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CHARACTERIZATION OF NORMAL FUZZY SUBGROUP

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ABSTRACT

In this article, we discuss the characterization of a normal fuzzy subgroup of classical group G. The discussion of this characterization is carried out using Abelian properties, fuzzy conjugate subgroups, fuzzy normalizers, α-level sets, and fuzzy cosets. The result shows that a sufficient and necessary condition for a normal fuzzy subgroup is the fulfilment of the Abelian condition in the fuzzy subgroup. Then, the equality between of the membership value of all element of G and its conjugate elements is also a sufficient and necessary condition for normal fuzzy subgroup of G. Moreover, sufficient and necessary conditions of the normal fuzzy subgroup are the normalizer of this fuzzy subgroup is equal to G. Henceforth, the sufficient and necessary conditions of a normal fuzzy subgroup of G is its α-level set is a normal subgroup of G. Meanwhile, the similarity of the fuzzy right coset and fuzzy left coset of the fuzzy subgroup is also a sufficient and necessary condition for the normal fuzzy subgroup. Furthermore, the normal properties of subgroups on classical groups are a special case of the normal properties in fuzzy subgroups.

Keywords: Normal Subgroup, Fuzzy Subgroup, Characterization.

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PRELIMINARY

The concept of sets in mathematics has developed since the introduction of the concept of fuzzy sets. In a crisp set A, the membership function can be expressed by a characteristic function that corresponds each object into 1 for all element of A and 0 for others (Alcântara et al., 2021). In a fuzzy set, the membership function corresponds each object into closed interval [0,1] (Chaira, 2019), (Hsien-Chung, 2023). Thus, a crisp set can be viewed as a special case of a fuzzy set since the membership value of the crisp set is a subset of the membership value of the fuzzy set

A group is a mathematical system $(G,*)$ that fulfils associative properties, the existence of identity element, and the existence of inverse for each element of G. A subgroup of G is a non-empty subset of G, which is also a group. The normal subgroup is a subgroup whose right coset is equal to the left coset. The discussion of group theory can be referred to (Fraeligh, 2014), (Judson, 2022), (Ford, 2020), (Hotta, 2018), (Lal, 2017).

Then, the development of a group in the ideal of matrices over a commutative ring is also discussed in (Mugi et al., 2023).

A fuzzy subset ρ of a set X which corresponds all elements of X into [0,1]. A fuzzy subgroup ρ of a group G is a fuzzy subset which satisfies $\rho(a * b) \geq \min\{\rho(a), \rho(b)\}$ and $\rho(a^{-1}) \geq \rho(a)$ for all $a, b \in G$. Therefore, the classical group structure can be viewed as a special case of fuzzy subgroups via its characteristic function. The discussion of fuzzy subgroup theory and its properties can be referred to (Riandani et al., 2016), (Deepak, 2017), (Abdy et al., 2019) and (Rozi et al., 2014). The development of fuzzy group into some other algebra structure has been done in, among others (Tarmizi et al., 2019), (Oktaviani & Habiburrohman, 2023), (Pratama, 2022), (Oktaviani & Habiburrohman, 2023), (Abdullah et al., 2015), (Mihsin., 2015) and (Onasanya, 2016). Meanwhile, cosets in fuzzy subgroup sense is also discusses in (Yanwar et al., 2022), (Islam, 2021).

In the classical group, the normal subgroup has an important role in factor group. Characterization of normality of subgroup in classical group can be done through cosets in a group (Onasanya & Ilori, 2014). As in classical group theory, fuzzy normal subgroups have the potential to have an important role in fuzzy factor subgroups. Several studies have been carried out on fuzzy normal subgroup studies, was carried out in construction of normal fuzzy subgroups as well as in classical group theory.

In this research, we discuss characterization of normal fuzzy subgroup. This is different from characterization normal subgroup in classical theory group which characterizes normal subgroup using cosets, we characterizes normal subgroup fuzzy using Abelian properties, fuzzy conjugate, fuzzy normalizer, α-level, and fuzzy cosets.

METHODS

The method used in this research is a literature study which develops the previous research. This research focuses on characterizing the normal subgroup in the fuzzy group. The procedure in this research as follows :

1. Explain the definition of normal fuzzy subgroups through Abel's fuzzy subsets,
2. Show the characterization of fuzzy normal subgroups through conjugate subgroups,
3. Show the characterization of fuzzy normal subgroups through fuzzy normalizers,
4. Show the characterization of fuzzy normal subgroups through α-level, and
5. Show the characterization of fuzzy normal subgroups through fuzzy cosets.

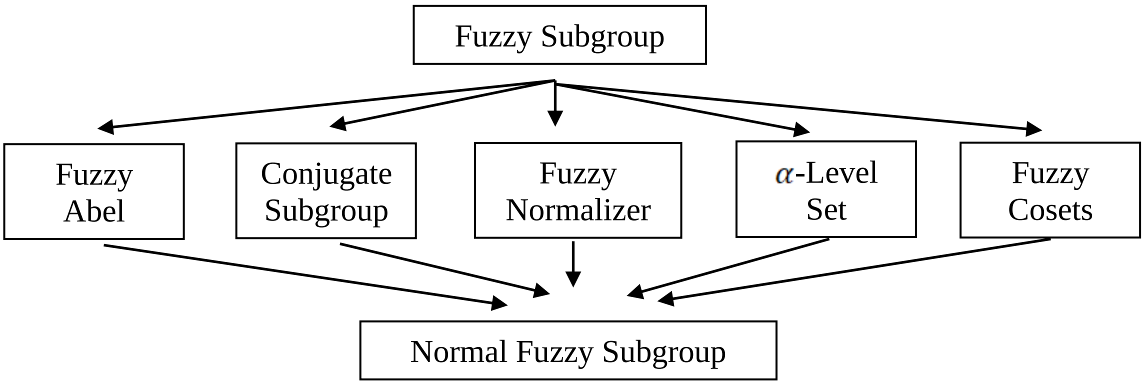

Figure 1. Procedure of Reseach

RESULT AND DISCUSSION

Before forming a fuzzy subgroup, the fuzzy subset of a classical group must first be defined. Suppose G is a classical group, any function $\rho: G \to [0,1]$ that assigns each element of G into $[0,1]$ is called a fuzzy subset of G. The set of all fuzzy subsets of G denoted by $FP(G)$. For each $a \in G$, $\rho(a)$ represents the membership value of a in the fuzzy subset ρ. Let G is a classical group and $\rho \in FP(G)$, then the function $\rho: G \to [0,1]$ is called a fuzzy subgroup of G if $\rho(a * b) \geq \min\{\rho(a), \rho(b)\}$ and $\rho(a^{-1}) \geq \rho(a)$ for all $a, b \in G$. Then, the set of all fuzzy subgroups of G is denoted by $F(G)$.

Characterization Normal Fuzzy Subgroup using Abelian Fuzzy Subsets

This section discusses about characterization normal fuzzy subgroup using Abelian fuzzy subsets. In fuzzy group theory, fuzzy normal subgroups can be formed by Abelian fuzzy subsets. The following theorem discusses about characterization normal fuzzy subgroup using Abelian fuzzy subsets.

Theorem 1. *Let G be a group and $\rho \in FP(G)$. For all $x, y \in G$, the following statements are equivalent :*

1. $\rho(x * y) = \rho(y * x)$

2. $\rho(x * y * x^{-1}) = \rho(y)$

3. $\rho(x * y * x^{-1}) \geq \rho(y)$

4. $\rho(x * y * x^{-1}) \leq \rho(y)$

Proof.

Let arbitrary elements $x, y \in G$.

$(1) \Rightarrow (2)$. Let $\rho(x * y) = \rho(y * x)$ Note that,

$\rho(x * y * x^{-1}) = \rho(x * (y * x^{-1})) = \rho((y * x^{-1}) * x) = \rho(y * x^{-1} * x)$
$= \rho(y * e) = \rho(y)$.

(2) $\Rightarrow$ (3). Let $\rho(x * y * x^{-1}) = \rho(y)$. Since $\rho: G \to [0,1]$ and $\rho(x * y * x^{-1}) = \rho(y)$, then $\rho(x * y * x^{-1}) \geq \rho(y)$.

(3) $\Rightarrow$ (4). Let $\rho(x * y * x^{-1}) \geq \rho(y)$, and it will be proven that $\rho(x * y * x^{-1}) \leq \rho(y)$. Suppose that,

$$\rho(x * y * x^{-1}) \leq \rho(x^{-1} * (x * y * x^{-1}) * (x^{-1})^{-1}) = \rho((x^{-1} * x) * y * (x^{-1} * (x^{-1})^{-1})) = \rho(y).$$

(4) $\Rightarrow$ (1). Let $\rho(x * y * x^{-1}) \leq \rho(y)$. It will be shown that $\rho(x * y) = \rho(y * x)$. Suppose that

$$\rho(x * y) = \rho(x * y * e) = \rho(x * y * x * x^{-1}) = \rho(x * (y * x) * x^{-1}) \leq \rho(y * x)$$

and

$$\rho(y * x) = \rho(y * x * e) = \rho(y * x * y * y^{-1}) = \rho(y * (x * y) * y^{-1}) \leq \rho(x * y).$$

Since $\rho(x * y) \leq \rho(y * x)$ dan $\rho(y * x) \leq \rho(x * y)$, then $\rho(x * y) = \rho(y * x)$, for all $x, y \in G$ ■

Then, Abelian properties can be characterized as in this following theorem.

Theorem 2. *Let G be a group and $\rho \in FP(G)$. For all $x, y \in G$, $\rho(x * y) = \rho(y * x)$ if and only if $\rho \circ v = v \circ \rho$ for all $v \in FP(G)$.*

Proof.

(⇒) Let $\rho(x * y) = \rho(y * x)$, for all $x, y \in G$. It will be shown $\rho \circ v = v \circ \rho$, for all $v \in FP(G)$. According to definition of $\circ$ in $FP(G)$, for all $c \in G$ satisfies

$$(\rho \circ v)(c) = \sup\{\min\{\rho(a), v(b)\} \mid a * b = c, \text{for } a, b \in G\}.$$

Note that,

$$\begin{aligned}(\rho \circ v)(x) &= \sup_{y \in G}\{\min\{\rho(x * y^{-1}), v(y)\}\} \\ &= \sup_{y \in G}\{\min\{v(y), \rho(x * y^{-1})\}\} \\ &= \sup_{y \in G}\{\min\{v(y), \rho(y^{-1} * x)\}\} \\ &= (v \circ \rho)(x).\end{aligned}$$

Therefore, $(\rho \circ v)(x) = (v \circ \rho)(x)$, for all $x \in G$. Consequently, $\rho \circ v = v \circ \rho$.

(⇐) Let $\rho \circ v = v \circ \rho$, and it will be shown $\rho(x * y) = \rho(y * x)$. Note that, every $x, y \in G$,

$$(1_{\{y^{-1}\}} \circ \rho)(x) = \sup\{\min\{1_{\{y^{-1}\}}(y^{-1}), \rho(y * x)\}\} = \sup\{\min\{1, \rho(y * x)\}\} = \sup\{\rho(y * x)\} = \rho(y * x)$$

and

$$(\rho \circ 1_{\{y^{-1}\}})(x) = \sup\{\min\{\rho(x*y), 1_{\{y^{-1}\}}(y^{-1})\}\} = \sup\{\min\{\rho(x*y), 1\}\}$$
$$= \sup\{\rho(x*y)\} = \rho(x*y).$$

Since $(1_{\{y^{-1}\}} \circ \rho)(x) = (\rho(x) \circ 1_{\{y^{-1}\}})(x)$, consequently $\rho(y*x) = \rho(x*y)$. ■

The following example discusses an example of Abelian fuzzy subset, which can be used to characterize a normal fuzzy subgroup.

Example 3. Given a permutation group P_3 equipped with a composition operation. Let a fuzzy subset of the group P_3, i.e $p: P_3 \to [0,1]$ where $p(\sigma) = \frac{1}{ord(\sigma)}$ and $ord(\sigma)$ is an order of σ, for all $\sigma \in P_3$. Since $p(\sigma_a \circ \sigma_b) = p(\sigma_b \circ \sigma_a)$ for all $\sigma_a, \sigma_b \in P_3$, then p is the Abelian fuzzy subset of P_3. ■

A normal fuzzy subgroup can be obtained from the Abelian fuzzy subset if it satisfies conditions as in the following definition.

Definition 4. *Let $\rho \in F(G)$. If ρ is an Abelian fuzzy subset of G, then ρ is called a normal fuzzy subgroup of G. Furthermore, the set of all normal fuzzy subgroups of G is denoted by $NF(G)$.*

The following example discusses the normality of the fuzzy subgroup, which is characterized using Abelian fuzzy subsets.

Example 5. Let $S = \{\bar{0}, \bar{2}, \bar{4}, \bar{6}\}$ and $(S, \oplus_8)$ be a group. Suppose a fuzzy subset $\gamma: S \to [0,1]$ where $\gamma(a) = \left(\frac{1}{2}\right)^{ord(a)}$ for all $a \in S$. Since $\gamma(a \oplus_8 b) = \gamma(b \oplus_8 a)$ for all $a, b \in S$, then fuzzy subset γ is Abelian, consequently, γ is a normal.■

Characterization Normal Fuzzy Subgroup using Fuzzy Conjugate

This section discusses the characterization of normal fuzzy subgroups using fuzzy conjugate. The following is a definition of conjugate subgroups in the context of fuzzy subgroups.

Definition 6. *Suppose G is a group and $\rho, v \in F(G)$. Then, ρ and v is called fuzzy conjugate subgroup (over u), if there is $u \in G$ such that $\rho(x) = v(u*x*u^{-1})$ for all $x \in G$. Furthermore, it is denoted by $\rho = v^u$ where $v^u(x) = v(u*x*u^{-1})$ for all $x \in G$.*

Example 7. Let β be a fuzzy subgroup of S as in Example 5

$$\beta(x) = \begin{cases} \dfrac{1}{2(ord(x))}, untuk\ x = \bar{0}, \bar{4} \in S \\ \dfrac{1}{4(ord(x))}, untuk\ x = \bar{2}, \bar{6} \in S \end{cases}$$

for al $x \in S$. Because for all $x \in S, \gamma(x) = \beta(uxu^{-1})$ where $u = 2 \in S$, it is concluded that γ and β is conjugate fuzzy subgroup (over $u = 2$), and it is denoted $\gamma = \beta^2$. ■

The condition when the membership value of each element in G is equal to the membership value of its conjugate elements is a sufficient and necessary condition for fuzzy subgroup ρ is normal. The following theorem is presented, which discusses the sufficient and necessary conditions for the normality of a fuzzy subgroup in terms of its conjugate.

Theorem 8. *Suppose ρ is a fuzzy subgroup of G. Then, ρ is a normal fuzzy subgroup if and only if $\rho = \mu^z$ for all $z \in G$.*

Bukti. $(\Rightarrow)$ Suppose ρ is normal fuzzy subgroup, it means $\rho(x * y) = \rho(y * x)$ for every $x, y \in G$. It will be shown $\rho = \mu^z$ for all $z \in G$. Note that

$$\rho^z(x) = \rho(z * x * z^{-1}) = \rho(z^{-1} * z * x) = \rho(e * x) = \rho(x).$$

Since this equation satisfies for every $x \in G$, then $\rho = \rho^z$.

$(\Leftarrow)$ Suppose $\rho = \rho^z$ for each $z \in G$. Note that, $\rho^z(x) = \rho(z * x * z^{-1}) = \rho(x)$. This condition satisfies (2) in Theorem 1, i.e fuzzy subset ρ is an Abelian, and therefore, ρ is also normal. ■

Characterization Normal Fuzzy Subgroup using Fuzzy Normalizer

This section discusses the characterization of normal fuzzy subgroups using a fuzzy normalizer. The normalizer in the context of a fuzzy subgroup is a set of elements of a classical group which its membership value elements is equal to its conjugate elements. The following is a definition of a fuzzy normalizer.

Definition 9. *Suppose ρ be a fuzzy subgroup of G. Normalizer of ρ is*

$$N(\rho) = \{x \in G | \rho(x * y * x^{-1}) = \rho(y), y \in G\}.$$

The following property is a theorem that explains the sufficient and necessary conditions for a fuzzy normal subgroup, which is viewed based on its normalizer.

Theorem 10. *Suppose ρ is a fuzzy subgroup of G. Then, ρ is a normal if and only if $N(\rho) = G$.*

Proof.

$(\Rightarrow)$ Let ρ is normal, it will be shown that $N(\rho) = G$. According to Definition 9, $N(\rho) \subseteq G$. Conversely, let $x \in G$. Since ρ is normal, then $\rho(x * y * x^{-1}) = \mu(y)$ for every $y \in G$. Therefore, for every $x \in G$ satisfy $x \in N(\rho)$, and consequently $G \subseteq N(\rho)$. Furthermore, $N(\rho) = G$.

($\Leftarrow$) Let $N(\rho) = G$, it will be shown that ρ is normal. Note that

$$N(\rho) = \{x \in G | \rho(x * y * x^{-1}) = \rho(y)\}$$

for every $y \in G$. Since $N(\rho) = G$, $\rho(x * y * x^{-1}) = \rho(y)$ for every $x, y \in G$. According to Condition 2 in Theorem 1, it is obtained that ρ is Abelian fuzzy subset. Therefore, ρ is normal. ■

Characterization Normal Fuzzy Subgroup using α-Level

Suppose ρ be fuzzy subsets of X. The a-level of ρ, denoted by ρ_a, is a set of all elements whose membership is more than or equal to a, for $a \in [0,1]$. Therefore,

$$\rho_a = \{x \in X | \rho(x) \geq a\}.$$

The following theorem discusses the characterization of normal fuzzy subgroups using an α-level set.

Theorem 11. *Suppose* $\rho \in FP(G)$. *Then,* ρ *is normal if and only if* ρ_a *is a normal subgroup for every* $a \in \rho(G) \cup \{b \epsilon [0,1] | b \leq \rho(e)\}$.

Proof.

($\Rightarrow$) Let $\rho \in NF(G)$ and $a \in \rho(G) \cup \{b \epsilon [0,1] | b \leq \rho(e)\}$. Since $\rho \in F(G)$, then ρ_a is a subgroup of G. Let $x \in G$ and $y \in \rho_a$, it is obtained $x * y * x^{-1} \in G$. Since $y \in \rho_a$, then $\rho(y) \geq a$. Note that, according to Theorem 1, $\rho(x * y * x^{-1}) = \rho(y) \geq a$. Consequently, $x * y * x^{-1} \in \rho_a$, and ρ_a is a subgroup of G and it is normal.

($\Leftarrow$) Let ρ_a be a normal subgroup of G and $a \in \rho(G) \cup \{b \epsilon [0,1] | b \leq \rho(e)\}$. It will be shown that $\rho \in NF(G)$. Since ρ_a is a subgroup of G, then it is obtained that ρ is a fuzzy subgroup of G. Let $x, y \in G$ where $\rho(y) = a$ such that $y \in \rho_a$. So, we have $x * y * x^{-1} \in \rho_a$. It is obtained $\rho(x * y * x^{-1}) \geq a = \rho(y)$, and ρ satisfies condition 3 in Theorem 1. So, ρ is a normal fuzzy subgroup of G. ■

The support of a fuzzy subset ρ, denoted by ρ^*, is defined as follows

$$\rho^* = \{x \in X | \rho(x) > 0\}.$$

The normality of a fuzzy subgroup affects the normality of the support, as presented in the following theorem.

Theorem 12. *If* $\rho \in NF(G)$ *then* ρ^* *is a normal subgroup of* G.

Proof.

Suppose $\rho^* = \{x \in G | \rho(x) > 0\}$. Note that $\rho^* \neq \emptyset$. If $\rho^* = \emptyset$, then for evry $x \in G$ satisfies $\rho(x) = 0$, that means $G = \emptyset$. This condition contradicts, since G is a group. Let

$x, y \in \rho^*$, then $\rho(x) > 0$ and $\rho(y) > 0$. We will show that $x * y^{-1} \epsilon \rho^*$. Since ρ is a fuzzy subgroup, then $\rho(x * y^{-1}) \geq \min\{\rho(x), \rho(y)\} > \min\{0,0\} = 0$. Therefore, $x * y^{-1} \epsilon \rho^*$ and consequently, ρ^* is subgroup of G.

Next, suppose $a \in G$ and $b \in \rho^*$ such that $\rho(b) > 0$. Note that $\rho(a * b * a^{-1}) = \rho(b) > 0$. Consequently, $a * b * a^{-1} \in \mu^*$. For arbitrary element of ρ^* satisfy condition in Theorem 1, so ρ^* is Abelian fuzzy subset. Furthermore, ρ^* is a normal subgroup of G. ■

Characterization Normal Fuzzy Subgroup using Fuzzy Cosets

Analogous to classical group theory, the similarity of the left and right cosets is a sufficient and necessary condition of the fuzzy subgroup ρ is normal. The definition of fuzzy cosets is presented as follows.

Definition 13. *Suppose ρ is a fuzzy subgroup of G and $a \in G$. Then left coset a of ρ is defined as $(a\rho)(x) = \rho(a^{-1} * x)$ for every $x \in G$. Meanwhile, right coset a of ρ is defined as $(\rho a)(x) = \rho(x * a^{-1})$ for every $x \in G$.*

The following theorem discusses about characterization of normal fuzzy subgroup using fuzzy cosets.

Theorem 14. *Suppose ρ is a fuzzy subgroup of G, then ρ is normal if and only if $a\rho = \rho a$ for every $a \in G$.*

Proof.

($\Rightarrow$) Let $x \in G$. Since ρ is normal then $(a\rho)(x) = \rho(a^{-1} * x) = \rho(x * a^{-1}) = (\rho a)(x)$ for every $a \in G$.

($\Leftarrow$) Let $x \in G$. Since $a\rho = \rho a$ for every $a \in G$, then $(a\rho)(x) = \rho(a^{-1} * x) = \rho(x * a^{-1}) = (\rho a)(x)$. This condition shows that ρ is Abelian. Therefore, ρ is normal.■

Based on the definition of fuzzy cosets and the previous example, it can be said that fuzzy cosets change the membership value of each element in the group. In classical group theory, a normal subgroup is defined based on the similarity of all the elements in its right and left cosets. In fuzzy group theory, coset similarity is seen as the similarity of the membership values for each element in the right and left cosets. This makes the normal subgroup in the classic group a special occurrence of the normal subgroup in the fuzzy subgroup. Given a classical group G, the membership value for each $x \in G$ is 1, denoted

$\rho(x) = 1, \forall x \in G$. Furthermore, ρ can be viewed as a fuzzy subgroup so that for every $a, x \in G$ we obtain $a\rho(x) = 1 = \rho a(x)$. In other words, all left fuzzy cosets are the same as right fuzzy cosets so that ρ is normal.

CONCLUSION

The normality of a fuzzy subgroup can be shown based on several things, i.e through the Abelian fuzzy subset, fuzzy conjugate subgroup, fuzzy normalizer, α-level set and fuzzy coset. Furthermore, subgroup normality in classical group theory can be said to be a special occurrence of subgroup normality in fuzzy group theory. For further research, we can examine fuzzy factor subgroups and homomorphisms in fuzzy subgroups.

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