

LEARNING DESIGN FOR LINEAR EQUATIONS IN TWO VARIABLES BASED ON THE PMRI APPROACH USING THE GOJEK CONTEXT FOR EIGHTH-GRADE STUDENTS

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ABSTRACT

This study aims to develop a learning trajectory to stimulate students in solving problems related to the topic of systems of linear equations in two variables (SPLDV), packaged in the form of student activity sheets. This research employs a design research method of the validation study type, consisting of *preparing for the experiment*, *the design experiment*, and *retrospective analysis*. The research was conducted with 37 students from class VIII.1 at SMPN 9 Palembang. Data were collected through documentation, field notes, student activity sheets, and interviews. The collected data were analyzed through data reduction, descriptive presentation, and conclusion drawing. This study resulted in the following learning trajectory: (1) Observing contextual problems related to SPLDV using the GoJek context, specifically the additional bonus for GoCar partners, (2) Students presenting data, including what is known and what is being asked from the problem. (3) Students create possible scenarios for the distances traveled by GoCar partners on weekdays (Monday–Friday) and Saturdays, (4) Students solve the problem, predict alternative solutions, and draw conclusions. The learning trajectory utilizing the GoJek context effectively aids students in understanding and resolving issues related to SPLDV.

Keywords : SPLDV, PMRI , GoJek, Design Research.

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PRELIMINARY

Mathematics learning is a process that provides learning experiences for students to deepen their understanding, enabling them to acquire competencies related to the mathematical material being studied (Fadilla et al., 2021). Mathematics is a process to enhance knowledge, skills, attitudes, and values (Gustin et al., 2020). Mathematics learning allows students to participate actively, such as asking questions and expressing opinions, to develop their mathematical abilities (Gusteti & Neviyarni, 2022). Through learning mathematics, students can think critically, become skilled in calculations, and possess the ability to implement basic mathematical concepts both in other subjects and real-life situations (Afsari et al., 2021).

Previous research indicates that students still struggle to understand mathematics learning (Septiani et al., 2023). Generally, students perceive the learning process in

mathematics as quite tricky due to the numerous formulas involved (Sumantri & Sari, 2022). This is attributed to the material studied being unrelated to everyday life and the teaching process primarily consisting of formulas presented by teachers without training students to understand concepts and solve problems critically, logically, and accurately. If this situation occurs repeatedly, it can lead to monotonous learning experiences (Rafiela & Andhany, 2023). Additionally, it affects students' problem-solving abilities and learning outcomes (Chandra & Hidayati, 2023).

To address students' difficulties in mathematics learning, researchers need to design more meaningful educational experiences. This study will conduct design research that develops a series of activities using the Indonesian Realistic Mathematics Education (PMRI) approach, which emphasizes the significance of students in solving mathematical problems. The researcher will modify the mathematics learning design by relating it to everyday life through the PMRI approach to overcome learning difficulties. The PMRI approach is highly suitable for mathematics, which requires understanding and the ability to solve mathematical problems correctly and accurately (Mandasari & Kusnanto, 2024). By linking everyday life contexts to students at the beginning of the learning process through PMRI, it is hoped that students will gain meaningful learning experiences and improved learning outcomes.

PMRI is an educational approach adapted from the Realistic Mathematics Education theory developed by Freudenthal in the Netherlands (Yunita Sari et al., 2024). The uniqueness of PMRI lies in its use of realistic situations in learning; these situations not only focus on real-world contexts but also include contexts that students can imagine (Zulkardi & Putri, 2019). In PMRI learning, instruction begins with real situations or contexts from students' daily lives, serving as a bridge to connect informal stages to formal mathematics stages (Mubharokh et al., 2022). Thus, PMRI makes mathematics relevant to students (Mubharokh et al., 2022).

The topic of systems of linear equations in two variables (SPLDV) is frequently encountered in everyday life, such as in financial problems, age calculations, business scenarios, and shopping. Mathematically, SPLDV is implemented to determine the intersection coordinates of two straight lines and to identify solution sets and variables within an equation (Nurhayati et al., 2021). The researcher has chosen this topic because students still face challenges in solving SPLDV-related problems, such as difficulty identifying each element in the given problems (Prabawati et al., 2021). This results in students needing help understanding subsequent material since SPLDV is a prerequisite

topic. Therefore, PMRI learning, characterized by its use of real-life contexts, is expected to enhance students' understanding when solving SPLDV problems.

Several previous studies have designed SPLDV materials using real contexts. For instance, Khotimah and Sari (2020) utilized environmental contexts to develop HOTS-based worksheets, employing a traditional context related to students' surroundings. Hidayati et al. (2022) used contexts involving canned drinks, cakes, and bags; however, their testing of the hypothetical learning trajectory (HLT) was limited to a small sample size of three students. Fitriyah et al. (2023) also employed Palembang culinary contexts to assess students' representation abilities in solving SPLDV questions based on local culture. Then, Hayati (2024) used the floating market context in Banjarmasin as a local cultural icon while developing instructional materials in video format for SPLDV content. Based on prior research, no studies have designed SPLDV materials using modern contexts that are more relevant and engaging for students in the digital era, such as using GoJek contexts based on PMRI to assist students in understanding and solving SPLDV-related problems.

GoJek is an online transportation service rapidly expanding across Indonesia, including Palembang. The researcher selected this context because GoJek is familiar with students' daily lives; thus, it is expected to increase their interest and curiosity during lessons. Furthermore, GoJek can solve various mathematical problems related to fares, income, bonuses, or optimization using SPLDV. Thus, Gojek can be used as a realistic context in SPLDV learning to be closer to the student's experiences. This is expected to make learning more interesting and meaningful, and can encourage students to think systematically and critically in solving problem.

In addition to contributing to overcoming student's difficulties in understanding SPLDV material, the results of this study are expected to provide a real contribution to the broader learning designs using modern contexts such as Gojek, teachers have alternative innovative and contextual approaches, in accordance with the times. This can enrich the variety of learning strategies that are not only oriented towards memorizing formulas, but also encourage understanding of concepts and applications in real situations. Then the results of study can be a reference in development of teaching tools, teacher training, and curriculum based on relevant local-modern contexts. So that the impact of this study is not only limited to one class or school. However, it can also provide a positive contribution to improving the quality of mathematics learning in Indonesia. Therefore, this research aims to develop a learning trajectory that stimulates students to solve problems related to systems of linear equations in two variables presented through student activity sheets.

METHODS

This research employs a design research method with a validation study type, aiming to describe the process of designing a Hypothetical Learning Trajectory (HLT) for the topic of systems of linear equations in two variables (SPLDV) and producing a Local Instruction Theory (LIT). The study is conducted during the odd semester of the 2024/2025 academic year, involving 37 students from class VIII.1 at SMPN 9 Palembang. According to Gravemeijer & Cobb (2006), design research consists of three phases: preparing for the experiment, the design experiment, and retrospective analysis.

1. Preparing for the Experiment

In this phase, a literature review is conducted regarding the learning material, specifically SPLDV, the PMRI approach, and the learning design. This is a foundation for formulating initial hypotheses about students' strategies in learning SPLDV. The HLT will be designed to develop a series of learning activities focused on SPLDV using the PMRI approach, which includes hypotheses of learning objectives, learning activities, and tools to aid the learning process. These hypotheses guide anticipations regarding students' strategies and thought processes that may arise and evolve during the learning activities. The designed HLT is dynamic, forming a cyclic process that can change and develop throughout the teaching experiment. To ensure the quality of the design, several indicators of success can be used such as suitability to learning objectives, integration of the Gojek context with the material, a logical sequence of activities and in accordance with the PMRI principles, namely from informal to formal, and predictions of possible student strategies. Then this initial design is validated through discussions with experts to comply with the PMRI principles and is ready to be tested at the experimental stage.

2. Design Experiment

This phase consists of preliminary teaching experiments (pilot experiments) and teaching experiments. During the preliminary teaching experiment (pilot experiment), six students with varying abilities participated as subjects while the researcher acted as the teacher. The results from this stage are used to revise the initial HLT, which will be tested again in one class designated as the research subject. The teaching experiment phase involves testing the revised HLT from the pilot experiment in the selected class. In this stage, the researcher assumes teacher and observer roles in learning activities. Evaluation of HLT changes is carried out using indicators such as the suitability between predictions and implementation of learning, achievement of learning objectives, student responses to

activities, changes in student strategies and understanding, and the effectiveness of learning aids. In addition, interactions between teachers and student involvement in the Gojek context are also observed to evaluate student motivation and participation.

3. Retrospective Analysis

In this phase, data obtained from the teaching experiment are analyzed, and these analyses are used to refine future learning activity designs. The goal is to develop Local Instructional Theory (LIT). To address the research problem, the HLT will be compared with students' actual learning trajectories (Actual Learning Trajectory/ALT). Then, the indicators used to assess the suitability between HLT and ALT include the achievement of learning objectives, strategies used by students, and errors or obstacles they encounter in solving problems. Based on the analysis of student success and difficulties, improvements to the learning design can be made to increase effectiveness in future learning activities or in further research.

Data collection techniques employed by the researcher include photographic documentation, field notes, student worksheets (LAS) collection, and student interviews. Photographic documentation occurs during both pilot and teaching experiments. Field notes are taken to record information related to learning that may indicate problems or guide subsequent steps; these notes are compiled during both pilot and teaching experiments. The LAS results from both phases aim to observe how students transform problems into mathematical models and solve presented issues. Unstructured interviews are conducted during both phases to confirm LAS responses and gather additional information not captured during instruction. Subsequently, HLT will be compared with ALT for retrospective analysis to determine whether students learned from the designed activities.

The collected data from LAS results and interviews are reduced by reviewing and selecting student responses; this reduced data is then presented descriptively to facilitate the researcher's conclusion. Finally, conclusions are drawn based on this data analysis.


RESULT AND DISCUSSION

Preparing for the Experiment

In this phase, the researcher conducts a literature review, specifically analyzing the curriculum in place at the school. This ensures that the researcher can prepare teaching materials that align with the current curriculum. Subsequently, the researcher collaborates

with teachers to divide students into groups for the pilot experiment, comprising six students and one class for the teaching experiment. This ensures that the HLT is suitable for implementation at the school due to the alignment of the material being studied with the curriculum and standards. An overview of the designed Hypothetical Learning Trajectory (HLT) can be seen in Table 1 below.

Table 1. HLT Sistem Persamaan Linear Dua Variabel

Goals	Activity	Student Answer Conjecture
Understand the importance of observing the problems through GoCar partner incentives.	<p>1. Students observe the problems presented; the problems show GoCar partner incentives on November 19-25, 2021, starting at 16.00-21.59 WIB.</p> 	<p>1. Students can read but need help understanding the problems presented.</p> <p>2. Students can read and understand the problems presented.</p>
Understand how to analyze, identify, and collect information based on the problems.	<p>1. Students collect information regarding GoCar partner incentives through question editorials and images.</p>	<p>1. Students can collect information in the form of what is known and asked based on the wording of the questions and the images presented.</p> <p>2. Students cannot collect information in the form of what is known and asked based on the wording of the questions and the images presented.</p>
Understand how to create possible travel distances to pick up customers on weekdays (Monday-Friday) and Saturdays.	<p>1. Students create possibilities regarding the distance travelled to pick up customers on weekdays (Monday-Friday) and Saturdays.</p>	<p>1. Students can present data by creating possible travel distances on weekdays (Monday-Friday), Saturdays, and</p>

		<p>including Sundays where the distance is known.</p> <ol style="list-style-type: none"> 2. Students can present data by creating possible travel distances on weekdays (Monday-Friday) and Saturdays only. 3. Students can present data by creating possible travel distances on weekdays (Monday-Friday), Saturdays, and Sundays and directly calculate the bonus received. 4. Students cannot present by creating possible travel distances on weekdays (Monday-Friday), Saturdays, and including Sundays where the distance is known.
Understand how to solve problems using effective strategies or methods.	<ol style="list-style-type: none"> 1. Students solve problems using their strategies or methods. 2. Students predict whether there are other ways to solve the problems presented, along with supporting arguments. 3. Students make a conclusion using their language 	<ol style="list-style-type: none"> 1. Students can solve problems using trial-and-error strategies. 2. Students can solve problems using the SPLDV substitution or elimination methods or substitution elimination methods. 3. Students need help finding other ways to include supporting arguments in solving problems. 4. Students can find other ways to solve problems using the SPLDV or elimination or substitution elimination

methods.

5. Students can find other ways to solve problems using trial-and-error strategies.
6. Students can present conclusions based on the results obtained using their language.
7. Students need help solving problems and finding other ways using trial and error strategies or SPLDV.
8. Students need help to make conclusions in their language.

The designed Hypothetical Learning Trajectory (HLT) was implemented to develop the Student Activity Sheets (LAS), which included four activities corresponding to the HLT. Additionally, the researcher prepared an interview guide to gather information on students' learning trajectories. Three validators—two university lecturers and one mathematics teacher—validated the LAS. The validation results indicated that the LAS was valid and ready for use.

Pilot Experiment (Cycle 1)

This phase took place on Monday, October 21, 2024, at SMPN 9 Palembang in class VIII, involving six students divided into two groups, each consisting of students with low, medium, and high abilities. The core of this learning activity involved students completing the LAS, which comprised four activities outlined in the HLT. The responses from the students were used as feedback for refining the HLT. Based on the analysis of student responses for each activity, only the answers to activity 2 indicated that the questions needed improvement. The results of the students' LAS responses, which will serve as material for HLT revisions, are presented in the following figure.

1. Berdasarkan permasalahan informasi apa saja yang didapat?

Figure 1. Questions on Activity 2

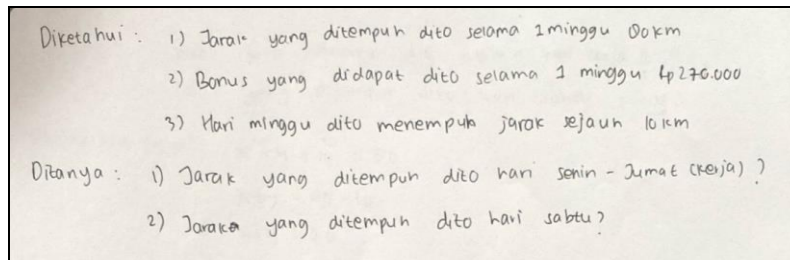


Figure 2. Group 1 Answer to Activity 2

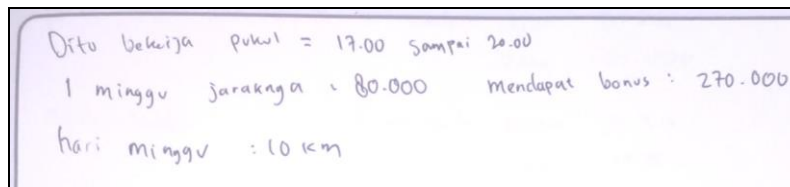


Figure 3. Group 2 Answer to Activity 2

In Activity 2, students were asked to write down the information they gathered based on the problem presented in Activity 1. Group 1 identified that the information in the problem included known and unknown elements. However, Group 1 only noted information based on the wording of the problem and needed to include details from an accompanying image that contained relevant information. Group 2 similarly concluded that the information in the problem was limited to what was stated in the text. They also made an error by writing “80.000” for the distance travelled in one week instead of “80 km.” Furthermore, Group 2 should have mentioned any information from the image or what was being asked.

After conducting a deeper exploration through interviews, it was found that students needed help understanding the question's meaning. Below are excerpts from interviews with both groups:

- Researcher : *Why did many of you not answer your LAS?*
 Group 1 : *We need clarification, Miss.*
 Group 2 : *We need help understanding what the question means, Miss.*
 Researcher : *Which part is confusing?*
 Group 1 : *The part in activity 2, Miss, we don't know what "information" means.*
 Group 2 : *Yes, Miss, our group also took a long time thinking about activity 2; we don't know what to write.*
 Researcher : *Look at the problem in activity 1; what do you see there?*
 Group 1 : *The distance travelled is 80 km in one week, with a bonus of Rp270,000 for one week and 10 km on Sunday. Then, it asks how far is travelled on weekdays (Monday to Friday) and Saturday.*
 Group 2 : *In the problem, it states Dito works from 5 PM to 8 PM; then there is the distance travelled for a week, bonuses earned, and distance travelled on Saturday.*
 Researcher : *So usually, what do you call that when you learn?*
 Group 1 : *What is “known,” Miss.*

- Researcher : *Besides what is known, what else is there?*
Group 2 : *What is being asked, Miss.*
Researcher : *That is what is meant by "information." Look again; is there only information in the text?* Group 1 : *Ooo iyo miss, ado jugo yang di gambar.*
Group 1 : *Oh right, Miss, something is in the picture.*
Researcher : *Good! Now, do you understand? Let us continue working.*
Group 2 : *Okay, Miss.*

Retrospective Analysis (Cycle 1)

Based on the activities conducted by students in Cycle 1, a retrospective analysis was performed to understand why students only recorded information based on the wording of the questions and why some groups needed to document the information presented in the problems entirely. In this case, students had yet to grasp the meaning and purpose of the questions presented. This aligns with Pulungan (2022), who states that students are considered not to understand a problem if they do not write down the known and unknown information in their answers. Therefore, it is necessary to improve the questions in Activity 2 to help students better comprehend the meaning and intent of the questions. The revised version of Activity 2 is provided below.

Table 2. LAS Revision Results

Activity	Before Revision	After Revision
2	Based on the problem, what information was obtained?	What is known and asked based on the problem above?

Teaching Experiment (Cycle 2)

Cycle 2 was conducted on Friday, October 25, 2024, with research subjects from class VIII.1 at SMPN 9 Palembang, consisting of 37 students. In this phase, the researcher, acting as the teacher, divided the students into seven groups with relatively balanced skill levels. The learning activities progressed according to the tasks assigned to the students, starting with activity 1, which involved presenting a contextual problem in line with PMRI principles. Below is a figure of activity 1.

Aktivitas 1

Amati dan pahami masalah permasalahan di bawah ini.

Dito bekerja sebagai driver GoCar part time mulai pukul 17.00-20.00 WIB. Dalam 1 minggu ia berhasil menempuh jarak 80 km saat menjemput pelanggan, dan mendapatkan bonus sebesar Rp270.000. Jika pada hari Minggu ia menempuh jarak 10 km saat menjemput pelanggan, maka berapa jarak yang ditempuh Dito saat menjemput pelanggan di hari kerja (Senin-Jumat) dan hari Sabtu?

Figure 4. Problems in LAS Activity 1

In activity 1, based on the HLT, students could read and understand the problem. For activity 2, students were asked to present the information obtained from the problem accurately. The responses from students regarding activity two are shown below.

1. Apa yang diketahui dan ditanyakan berdasarkan permasalahan diatas?

Dik: Minggu = 80 km
 Bonus = 270.000
 Hari Minggu = 10 km
 Bonus = 270.000
 Dit: Jarak hari kerja (Senin-Jumat) dan Sabtu?

(a) Jawaban Kelompok 1

1. Apa yang diketahui dan ditanyakan berdasarkan permasalahan diatas?

1 minggu = 270.000 dan jarak 80 km
 Hari minggu 10 km = 10 x 5.500 = 55.000 Rp.
 Senin - Jumat / km = 3 ribu rupiah
 Sabtu / km = 3,5 ribu
 270000 (1 minggu) - 55.000 (hari minggu)
 = 215.000.

(b) Jawaban Kelompok 3

1. Apa yang diketahui dan ditanyakan berdasarkan permasalahan diatas?

Dik: 1 minggu = 80 km
 1 minggu bonus = 270.000
 Hari minggu = 10 km
 Bonus yang didapat dalam 1 minggu = 270.000
 Harga Senin - Jumat = 3.000
 Harga Sabtu = 3.500
 Harga Minggu = 5.500
 Dit: Berapa yang ditempuh Dito saat menjemput pelanggan (Senin-Jumat) dan hari Sabtu?

(c) Jawaban Kelompok 5

1. Apa yang diketahui dan ditanyakan berdasarkan permasalahan diatas?

Dik: Dalam 1 minggu menempuh Jarak 80 km
 Bonus yang didapat dalam 1 minggu = 270.000
 Hari minggu = 10 km
 Bonus = 5.500 / km
 Harga (Senin - Jumat) = 3.000 / km
 Sabtu = 3.500 / km
 Dit: Jarak tempuh hari kerja (Senin-Jumat) dan Sabtu?

(d) Jawaban Kelompok 7

Figure 5. Student Questions and Answers in Activity 2

In Figure 5, students could correctly present information based on the given problem. This aligns with the researcher's expectations outlined in the HLT. During interviews, students could articulate what was known from the text and images in the problem and clearly state what was being asked or what problem needed solving. Following this, in activity 3, students were tasked with estimating the possibility of picking up customers during workdays (Monday-Friday) and on Saturdays. The responses related to activity three are as follows.

2. Buatlah kemungkinan mengenai jarak tempuh untuk menjemput pelanggan pada hari kerja (Senin-Jumat) dan hari Sabtu.

(a) Jawaban Kelompok 1

2. Buatlah kemungkinan mengenai jarak tempuh untuk menjemput pelanggan pada hari kerja (Senin-Jumat) dan hari Sabtu.

(b) Jawaban Kelompok 3

2. Buatlah kemungkinan mengenai jarak tempuh untuk menjemput pelanggan pada hari kerja (Senin-Jumat) dan hari Sabtu.

(c) Jawaban Kelompok 5

2. Buatlah kemungkinan mengenai jarak tempuh untuk menjemput pelanggan pada hari kerja (Senin-Jumat) dan hari Sabtu.

(d) Jawaban Kelompok 7

Figure 6. Student Questions and Answers in Activity 3

From Figure 6, it can be observed that students employed two strategies. In Figures 6(a) and 6(b), students created three possibilities and directly multiplied them by the known bonus amounts. For example, they calculated $40 + 30 + 10 = 80$ and multiplied this by the bonuses for weekdays (Monday-Friday), Saturday, and Sunday: $40(3000) + 30(3.500) + 10(5.500) = 280.000$. Figures 6(c) and 6(d) show that some students generated five possibilities and seven possibilities regarding distances travelled during weekdays (Monday-Friday), Saturday, and Sunday. Based on interviews, students explained their reasoning for these possibilities using assumptions or trial-and-error based on a distance of 10 km travelled on Sunday and a total distance of 80 km over the week. In activity 4, there were three questions: for the first question, students were asked to solve a problem; for the second question, they needed to consider alternative methods for solving it; and for the final question, they were required to provide a conclusion regarding their findings from the presented problems. Below are student responses related to activity 4.

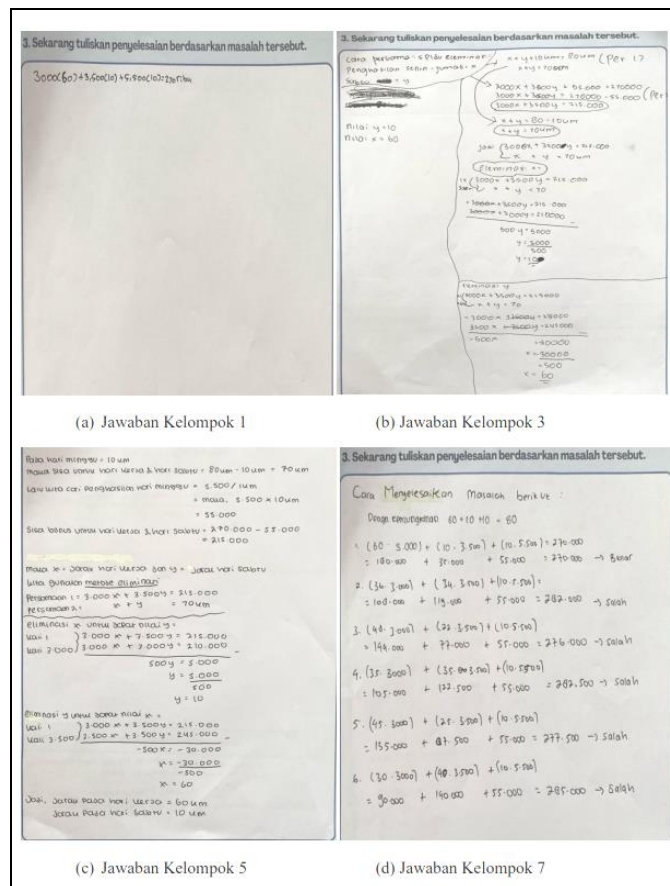


Figure 7. Student Questions and Answers in Activity 4 No. 1

In Figure 7, it can be seen that students employed various strategies to solve problems. In Figure 7(a), one group wrote their answer using a trial-and-error strategy based on possibilities but only noted one possibility while they had identified three in Activity 2. Figure 7(b) shows that another group solved problems using SPLDV elimination methods. Figure 7(c) shows another group utilizing SPLDV elimination to address issues. In Figure 7(d), another group solved problems using trial-and-error strategies by multiplying equations derived from activity three by bonuses for each day category, yielding different results for each equation. Based on student responses, it is evident that they could solve problems effectively using their respective strategies. Following this, interviews were conducted with students; here are excerpts from those interviews:

- Researcher : *What method did you use to solve this problem?*
 Group 1 : *Our group tried multiplying the equations we derived by their bonuses.*
 Researcher : *Okay, in activity 2, you obtained three equations; why did you only write one equation in activity 4?*
 Group 1 : *We already calculated it in activity 2. After finding our possibilities, we multiplied by its bonus and got our result. Then, in Activity 4, we just wrote down what was correct based on our findings in Activity 2.*
 Researcher : *I see. Next, what strategies did groups 3 and 5 use?*

- Group 3 : Our group used SPLDV elimination; we derived two equations: (1) $3000x + 3.500y = 215.000$ and (2) $x + y = 70$ km. We eliminated x first to find $y = 10$. Then, we eliminated yy from both equations to find $x = 60$.
- Group 5 : Our group also used SPLDV elimination; we first eliminated x , which gave us $y = 60$, and then eliminated y , which resulted in $x=10$.
- Researcher : Why did you choose to solve this problem using SPLDV?
- Group 5 : Because it asks for distances travelled during workdays and Saturdays, we defined workdays as x and Saturdays as y . Each day's category image also reflects a bonus earned over a week. Thus, we had two equations: distance travelled and one for bonuses, so we used SPLDV.
- Group 3 : There are two equations here with two variables: workdays as x and Saturday as y . Since we thought it was SPLDV, we used SPLDV elimination.
- Researcher : Great! Now, how about group 7?
- Group 7 : Our group answered by trying different distance possibilities multiplied by their corresponding bonuses; we created many possibilities and found that $60(3000) + 10(3.500) + 10(5.500) = 270.000$, which matched our weekly bonus.

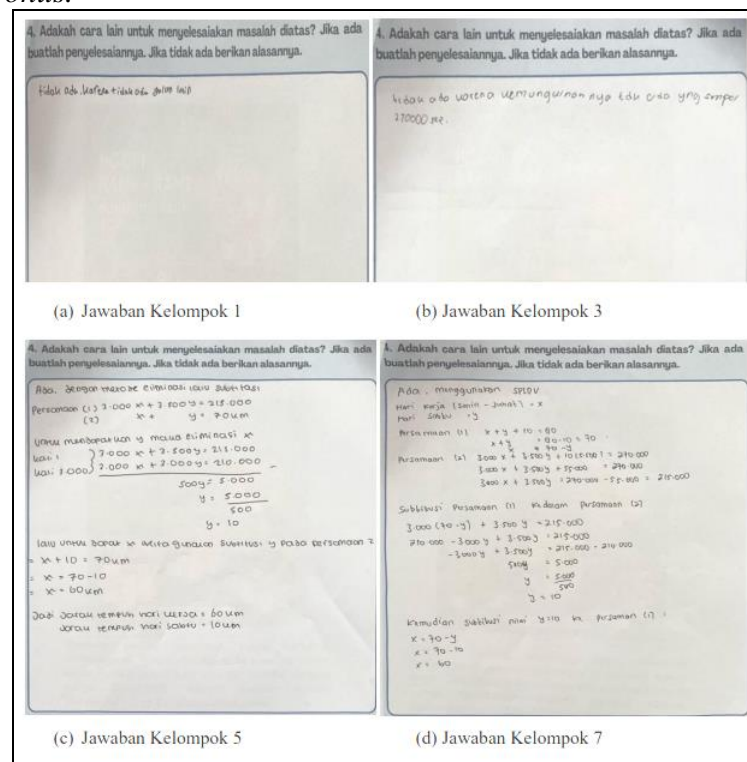


Figure 8. Student Questions and Answers in Activity 4 No. 2

In Figure 8, students were asked to think of alternative ways to solve the problems presented, based on Figures 8(a) and 8(b), students stated that no other methods were available to solve their problems. During the interviews, Group 1 mentioned that, according to their group, they could only use trial and error by calculating possible distances multiplied by the bonuses received. Group 3 stated that the only method they could use was SPLDV elimination, as using any other method would not yield a result of 270,000. However, in Figure 8(c), it can be seen that students discovered alternative

methods for solving the problems using both elimination and substitution methods within SPLDV. During the interview, Group 5 explained that within SPLDV, various methods are available; therefore, they used another method to solve the problem and obtained results consistent with their previous method, specifically finding values of $x = 50$ dan $y = 10$. In Figure 8(d), students found another way to solve the problem using the substitution method of SPLDV. During the interview, Group 7 mentioned that after completing the first problem in Activity 4, they realized that the possible distances and bonuses could be formulated into two equations. This led them back to SPLDV material, prompting them to use the substitution method and achieve results consistent with their answers to earlier questions.

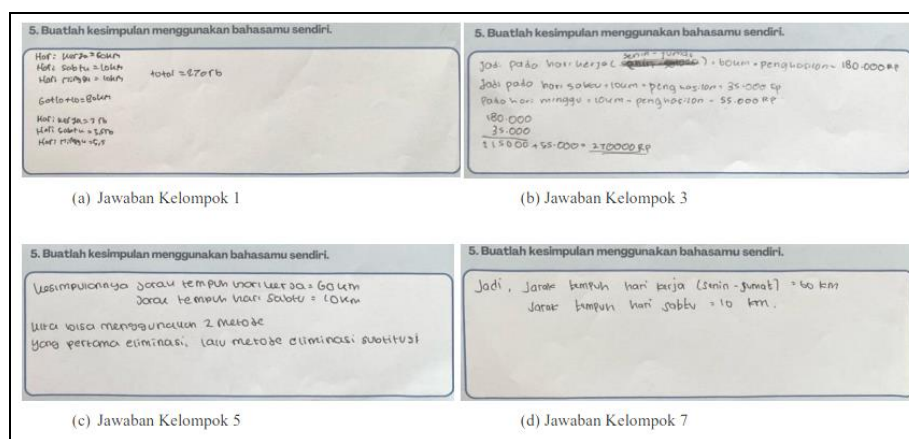


Figure 9. Student Questions and Answers in Activity 4 No. 3

In Figure 9, students were asked to summarize their findings. They were able to write conclusions in their own words based on the results they obtained from the given problems.

Retrospective Analysis (Cycle 2)

based on the learning activities from Cycle 2, where HLT was compared with ALT alongside previous theories or research related to systems of linear equations in two variables, The learning processes from both Cycle 1 and Cycle 2 resulted in a Local Instruction Theory (LIT) where students had to navigate through four HLT activities contained within student activity sheets (LAS). According to Abrika et al. (2023), This statement suggests that HLT (Hypothetical et al.) is a learning pathway designed to achieve specific educational goals. Therefore, the HLT that has been developed is considered successful in guiding students to engage in learning activities aligned with the principles of PMRI. Then, the learning trajectory for the topic of linear equations in two variables, utilizing the context of GoJek, proved effective in helping students solve problems related

to this concept. In Activity 1, students were introduced to a contextual problem linked to linear equations in two variables, explicitly focusing on the bonus system for GoCar partners. Students observed and understood the problem presented. In Activity 2, students presented data, identifying what was known and what was being asked in the problem. Through this activity, students gained experience analyzing, identifying, and gathering relevant information from the problem. Next, in activity three, students were asked to create possible scenarios for GoCar partners' travel distances on weekdays (Monday to Friday) and Saturdays. This activity allowed students to experiment and calculate possible travel distances. Finally, in Activity 4, students were required to solve the problem, explore alternative methods, and draw conclusions. This activity allowed students to solve problems, predict other possible solutions using their strategies, and arrive at conclusions. Through these activities, using the GoJek context to learn linear equations in two variables successfully guided students from an informal understanding to a formal mathematical approach.

The following is a comparison of learning outcomes and student abilities in cycle 1 and cycle 2 in applying HLT to SPLDV material.

Table 2. Comparison of Cycle 1 and Cycle 2

Aspects	Cycle 1	Cycle 2
Question Understanding	Many Students have difficulty understanding questions, especially in interpreting information from text and images. Some students only take note from the text.	After revising the questions, students are better able to understand the questions better and can identify information from the text and images completely.
Difficulties Experienced by Students	<ul style="list-style-type: none"> Students have difficulty identifying the information that is “known” and “asked” in the problem. Some students make mistakes in understanding the data presented, for example writing the number “80.000” as the distance traveled instead of “80 km”. 	<ul style="list-style-type: none"> Students are more fluent in identifying the information given in the problem. Difficulties decrease after the problem revision, and students can understand the context better.
Problem Solving Strategy	<ul style="list-style-type: none"> Students with high ability tend to use the SPLDV method. Students with low ability tend to use the less efficient trial-and-error method. 	<ul style="list-style-type: none"> The high ability group continues to use the SPLDV method more quickly and accurately. The low ability group continues to switch to using SPLDV or continues to use

		the more focused trial-and-error.
Method Use	<ul style="list-style-type: none"> Many students used trial-and-error to find solutions. Some group had difficulty in creating correct equations. 	<ul style="list-style-type: none"> The use of SPLDV method was more dominant and more precise. Some groups used other methods in SPLDV (such as elimination or substitution, or combination) with more confidence.
Student Engagement in Discussion	<ul style="list-style-type: none"> Students were more passive in discussions and needed a lot of help. Some groups did not understand the meaning of the questions well. 	<ul style="list-style-type: none"> Students were more active in discussions and were able to explain their solution well. Group engagement was higher, with some groups able to provide more complex explanations.
Student Success in Solving Problem	The high-ability group was faster in solving the problems, but the low-ability group had difficulty solving the problems correctly.	Overall, student success increased, especially in the low-ability group who were better at understanding the steps in solving problems in the questions.
Changes in Ability from Cycle 1 to Cycle 2	<ul style="list-style-type: none"> Students have not been able to utilize information well from text and images. The main difficulty lies in identifying relevant information. 	Students' abilities have increased in understanding information, and they are more able to switch to using appropriate mathematical methods such as SPLDV.
Reflection on HLT Design	HLT needs to be revised and the questions should be more explicit to help students understand the steps in solving problems.	The revision of the questions was successful, students were able to solve the questions more independently. HLT has proven effective in providing clearer directions to students.

The research findings indicate that the Indonesian Realistic Mathematics Education (PMRI) approach can assist students' Local Instruction Theory (LIT) in solving problems related to systems of linear equations in two variables. This is consistent with studies using design research, such as Suryabayu et al. (2022), which state that there is a relationship between Hypothetical Learning Trajectories (HLT) and educational theories or LIT that are tested directly. This suggests that PMRI, through the developed HLT, can enhance students' understanding and cognitive processes to higher levels (Hidayati et al., 2022). Therefore, the HLT on the topic of systems of linear equations in two variables, developed through PMRI using design research methods, can be categorized as effective because it aligns with existing educational theories.

CONCLUSION

Based on the research, the outcome is a learning design for the topic of systems of linear equations in two variables using the PMRI approach with the GoJek context for eighth-grade students, developed through pilot and teaching experiments. The learning trajectory for the topic of systems of linear equations using the GoJek context can help students understand and solve problems through four activities: 1) Observing contextual problems related to SPLDV material using the GoJek context, specifically the additional bonuses for GoCar partners; 2) Students presenting data regarding what is known and what is being asked from the problem; 3) Students estimating possible distances for GoCar partners during workdays (Monday-Friday) and Saturday; 4) Students solving problems, predicting alternative methods for problem-solving, and drawing conclusions.

The implemented learning trajectory, consisting of four activities that adhere to PMRI principles, contributes to the development of Local Instructional Theory (LIT) for systems of linear equations in two variables using the GoJek context. This approach facilitates students' progression from informal to formal in solving SPLDV problems. The researcher suggests that this HLT can be implemented in teaching systems of linear equations and further researched to assess its effectiveness.

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