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ANALYSIS OF FRACTIONAL LOGISTIC MODEL SOLUTION AND ITS SIMULATION ON HUMAN DEVELOPMENT INDEX DATA OF CILACAP DISTRICT

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ABSTRACT

This research discusses the formation of the Fractional Logistic model and its solution analysis, as well as its simulation to predict the Human Development Index in Cilacap Regency using the Fractional Logistic growth model. This study uses secondary data obtained from the official website of the Central Statistics Agency of Cilacap Regency. Based on these data, the growth of the Human Development Index in Cilacap Regency has increased relatively. This shows the Cilacap community's good quality of life. Based on the environmental carrying capacity value of 73, a relative growth rate per year of 0.14273269 is obtained. This model predicts the Human Development Index in Cilacap Regency in 2024 and 2025. The prediction results for 2024 of 71.47 and 2025 of 71.67 are achieved when the fractional derivative order is one. The best approximate solution is obtained when the fractional derivative order is 1, 0.95 0.90, 0.85, 0.80 and 0.75.

Keywords: Environmental Carrying Capacity, Human Development Index, Growth Rate, Fractional Logistic Model, Solution.

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PRELIMINARY

As is well known, the Human Development Index (HDI) is an indicator used to measure society's quality of life and welfare. So that the HDI can be used to determine the condition of a country and whether it is a developed, developing, or underdeveloped country (Hasibuan et al., 2023). The HDI consists of three main dimensions, namely life expectancy, education level and decent living standards (Raharti et al., 2020). The HDI can function as a benchmark for the success of the development of a country or region and provide an overview of the welfare of society as a whole.

Based on data released by the Central Statistics Agency (2023), the Human Development Index in Indonesia is 74.39. Meanwhile, Central Java province is ranked 20th with an HDI reaching 73.39. Meanwhile, in Cilacap Regency, the HDI figure is 71.83. The

HDI of Cilacap Regency from 2021 to 2023 continues to increase. It can be concluded that the quality of life in the community is improving every year.

Various factors influence the Human Development Index. According to Muhamad & Rahmi (2023), the use of technology, poverty and government spending affect the Human Development Index in West Java. Other factors that affect the HDI include per capita income (Ramadanisa & Triwahyuningtyas, 2022), population, and gross regional domestic product (Mutiara, 2023). Changes can influence the overall HDI figure in these factors because the HDI reflects various aspects of people's lives. HDI projections are very important because they can provide a picture of the future of human development and help us take the right steps to achieve better development goals.

Meanwhile, according to research (Anggreini et al., 2023) examining population projections in the Special Region of Yogyakarta using exponential and logistic models based on growth rates and carrying capacity. The results of this study are that the carrying capacity of the Special Region of Yogyakarta is 10,652,814 and the population growth rate for the logistic model is 0.02048. Research conducted by Widiyanti & Kurniawan (2024) related to population growth modelling of West Nusa Tenggara by comparing logistic and exponential models. The study results showed that the logistic model was better at representing the population of NTB with a MAPE value of 4.6315%.

Tang & Tang (2021) conducted research to project the Human Development Index in Alor Regency in 2030 using exponential and logistic models. The study results showed that using exponential model 1, exponential model 2, and exponential model 3, the results obtained for the number of HDI in 2030 were 64.12, 65.69, and 65.72. While using the logistic model, the number of HDI in 2030 was 64.98.

On the other hand, fractional calculus was born in 1695. Fractional calculus began when l'Hopital, one of the founders of calculus, wrote a letter to Leibnitz, the father of calculus, the meaning of $\frac{d^n y}{dx^n}$ for $n = \frac{1}{2}$. Leibnitz replied that it was a paradox that would be very useful one day (Mathai & Haubold, 2017). According to Xu et al. (2020), over the past three decades, materials on fractional calculus have found applications in many problems in various disciplines, such as fluid flow, diffusion, anomalous diffusion, reaction-diffusion, physics, chemistry, waves, statistical distribution theory, demography, and financial mathematics.

According to the research, the logistic model can calculate the human development index. Apart from that, there are still shortcomings in the logistic model, namely that it is less flexible in capturing real data. Hal This is especially true if the data collected experiences sharp fluctuations. The Logistic Model is very difficult to approximate due to the sharp jumps. So, a better model is needed, namely the fractional logistic model. The fractional logistic model is more flexible and smooth when approaching real data because it can vary the value of its fractional order. This model is a generalization of the logistic model. This results in modelling population growth or calculating the value of the human development index, which is more flexible and smoother.

Based on the background description, the formulation of the problem to be studied is how to apply the logistics growth model to the Human Development Index in Cilacap Regency and what are the predicted results of the Human Development Index in Cilacap Regency in 2024 and 2025 using the Fractional logistics growth model. According to the formulation of the problem, it can be used as the basis for the research objectives, namely to determine the application of the Fractional logistics growth model to the Human Development Index in Cilacap Regency and to determine the predicted results of the Human Development Index in Cilacap Regency in 2024 and 2025 using the Fractional logistics growth model.

METHODS

The research method used in this paper combines theoretical and applied approaches. The theoretical aspect is evident in the development of a fractional logistic model and its analytical solution determination. The applied aspect is demonstrated in a simulation using Cilacap Regency's HDI data from 2015 to 2023, along with an error analysis. The following is a flowchart that describes the stages of research carried out.

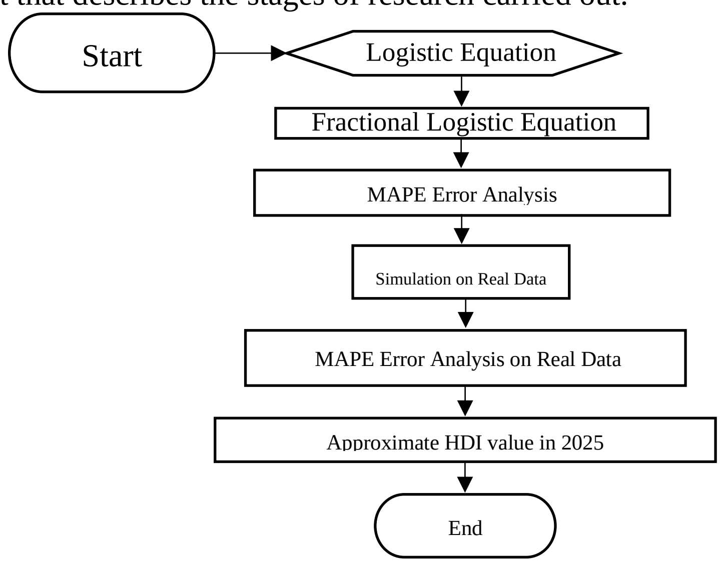

Figure 1. Research Flowchart

The following is a discussion of the concept and theorems of fractional derivatives and conformable fractional derivatives. This is used to form models and determine solutions to fractional logistic equations.

1. Logistic Differential Equation

The logistic growth model is a development of the exponential growth model first proposed by Malthus. The logistic population growth model is a refinement of the exponential growth model. The simplest form for the relative growth rate that accommodates these assumptions is:

$$\frac{1}{N}\frac{dN}{dt} = r\left(1 - \frac{N}{K}\right),$$

Both sides are multiplied by N, and a model for population growth is obtained which is known as the logistic differential equation:

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)$$

The logistic differential equation can be viewed as a special case of the fractional logistic differential equation with the order of its derivative being 1.

Theorem 1. Riemann-Liouville fractional integral operator (Sugandha et al., 2023)

The Riemann-Liouville fractional integral operator of order q of a function $\mu(x) \in K_\omega$ is given as follows:

$$J^q\psi(x) = \begin{cases} \frac{1}{\Gamma(q)}\int_0^x (x-\tau)^{q-1}\psi(\tau)d\tau, q > 0, x > 0, \\ \psi(x) \qquad , q = 0. \end{cases}$$

The J^q operator has the following properties:

1. $J^0\psi(x) = \psi(x)$.
2. $J^q j^\tau\ \psi(x) = J^{q+\tau}\psi(x)$.
3. $J^q j^\tau\ \psi(x) = J^\tau J^q\ \psi(x)$.
4. $J^q x^\zeta = \frac{\Gamma(\zeta+1)}{\Gamma(q+\zeta+1)} x^{q+\zeta}$.
5. $J^q C = \frac{C}{\Gamma(q+1)} x^q$.

Theorem 1 is a very popular form of the Riemann-Liouville fractional integral operator. Based on Theorem 1 and its properties, it plays a role in forming a fractional logistic model from a logistic model.

Definition 1. (Sugandha et al., 2024). The Caputo fractional derivative D^q of a function $\psi(x)$ of a real number q such that $m - 1 < q \leq m, m \in N$ for $x > 0$ and $\psi \in C^m_{-1}$ as

$$D^q\psi(x) = \begin{cases} \dfrac{1}{\Gamma(m-q)}\displaystyle\int_0^x (x-\tau)^{m-q-1}\psi^{(m)}(\tau)d\tau, & \\ \dfrac{\partial^m\psi(x)}{\partial x^m} & q = m. \end{cases}$$

Definition 1 is a form of the Caputo fractional derivative. Based on this definition 1, it can be used to determine the guarantee of the existence of a solution to the fractional logistic model.

2. Conformable Fractional Derivatives

Definition 2. (Khalil et al., 2014) Given a function $g: [0, \infty] \to \mathbb{R}$ and $t > 0$. The conformable fractional derivative of a function g of order α is defined as:

$$T_\alpha(g)(t) = \lim_{\varepsilon \to 0} \frac{g(t + \varepsilon t^{1-\alpha})}{\varepsilon},$$

For every $t > 0$ and $0 < \alpha \leq 1$. If g is α-differentiable on $(0, a), a > 0$ and $\lim_{t \to 0^+} f^{(\alpha)}(t)$ exists, then we have $f^{(\alpha)}(0) = \lim_{t \to 0^+} f^{(\alpha)}(t)$.

Theorem 2.

If $0 < \alpha \leq 1$ and g, h is α-differentiable at point $t > 0$, then it implies:

(i) $T_\alpha(ag + bh) = aT_\alpha(g) + bT_\alpha(h)$.

(ii) $T_\alpha(gh) = gT_\alpha(h) + hT_\alpha(g)$.

(iii) $T_\alpha\left(\frac{g}{h}\right) = \frac{hT_\alpha(g) - gT_\alpha(h)}{g^2}$.

(iv) If g is differentiable, then $T_\alpha(g)(t) = t^{1-\alpha}\frac{dg(t)}{dt}$.

As a consequence of **theorem 2** above, the following are the fractional derivatives of simple functions.

(i) $T_\alpha(t^n) = nt^{n-\alpha}$ for every $n \in \mathbb{R}$.

(ii) $T_\alpha(c) = 0$ for each constant function $g(t) = c$.

(iii) $T_\alpha(e^{ax}) = ax^{1-\alpha}e^{ax}$ for every $a \in \mathbb{R}$.

(iv) $T_\alpha(\sin bx) = bx^{1-\alpha}\cos bx$ for every $b \in \mathbb{R}$.

(v) $T_\alpha(\cos bx) = -bx^{1-\alpha}\sin bx$ for every $b \in \mathbb{R}$.

(vi) $T_\alpha\left(\frac{1}{\alpha}\right) = 1$.

(vii) $T_\alpha\left(\sin\frac{1}{\alpha}t^\alpha\right) = cos\frac{1}{\alpha}t^\alpha$.

(viii) $T_\alpha\left(\cos\frac{1}{\alpha}t^\alpha\right) = -sin\frac{1}{\alpha}t^\alpha$.

(ix) $T_\alpha\left(e^{\frac{1}{\alpha}t^\alpha}\right) = e^{\frac{1}{\alpha}t^\alpha}$.

Based on definition 2 and theorem 2, this paper is used to determine the analytical solution of the fractional logistic model.

RESULT AND DISCUSSION

The following section discusses the results of forming fractional logistic equations, solutions to fractional logistic equations, and their simulations of human development index data in the Cilacap district.

1. Differential Equation

Differential equations are a basic mathematical concept that describes the relationship between a function and its derivatives. Based on the number of independent variables, differential equations can be divided into two types, namely ordinary and partial differential equations. Ordinary differential equations only have one independent variable, while partial differential equations have more than one independent variable (Nubatonis, 2021)

A differential equation is an equation that involves an unknown function with its derivatives concerning one or more independent variables (Lestari et al., 2023). Differential equations can be classified based on the highest order of the derivatives.

2. Logistic Growth Model

The logistic model is a more realistic for describing population growth (Apriani et al., 2022). This model considers that a population's growth rate is not constant but is influenced by various factors. The logistic model was first introduced by a Dutch scientist, Pierre Franscois Verhulst, in 1838 (Nurmadhani & Faisol, 2022).

Pierre Franscois Verhulst developed the Malthusian model by adding a competition factor due to the limitations of environmental carrying capacity. Environmental carrying capacity is the ability of an environment to support the life of living things over a long period. The general form of the logistic growth model is:

$$N(t) = \frac{KN_0}{(K-N_0)e^{-rt}+N_0}\,.$$

With:

$N(t)$: population growth at the time

N_0 : initial value of population growth at time $t = 0$ r : intrinsic growth rate

K : carrying capacity

t : time

3. Formation of Fractional Logistics Model

To obtain the Fractional Logistic model based on the logistic equation model, it is brought to the population growth process in the fractal structure.

Suppose $N(t)$ is the population at time t. The total population growth rate $\bar{Y}(t)$ per unit time from time t_0 to t and the population growth value at time t, namely $N(t)$ must meet:

$$\int_{t_0}^{t} \bar{Y}(t')dt' = \mu \int_{t_0}^{t} H(\tau' - \tau)[N(t') - N(T)]d\tau', \tag{3.1}$$

with $H(\tau)$ as the transmission function, and μ is a constant. The equation above is called the conservation equation of the diffusion process of option values in fractal structures. The transmission function $H(\tau)$ is defined as:

$$H(\tau) = \frac{A_\alpha}{\Gamma(1-\alpha)t^\alpha}. \tag{3.2}$$

The next step is to differentiate the conservation equation (3.1) above concerning t' to obtain:

$$\bar{Y}(t') = \mu \frac{d}{dt}\left(\int_{t_0}^{t} H(\tau' - \tau)[N(t') - N(T)]d\tau'\right) \tag{3.3}$$

Conversely, it is defined as $\mu = \frac{1}{(2-\alpha)}$, $0 < \alpha \le 1$.

Based on the Logistic Equation, we obtain:

$$\bar{Y}(s,\tau) = rN\left(1 - \frac{N}{K}\right).$$

Combined with equation (3.3) above, we obtain:

$$\frac{d^\alpha N}{dt^\alpha} = (2-\alpha)rN\left(1 - \frac{N}{K}\right), \tag{3.4}$$

with :

N: Population growth value at time t.

r: Average maximum population growth.

K: Carrying capacity.

Equation (2.4) is called the Fractional logistic equation. For the value of α=1, the logistic equation is obtained. So, the Logistic equation is a special case of the Fractional logistic equation.

4. Solution of Fractional Logistic Equations

The definition of the Conformable Fractional Derivative theorem is used to obtain the solution of the Fractional Logistic equation. Based on equation (3.4), so that we obtain the following:

$$N(t) = N_0 + T_\alpha\left[(2-\alpha)rN\left(1-\frac{N}{K}\right)\right] \tag{3.5}$$

So according to equation (3.5), it is obtained:

$$\frac{d^\alpha N}{dt^\alpha} = t^{1-\alpha}\frac{dN}{dt},$$

$$rN(2-\alpha)\left(1-\frac{N}{K}\right) = t^{1-\alpha}\frac{dN}{dt},$$

$$\frac{KdN}{rN(2-\alpha)(K-N)} = t^{\alpha-1}dt. \tag{3.6}$$

Integrating both sides will yield:

$$N(t) = \frac{KN_0}{(K-N_0)e^{-\frac{r(2-\alpha)t^\alpha}{\alpha}} + N_0}, \tag{3.7}$$

Figure 2 is a graph for the exact solution, namely when $\alpha = 1$.

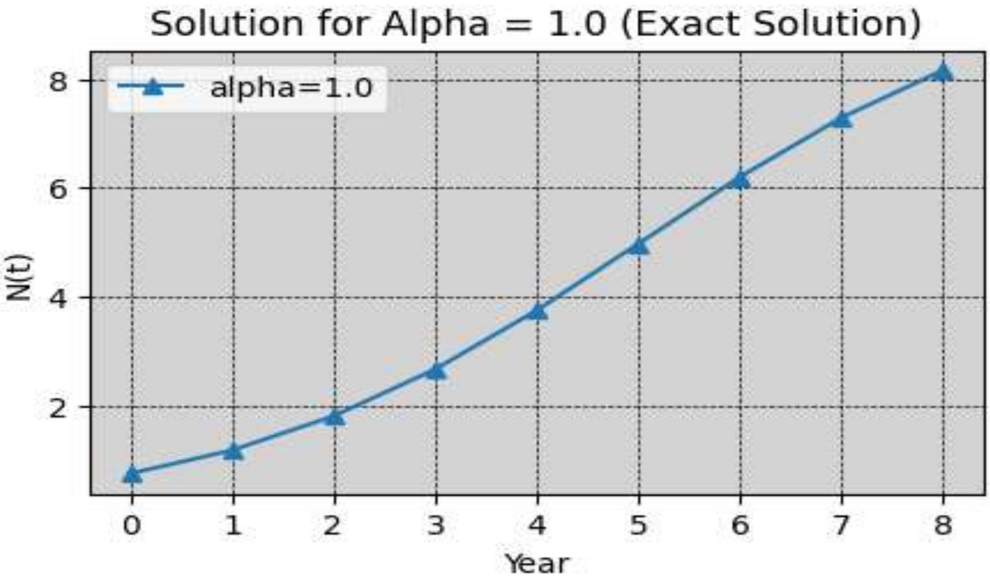

Figure 2. Graph of the exact solution of the Logistic equation

Based on Figure 2 for the value of $N_0 = 0{,}75$, $r = 0{,}5$, K=10, α=1 with the value of $0 \leq t \leq 8$. It appears that the resulting solution graph is an increasing function graph. Now consider the solution of the Fractional Logistic equation (3.7). If the limit of equation (3.7) is taken for time towards infinity, then we get:

$$\lim_{t\to\infty} N(t) = \lim_{t\to\infty} \frac{KN_0}{(K-N_0)e^{-\frac{r(2-\alpha)t^{\alpha}}{\alpha}}+N_0},$$

$$= \frac{KN_0}{N_0+0},$$

$$= K.$$

Thus, $N(t)$ will approach the environmental carrying capacity. This indicates that the population will be stable at the value of K in the long term.

Next, Figure 3 is a graph of the solution of the fractional logistic equation when the value of α is changed to α = 1, 0,95, 0,9, 0,85, 0,8 and α = 0,75.

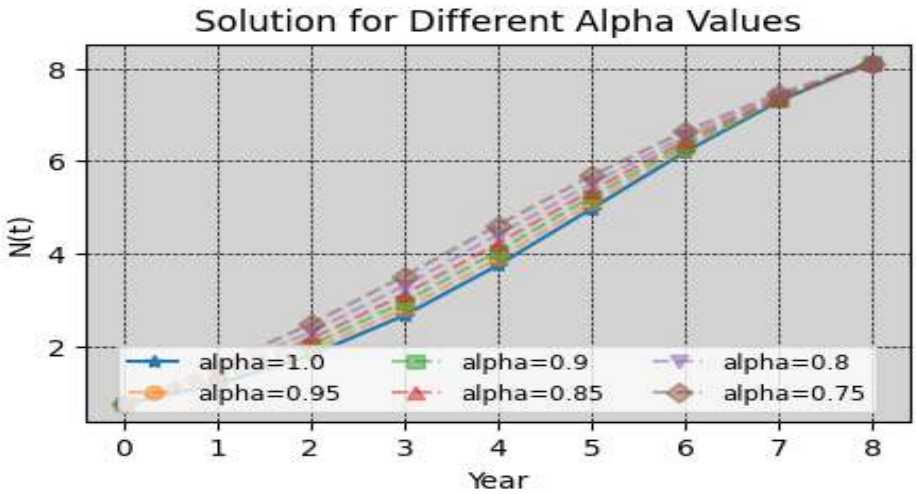

Figure 3. Graph of the solution of the fractional logistic equation.

Using the Phyton 3.7 program, the mean absolute error is obtained, namely 2.64%, 5.54%, 8.78%, 12.45% and 16.67% for the values α = 0,95, 0,90, 0,85, 0,80, 0,75, respectively. Based on the graph of the fractional logistic equation solution in Figure 3, a graph of the *Mean Absolute Percentage Error* (MAPE) is presented when compared to its exact solution. This can be presented in Figure 4 as follows:

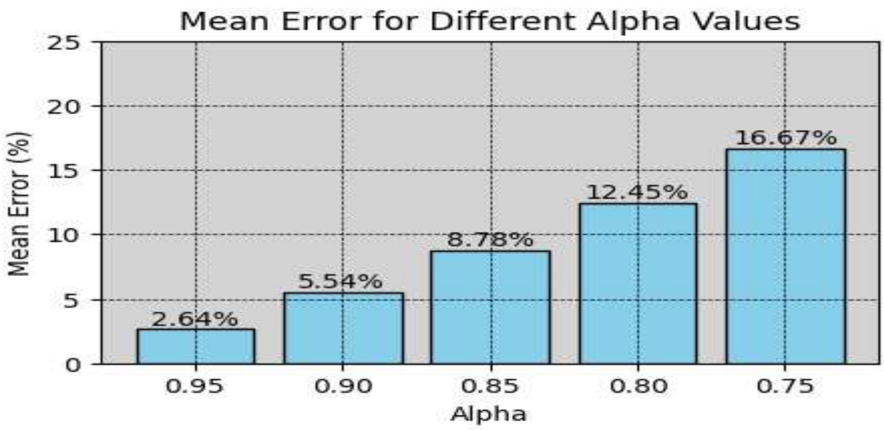

Figure 4. MAPE for α values when compared to the exact solution.

Based on Figure 4, it appears that a very good approximate solution is obtained when $\alpha = 0{,}95$, 0.90, 0.85. This is because the MAPE value obtained is 2,64% $<$ 10% (Moreno, 2013)

5. Simulation On Secondary Data of Human Development Index of Cilacap District

The following is a simulation of the solution of the Fractional Logistic equation applied to secondary data on the Human Development Index values of Cilacap Regency.

Research Data

The data used in this study is the Human Development Index data in Cilacap Regency for 2015-2023 obtained from the Central Statistics Agency of Cilacap Regency.

Table 1. Human Development Index Data in Cilacap Regency in 2015 – 2023

Year	Human Development Index
2015	67,77
2016	68,60
2017	68,90
2018	69,56
2019	69,95
2020	68,90
2021	70,42
2022	70,99
2023	71,83

Then, based on Table 1, the Human Development Index data of Cilacap district in 2015-2023 is approached by the Fractional Logistic equation solution. This can be seen in Figure 5.

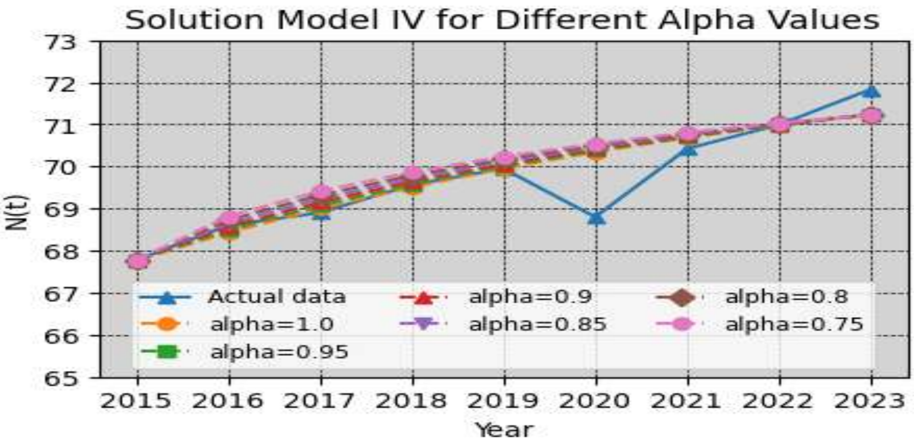

Figure 5. Primary Data of Human Development Index Value Approximated by Fractional Logistic Equation Solution

Based on Figure 5, the graph of the solution of the fractional Logistic equation is obtained for the values α = 1, 0,95, 0,90, 0,85, 0,80, 0,75. In comparison, the values were $N_0 = 67{,}77, K = 73, and\ r = 0{,}14273269$. It appears that in 2020, there was a significant decrease in the value of the Human Development Index. This is likely due to the large number of people affected by COVID-19 and the economic slowdown. **To get a good model overlay, it is necessary to vary the value of α.** Then, based on graph 5, the mean absolute error value is obtained, approximating the HDI data for Cilacap Regency in 2015-2023. This can be stated in Figure 6.

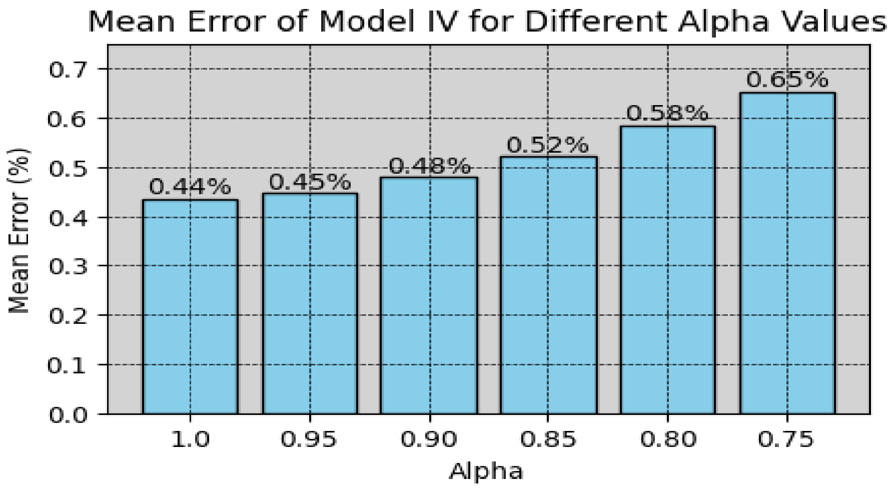

Figure 6. MAPE bar chart for each value of α

Based on Figure 6, it appears that a very good approximate solution from the Cilacap Regency Human Development Index data in 2015-2023 was obtained at values α= 0,95, 0,90, 0.85, 0.80 and 0.75. This is because the MAPE at the α value is less than 10%.

6. Prediction of Human Development Index In Cilacap Regency

To predict the HDI in Cilacap Regency with the logistic growth model, first assume the time (t) measured in years. Suppose $t = 0$ in 2015, then the initial requirement is $N(0)$=67.77. Next, determine the environmental carrying capacity value or K to be used. By taking the environmental carrying capacity value K=73, r=0.14273269 and α=1, the logistic growth model becomes**:**

$$N(t) = \frac{(73)(67{,}77)}{(73-67{,}77)e^{-rt}+67{,}77}$$

$$N(t) = \frac{4947{,}21}{5{,}23e^{-rt}+67{,}77}.$$

To predict the Human Development Index in Cilacap Regency in 2024 and 2025, take the value t=9 in 2024, then we get:

$$N(t) = \frac{4947{,}21}{5{,}23e^{-(0{,}14273269)9}+67{,}77}$$

$$N(t) = \frac{4947{,}21}{1{,}4474698+67{,}77}$$

$$N(t) = \frac{4947{,}21}{69{,}2174698}$$

$$N(t) \approx 71{,}47$$

Then, when t=10 in 2025, we obtain:

$$N(t) = \frac{4947{,}21}{5{,}23e^{-(0{,}14273269)10}+67{,}77}$$

$$N(t) = \frac{4947{,}21}{1{,}25493575+67{,}77}$$

$$N(t) = \frac{4947{,}21}{69{,}02493575}$$

$$N(t) \approx 71{,}67$$

So, the results of the calculation of the Human Development Index value in Cilacap Regency in 2024 were 71.47, while in 2025, it was 71.67.

CONCLUSION

Based on the results of the discussion, a mathematical model was obtained that can be used to predict the Human Development Index of Cilacap Regency in 2024 and 2025 and the Human Development Index data for 2015-2023 using the fractional logistic growth model. The best approximate solution obtained is when the value of α = 1, 0.95, 0.90, 0.85, 0.80 and 0.75. The predicted value of the Human Development Index of Cilacap Regency for 2024 is 71.47, and in 2025, it is 71.67.

Meanwhile, this paper is still limited to finding solutions for the fractional logistic growth model and its simulation. The issue of the solution's existence has not been addressed in this paper, making it a very interesting research gap.

FURTHER RESEARCH

Based on the results and discussion of this paper, several research topics that can be developed theoretically are the problem of the existence of solutions to the fractional logistic model, and then its numerical solution. Adam Basworth's method can be used as a reference for solving the fractional logistic model numerically.

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