divides the configuration into 3 parts. An image of RSS to show how he calculated can be seen in the following image.

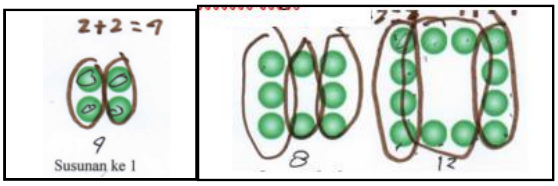


Figure 9. RSS answer for 5th configuration

When determining the 5th configuration, RSS drew points that resembled the sides of a quadrilateral then made curves on the left and right. After that, RSS wrote "6+6+8=20" at the top of the configuration and wrote "5th configurationt" at the bottom of the configuration. RSS subject image for the 5th configuration appears in the following image.

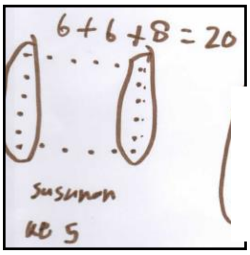


Figure 10. RSS drawing for 5th configuration

When determining the 50th configuration, RSS no longer draws points that resemble the sides of a quadrilateral. RSS made 4 (four) curves on the left, right, top and bottom then wrote the numbers 51, 49, 51 and 49 respectively on the inside of the curve and wrote "50th order" at the bottom of the configuration. RSS image for the 50th configuration appears in the following image.

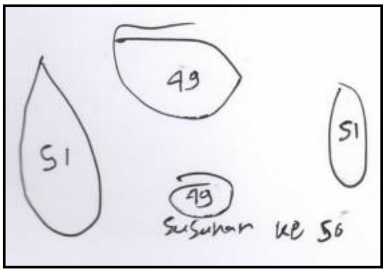


Figure 11. RSS drawing for 50th configuration

RSS was then asked how to determine the numbers he wrote, Subject Vo then stated "For the 1st arrangement where there are 2 vertical points, for the 2nd arrangement there are 3 points (on each side), in the 3rd arrangement there are 4 dots and in the 4th arrangement there are 5 dots, in the 5th arrangement there are 6 dots so in the 50th arrangement there are 50 upright dots. RSS then adds "If What is sought is the 10th arrangement, so there are 11 upright points. Ten (from the order of arrangement) plus one."

Far Generalization

To find the general rule of nth configuration, RSS made a list of the number of the balls in each configuration but focus on the number only. At this stage, it seems that RSS realize that there is a relation between the order of the configuration and the number mentioned in this operation. Figure 12 shows that RSS struggle with the process of generalization.

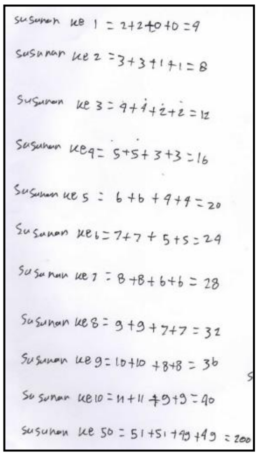


Figure 12. RSS calculation for generalization

The list of the operation brings him to the generalization. He wrote the formula of determining the number of the subject at figure13.

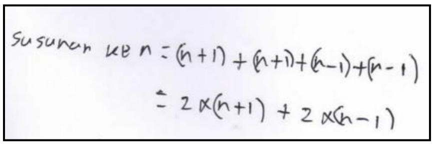


Figure 13. RSS answer for third problem

RSS, a student categorized with an object visualizer cognitive style, achieved success in solving all the problems presented. His approach exhibited a fascinating shift from reliance on visual segmentation to an emphasis on numerical manipulation as the tasks progressed from immediate to far generalization. During immediate generalization, RSS adopted a whole-object strategy. He segmented the initial configuration into two parts, followed by three parts for subsequent configurations. This approach involved meticulously counting the elements within each section and summing them to arrive at the total number of objects. This segmentation highlights a focus on the specific visual characteristics of the configurations.

As the tasks transitioned to near generalization, RSS's strategy evolved. He maintained the whole-object strategy, but the number of segments increased to four. This suggests a potential attempt to identify a more generalized pattern within the visual representation. For far generalization tasks, a remarkable shift occurred. RSS demonstrably abandoned the visual reliance and transitioned to functional strategy. He meticulously analyzed the problem, identifying constant and variable elements. Notably, he replaced the variable element with a symbolic representation, signifying a move towards algebraic reasoning. This finding underscores the potential of object visualizers to develop abstract problem-solving skills when presented with tasks demanding a shift beyond perceptual features.

From these three students we know that all student's success in immediate and near generalization but one of the students, RMZ failed in far generalization. The result of the study in line with (El Mouhayar & Jurdak, 2016)majority of the students' success in immediate and near generalization but faced difficulties in finding the rules for far generalization in which they make a rule or using variable and equation to represent the generalization.

The findings from this study reveal interesting patterns in students' approaches to generalization tasks, with some variations based on their cognitive styles. All three students demonstrated success in solving problems requiring immediate and near generalization. This aligns with previous research by El Mouhayar & Jurdak (2016), which suggests that students often perform well when dealing with familiar or slightly modified versions of presented patterns. However, the results diverge regarding far generalization. While FHS and RSS successfully transitioned to abstract reasoning and employed symbolic representations (variables and equations) to find general rules, RMZ encountered difficulties. This suggests that a purely visualizer cognitive style, as exhibited by RMZ, might pose limitations when dealing with significantly different configurations in far generalization tasks. RMZ's struggle aligns with the notion that translating visual patterns into abstract rules requiring variable manipulation can be challenging for this specific cognitive style.

The data analysis confirmed distinct approaches to problem-solving between verbalizer and visualizer subjects. As anticipated, the verbalizer subject did not rely on visualization for tasks. This aligns with the recent work of (Kozhevnikov, Hegarty, & Mayer, 2002) who found that verbalizers exhibited a preference for logic-based reasoning, focusing on manipulating symbolic representations. Conversely, the visualizer subjects consistently employed visualization to aid their reasoning processes. This supports the meta-analysis by (Carden & Cline, 2015) which highlighted the consistent use of mental imagery for problem-solving by visualizers. However, our study also revealed a distinction between the visualization strategies used by the spatial and object visualizers.

The spatial visualizer, like RMZ, appeared to focus on the spatial arrangement of elements within the configurations. This aligns with the concept of "viewer-centered representations" described by (Sternberg & Sternberg, 2006) emphasizes the visual information about an object as determined by its position and orientation relative to the observer. On the other hand, the object visualizer, like RSS, seemed to concentrate on identifying and manipulating individual objects within the configurations. This suggests a potential reliance on "object-based representations" (Vecera, 1998), where the focus lies on the inherent properties of the objects themselves. Reasoning capabilities are demonstrably enhanced by the incorporation of object-based representations. (Xu, Y., Li, W., et al, 2023)

This study highlights the importance of considering individual cognitive styles when investigating problem-solving strategies. Future research could explore the effectiveness of training interventions designed to promote the use of complementary strategies (visualization and verbalization) for different learning styles. Additionally, investigating the neural correlates of these cognitive styles during problem-solving tasks using modern neuroimaging techniques like functional magnetic resonance imaging (fMRI) could provide new insights into the underlying mechanisms.

CONCLUSION

The findings revealed that students with different cognitive style (verbalizer, spatial visualizer or object visualizer) have different strategies on functional thinking. FHS, a student with visualizer cognitive style successfully solved all the problems given. He consistently employed whole-object strategy throughout immediate, near, and far generalization tasks. RMZ, a student with spatial visualizer cognitive style successfully solved tasks requiring immediate and near generalization, but not in far generalization. He used various strategies depending on the generalization level: counting number for immediate generalization, a recursive strategy for near generalization, and whole-object strategy for far generalization. Meanwhile, RSS, a student with object visualizer cognitive style successfully solved all the problem given. RSS applied whole-object strategy in immediate and near generalization. However, for far generalization tasks, he shifted to functional strategy. We acknowledge that this study has several limitations: 1) We utilized the case study method, so the findings cannot be extrapolated to other cases. 2) The study's participants were exclusively male students from specific locations. 3) Our analysis focused on a single problem and linear pattern.

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