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OPTIMAL CONTROL OF THE SPREAD OF COVID-19 IN JAKARTA

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ABSTRACT

SARS-CoV-2 is the virus that causes COVID-19. In Indonesia, the highest number of COVID-19 cases is in the Jakarta province. It is necessary to restrict the virus's transmission. This research purposes to determine optimal control strategies (self-prevention, vaccination, and cure) of the SEAIQHRD model to reduce disease spread. Optimal control analysis is solved utilising Pontryagin's Minimum Principle. In this study, numerical simulations were conducted using COVID-19 outbreak data from Jakarta province from March 1 to August 31, 2022. Based on the analysis results, the basic reproduction number $\mathfrak{R}_0$ = 2,1316. Since $\mathfrak{R}_0 > 1$ at the EE point, the COVID-19 spread model is asymptotically stable, indicating that the virus persists in the population. The application of control steps combining all three strategies was shown to reduce the subpopulations of exposed, infected, hospitalized, and deceased individuals. Simultaneous optimal control is more effective at controlling the spread than using a control step. The simultaneous implementation of optimal controls has proven an effective strategy for reducing COVID-19 transmission in Jakarta.

Keywords: COVID-19, SEAIQHRD Model, Optimal Control, Disease Spread, Pontryagin's Minimum Principle

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PRELIMINARY

A contagious illness that strikes the human respiratory system is COVID-19. The disease is transmitted from person to person and spreads rapidly, reaching countries worldwide. On March 12, 2020, the World Health Organisation (WHO) formally designated COVID-19 as a pandemic wherefore its high contagionability (Susilo et al., 2020). Some individuals infected with COVID-19 reveal various kinds of indication, from mild to severe. Mathematical models are frequently used across various fields of science, particularly in the health sector, especially for managing the COVID-19 pandemic.

As time has progressed, many previous studies have discussed models of COVID-19 transmission that incorporate control measures. These include the SEIR model (Winarni et al., 2024), which considers vaccination and treatment as control variables (Mahardika et al., 2021), and the STQIR model, which considers self-protection, treatment, and quarantine as control variables (Fitriani et al., 2022). The SEAIQHRD model, which considers the use of medical masks, hospitalization rates for symptomatic individuals, rapid testing for asymptomatic individuals, encouraging self-quarantine, and improving the quality of medical care to accelerate recovery rates, is a control variable (Dipo Aldila et al., 2020). The SEAIQHR model, which considers vaccination and treatment as control variables (Ghosh et al., 2020). The SEIAHRD model incorporates prevention and management strategies for hospitalized individuals and accelerated recovery as control variables (Olaniyi, 2020). The SEAIQHRD model incorporates antiviral therapy for symptomatic, hospitalized infected individuals, and isolation measures for hospitalized individuals (Masud et al., 2021). The SVITRD model was developed by adding vaccination as a control variable (Chasanah et al., 2025). The SEAIQHRD model was developed by modifying its control variables. The study (Masud et al., 2021) used antiviral therapy for clinically infected and hospitalized cases, and isolation for hospitalized patients. This study uses self-prevention, vaccination, and cure as control variables. Therefore, several new parameters are included in our proposed model, which is novel in this study.

This research aims to analyse to the stability of the SEAIQHRD model and determine the optimal control (self-prevention, vaccination, and cure). The method used to determine the optimal control measures is the Pontryagin Minimum Principle. This study uses data from DKI Jakarta province (March 1–August 31, 2022), sourced from the website corona.jakarta.go.id. The authors hope this research can provide optimal control strategies for local governments to reduce COVID-19 cases, particularly in high-case areas such as Jakarta.

METHODS

This study involves the following steps:

1. Conduct a literature review by analyzing articles, journals, and previous studies related to COVID-19 epidemic models and optimal control measures in order to gain insights and establish a conceptual framework through these references.
2. The initial model formulation (Masud et al., 2021) is as follows:

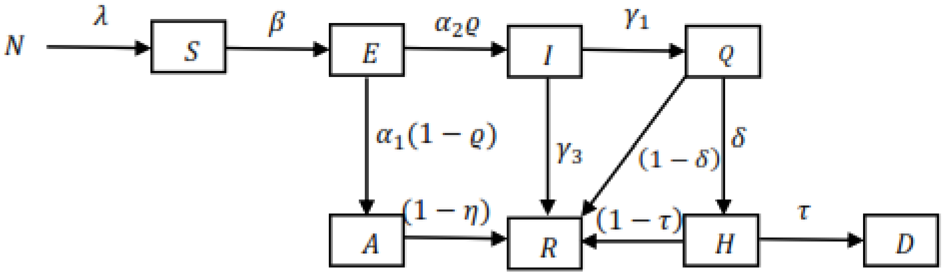

Figure 1. Diagram of the Transfer of the Spread of COVID-19 Disease.

The initial model formulation of the transmission of COVID-19 consists of 8 subpopulations, namely the susceptible (S) exposed (E), infected without symptoms (A), clinically ill and infections (I), quarantine (Q), hospitalized (H), recovered (R), and deceased (D). Based on Figure 1 (Masud et al., 2021), it is known that N is not a variable but the population size.

Furthermore, the mathematical model formulation of the transmission of COVID-19 was developed by adding several transmission shifts, adding and removing certain parameters, and modifying control variables. The following diagram illustrates the transmission pathways from the model development for COVID-19:

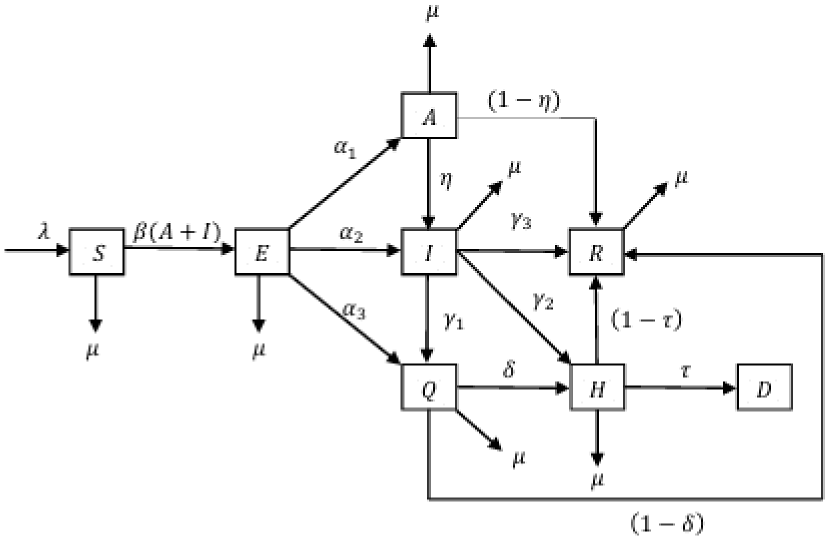

Figure 2. Modified COVID-19 Transmission Diagram

The following is a modified $SEAIQHRD$ dynamic model of COVID-19 transmission:

$$\frac{dS}{dt} = \lambda - \beta S(A+I) - \mu S$$

$$\frac{dE}{dt} = \beta S(A+I) - (\alpha_1 + \alpha_2 + \alpha_3)E - \mu E$$

$$\frac{dA}{dt} = \alpha_1 E - \eta A - (1-\eta)A - \mu A$$

$$
\begin{aligned}
\frac{dI}{dt} &= \alpha_2 E + \eta A - (\gamma_1 + \gamma_2 + \gamma_3) I - \mu I \\
\frac{dQ}{dt} &= \alpha_3 E + \gamma_1 I - \delta Q - (1-\delta) Q - \mu Q \\
\frac{dH}{dt} &= \gamma_2 I + \delta Q - \tau H - (1-\tau) H - \mu H \\
\frac{dR}{dt} &= (1-\eta) A + \gamma_3 I + (1-\delta) Q + (1-\tau) H - \mu R \\
\frac{dD}{dt} &= \tau H
\end{aligned} \tag{1}
$$

This research represent a mathematical model of COVID-19 transmission involving eight subpopulations: susceptible (Susceptible (S)), exposed (Exposed (E)), asymptomatic infected (Asymptomatic (A)), symptomatic infected (Symptomatic (I)), quarantine (Quarantine (Q)), hospitalized (Hospitalized (H)), recovered (Recovery (R)), and deceased subpopulation (Deceased (D)).

3. Model Analysis (Ariyani et al., 2023).
    a. Positive and limited solution analysis.
    b. Determining the equilibrium point of the COVID-19 spread model. It is the EE and DFE points.
    c. Determining the basic reproduction number utilises the NGM method.
    d. Investigating the stability of the equilibrium point using the Routh-Hurwitz criterion and Lyapunov method (Ariyani et al., 2023; Sundari et al., 2017; Goh et al., 1977).
4. Formularization of the optimal control problem. In the initial $SEAIQHRD$ model formulated, optimal control is applied using three control measures: self-prevention control to minimise the exposed subpopulation, vaccination control to minimise the asymptomatic and symptomatic infected subpopulations, and cure control to accelerate recovery. In the formulation stage of the optimal control problem, the initial steps are:
    a. Determining the objective function.

    Given the objective function of (Masud et al., 2021) model is

$$\mathcal{J}(u_a, u_b, u_c) = min \int_0^T W_1 I + W_2 Q + W_3 H + \frac{q_1}{2} u_a^2 + \frac{q_2}{2} u_b^2 + \frac{q_3}{2} u_c^2,$$

    Then, given the objective function in this study is

$$J(u_1, u_2, u_3) = \int_0^{Tf} \left[W_1 E + W_2 A + W_3 I + \frac{1}{2}(q_1 u_1^2 + q_2 u_2^2 + q_3 u_3^2)\right] dt \tag{2}$$

b. Determine the constraint function.

The constraint function of (Masud et al., 2021) model can be seen in (Masud et al., 2021). Then, the constraint function in this study is

$$\frac{dS}{dt} = \lambda - \beta SA(1-u_1) - \beta SI(1-u_1) - (\mu + u_2)S$$
$$\frac{dE}{dt} = \beta SA(1-u_1) + \beta SI(1-u_1) - (\alpha_1 + \alpha_2 + \alpha_3)E - \mu E$$
$$\frac{dA}{dt} = \alpha_1 E - \eta A - (1-\eta)A - \mu A$$
$$\frac{dI}{dt} = \alpha_2 E + \eta A - \big(\gamma_1 + \gamma_2 + (\gamma_3 + u_3)\big)I - \mu I$$
$$\frac{dQ}{dt} = \alpha_3 E + \gamma_1 I - \delta Q - (1-\delta)Q - \mu Q$$
$$\frac{dH}{dt} = \gamma_2 I + \delta Q - \tau H - (1 - \tau + u_3)H - \mu H$$
$$\frac{dR}{dt} = (1-\eta)A + (\gamma_3 + u_3)I + (1-\delta)Q + (1-\tau+u_3)H - \mu R + u_2 S$$
$$\frac{dD}{dt} = \tau H \qquad (3)$$

with the initial conditions

$$S(0) > 0, E(0) > 0, A(0) > 0, I(0) > 0, Q(0) > 0, H(0) > 0, R(0) > 0, D(0) > 0$$

Determine whether the controls u_1^*, u_2^*, u_3^* are valid.

$$J(u_1^*, u_2^*, u_3^*) = min\{J(u_1, u_2, u_3), u_1, u_2, u_3 \in U\}$$

with $\coloneqq \{u_1, u_2, u_3 | 0 \le u_i(t) \le 1, i = 1,2,3, t \in (0, T)\}$

c. Solving optimal control problems utilising Pontryagin's Maximum Principle (PMP).

5. Conducting a Case Study. To illustrate the dynamics of COVID-19 transmission based on a modified model using data from the Jakarta province available on the official website corona.jakarta.go.id, from March 1, 2022, to August 31, 2022. Parameter fitting was then performed using the Least Squares Method (Daniya et al., 2020; Nakamura et al., 2006; Cao et al., 2012). This case study was conducted to obtain a simulation depicting the dynamics of COVID-19 dispersion. Using MATLAB, graphs of COVID-19 transmission dynamics were generated, both with and without control measures such as self-prevention, vaccination, and cure.

RESULT AND DISCUSSION

In this section, we modify the dynamic model using control measures tailored to the pandemic conditions in the Jakarta province. A human population consists of eight classes: susceptible (S), exposed (E), asymptomatic infected (A), symptomatic infected (I), quarantined (Q), hospitalized (H), recovered (R), and deceased subpopulation (D). Given the following dynamic model without control as contained in equation (II.1).

Since the variables R and D in equations $\frac{dR}{dt}$ and $\frac{dD}{dt}$ do not affect the results of the analysis of equations $\frac{dS}{dt}$ through $\frac{dH}{dt}$, they can be eliminated from the subsequent analysis, leaving only $S, E, A, I, Q,$ and H (Musafir et al., 2021; Fitriani et al., 2021). This yields the following equation:

$$\begin{aligned}
\frac{dS}{dt} &= \lambda - \beta S(A+I) - \mu S\\
\frac{dE}{dt} &= \beta S(A+I) - (\alpha_1+\alpha_2+\alpha_3)E - \mu E\\
\frac{dA}{dt} &= \alpha_1 E - \eta A - (1-\eta)A - \mu A\\
\frac{dI}{dt} &= \alpha_2 E + \eta A - (\gamma_1+\gamma_2+\gamma_3)I - \mu I\\
\frac{dQ}{dt} &= \alpha_3 E + \gamma_1 I - \delta Q - (1-\delta)Q - \mu Q\\
\frac{dH}{dt} &= \gamma_2 I + \delta Q - \tau H - (1-\tau)H - \mu H
\end{aligned} \qquad (4)$$

with initial state (S(0) until H(0) ≥ 0).

Basic Reproduction Number

The basic reproduction number is the average number of new cases caused by an individual capable of transmitting a disease. The Next Generation Matrix (NGM) method is used to determine the basic reproduction number.

For example,

$$\frac{dx}{dt} = F(x) - V(x),$$

With,

$$x = [E, A, I, Q, H]^T$$

$$\frac{dE}{dt} = \beta S(A+I) - (\alpha_1+\alpha_2+\alpha_3)E - \mu E$$

Do the same for $\frac{dA}{dt}, \frac{dI}{dt}, \frac{dQ}{dt}$

$$\frac{dH}{dt} = \gamma_2 I + \delta Q - \tau H - (1-\tau)H - \mu H$$

Where

$$F(x)=\begin{bmatrix}F_1\\F_2\\F_3\\F_4\\F_5\end{bmatrix}=\begin{bmatrix}\beta S(A+I)\\0\\0\\0\\0\end{bmatrix},V(x)=\begin{bmatrix}V_1\\V_2\\V_3\\V_4\\V_5\end{bmatrix}=\begin{bmatrix}(\alpha_1+\alpha_2+\alpha_3)E+\mu E\\-\alpha_1 E+(1+\mu)A\\-\alpha_2 E-\eta A+(\gamma_1+\gamma_2+\gamma_3)I+\mu I\\-\alpha_3 E-\gamma_1 I+(1+\mu)Q\\-\gamma_2 I-\delta Q+(1+\mu)H\end{bmatrix},$$

Suppose $\mathcal{F}$ and $\mathcal{V}$ are the Jacobian matrices of $F(x)$ and $V(x)$ respectively at the DFE point $\mathcal{E}_0(S_0,E_0,A_0,I_0,Q_0,H_0)=\left(\frac{\lambda}{\mu},0,0,0,0,0\right)$ (Ariyani et al., 2023). Next, to obtain the basic reproduction number, the largest eigenvalue of the NGM matrix is sought as follows: $NGM=\mathcal{F}(\mathcal{E}_0).\mathcal{V}^{-1}$.

$$\mathrm{F}\left(\mathrm{E}_0\right)=\begin{bmatrix}0&\frac{\beta\lambda}{\mu}&\frac{\beta\lambda}{\mu}&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\end{bmatrix},\quad \mathcal{V}^{-1}=\begin{bmatrix}v_{11}&v_{12}&v_{13}&v_{14}&v_{15}\\v_{21}&v_{22}&v_{23}&v_{24}&v_{25}\\v_{31}&v_{32}&v_{33}&v_{34}&v_{35}\\v_{41}&v_{42}&v_{43}&v_{44}&v_{45}\\v_{51}&v_{52}&v_{53}&v_{54}&v_{55}\end{bmatrix},$$

Then the basic reproduction number obtained in model (III.1) is as follows:

$$\Re_0=\frac{\beta\lambda\big((\eta+\mu+\gamma_1+\gamma_2+\gamma_3)\alpha_1+(1+\mu)\alpha_2\big)}{\mu(1+\mu)(\gamma_1+\gamma_2+\gamma_3+\mu)(\alpha_1+\alpha_2+\alpha_3+\mu)} \qquad (5)$$

Dynamic Model of COVID-19 Transmission with Control Measures

Then, a modified dynamic model with control is given as follows:

$$\begin{aligned}
\frac{dS}{dt}&=\lambda-\beta SA(1-u_1)-\beta SI(1-u_1)-(\mu+u_2)S\\
\frac{dE}{dt}&=\beta SA(1-u_1)+\beta SI(1-u_1)-(\alpha_1+\alpha_2+\alpha_3)E-\mu E\\
\frac{dA}{dt}&=\alpha_1 E-\eta A-(1-\eta)A-\mu A\\
\frac{dI}{dt}&=\alpha_2 E+\eta A-\big(\gamma_1+\gamma_2+(\gamma_3+u_3)\big)I-\mu I\\
\frac{dQ}{dt}&=\alpha_3 E+\gamma_1 I-\delta Q-(1-\delta)Q-\mu Q\\
\frac{dH}{dt}&=\gamma_2 I+\delta Q-\tau H-\big((1-\tau)+u_3\big)H-\mu H\\
\frac{dR}{dt}&=(1-\eta)A+(\gamma_3+u_3)I+(1-\delta)Q+\big((1-\tau)+u_3\big)H-\mu R+u_2 S\\
\frac{dD}{dt}&=\tau H
\end{aligned} \qquad (6)$$

where u_1 is the control variable for self-prevention, u_2 is the control variable for vaccination, and u_3 is the control variable for cure.

Formulation of the Optimal Control Model

In the initial SEAIQHRD model, optimal control is implemented through three control variables: self-prevention control by using masks (u_1) to minimise the exposed subpopulation, vaccination control (u_2) to minimise the subpopulation of asymptomatic and symptomatic infected persons, and cure control (u_3) to accelerate recovery.

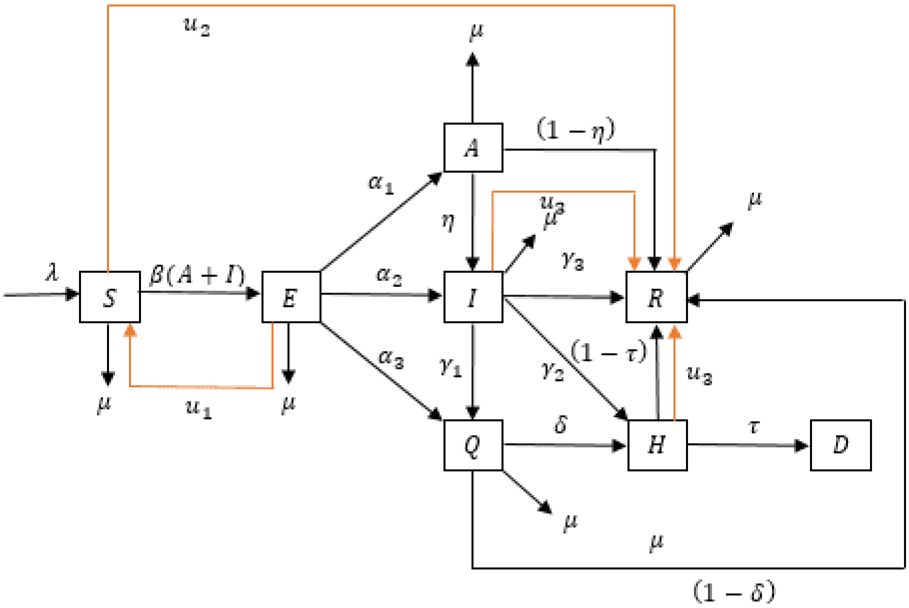

Figure 3. Schematic of COVID-19 Spread with Control Measures.

Objective Function

The optimal control problem goals to minimise the number of subpopulations exposed to and infected with asymptomatic and symptomatic COVID-19, accelerate recovery, and keep the costs of implementing control measures as low as possible. The time variable (t) in $u_1(t), u_2(t), u_3(t)$ is omitted to simplify the notation, resulting in u_1, u_2, u_3. The objective function of this optimal control can be written in the following form:

$$\mathcal{J}(u_1, u_2, u_3) = min \int_0^{Tf} \left[W_1 E + W_2 A + W_3 I + W_4 H + \frac{1}{2}(q_1 u_1^2 + q_2 u_2^2 + q_3 u_3^2)\right] dt \quad \text{(III.4)}$$

with

W_1= The relative weight of the exposed subpopulation.

$W_1 E$ = cost function associated with the exposed subpopulation.

W_2= The relative weight of the asymptomatic infected subpopulation.

$W_2 A$ = cost function associated with the asymptomatic infected subpopulation.

W_3= The relative weight of the symptomatic infected subpopulation.

$W_3 I$ = cost function associated with the symptomatic infected subpopulation.

W_4= The relative weight of the hospitalized subpopulation.

$W_4 H$ = cost function associated with the hospitalized subpopulation.

q_1= relative weight associated with self-prevention costs.

$q_1 u_1^2$ = cost function for self-prevention.

q_2= relative weight associated with vaccination costs.	$q_2u_2^2$ = cost function for vaccination.
q_3 = relative weight associated with cure costs.	$q_3u_3^2$ = cost function for cure.

The first component of $\mathcal{J}$, is $\int_0^{T_f}[W_1E + W_2A + W_3I + W_4H]\,dt$, representing the costs associated with the size of the subpopulation in the field. These costs are not related to control variables, but rather to economic costs. The second component is $\int_0^{T_f}\left[\frac{1}{2}(q_1u_1^2 + q_2u_2^2 + q_3u_3^2)\right]dt$ representing the costs associated with implementing steps to restrain the dispersion of COVID-19.

Constraint Function

Build upon Figure 3, the constraint function of the COVID-19 spread model with control measures is as follows :

Given equation (III.3) and the initial conditions :

$$S(0) \geq 0, E(0) \geq 0, A(0) \geq 0, I(0) \geq 0, Q(0) \geq 0, H(0) \geq 0, R(0) \geq 0, D(0) \geq 0.$$

Suppose the optimal control functions in equations (III.3) and (III.4) are u_1^*, u_2^*, u_3^*, then the following holds :

$$\mathcal{J}(u_1^*, u_2^*, u_3^*) = min\{J(u_1, u_2, u_3), u_1, u_2, u_3 \in U\} \tag{7}$$

with $U := \{u_1, u_2, u_3 | 0 \leq u_i(t) \leq 1, i = 1,2,3, t \in (0,T)\}$.

By reformulating equation (III.3) as an optimal control problem with a constraint function, we obtain:

$$(P_c)\begin{cases} Minimize\ \mathcal{J}(x,u) = \int_0^T L\big(x(t),u(t)\big)dt, \\ constraint, \\ \dot{x}(t) = f\big(x(t)\big) + g\big(x(t)\big)u(t), \forall t \in [0,T], \\ u(t) \in U(t), \forall t \in [0,T], \\ x(0) = x_0, \end{cases} \tag{8}$$

where

$$x(t) = \begin{bmatrix} S(t) \\ E(t) \\ A(t) \\ I(t) \\ Q(t) \\ H(t) \\ R(t) \\ D(t) \end{bmatrix}, f\big(x(t)\big) = \begin{bmatrix} \lambda - \beta S(A+I) - \mu S \\ \beta S(A+I) - (\alpha_1 + \alpha_2 + \alpha_3)E - \mu E \\ \alpha_1 E - \eta A - (1-\eta)A - \mu A \\ \alpha_2 E + \eta A - (\gamma_1 + \gamma_2 + \gamma_3)I - \mu I \\ \alpha_3 E + \gamma_1 I - \delta Q - (1-\delta)Q - \mu Q \\ \gamma_2 I + \delta Q - \tau H - (1-\tau)H - \mu H \\ (1-\eta)A + \gamma_3 I + (1-\delta)Q + (1-\tau)H - \mu R \\ \tau H \end{bmatrix},$$

$$g(x(t)) = \begin{bmatrix} \beta S(t)A(t) + \beta S(t)I(t) & -S(t) & 0 \\ -\beta S(t)A(t) - \beta S(t)I(t) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -I(t) \\ 0 & 0 & 0 \\ 0 & 0 & -H(t) \\ 0 & S(t) & I(t) + H(t) \\ 0 & 0 & 0 \end{bmatrix}, u(t) = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix},$$

and the integral of the objective function may be drawed up in the subsequent form:

$$L(x,u) = W_1 E + W_2 A + W_3 I + W_4 H + \frac{1}{2}(q_1 u_1^2 + q_2 u_2^2 + q_3 u_3^2).$$

Optimal Control Solution

Before determining optimal control, the Hamiltonian function is first derived using PMP. The application of PMP aims to select the optimal levels of self-prevention through mask-wearing, vaccination, and cure. For example, if $x = (S, E, A, I, Q, H, R, D)$ is a state variable and $u = (u_1, u_2, u_3)$ is a control variable, then the following Hamiltonian function is obtained:

$$H(x, u, \Lambda) = L(x,u) + \Lambda g(x,u)$$

Where

$$L(x,u) + W_1 E + W_2 A + W_3 I + W_4 H + \frac{1}{2}(q_1 u_1^2 + q_2 u_2^2 + q_3 u_3^2)$$

$$\Lambda = (\Lambda_1, \Lambda_2, \Lambda_3, \Lambda_4, \Lambda_5, \Lambda_6, \Lambda_7, \Lambda_8)$$

$$g(x,u) = \big(g_1(x,u), g_2(x,u), g_3(x,u), g_4(x,u), g_5(x,u), g_6(x,u), g_7(x,u), g_8(x,u)\big)$$

With

$$g_1(x,u) = \lambda - \beta SA(1 - u_1) - \beta SI(1 - u_1) - (\mu + u_2)S$$

Do the same for $g_2, g_3, g_4, g_5, g_6, g_7$ until

$$g_8 = \tau H$$

The Langrangian and Hamiltonian functions are useful for finding optimal solutions to the optimal control problems (III.3) and (III.4). The Langrangian function in the optimal control problem can be written as:

$$L = W_1 E + W_2 A + W_3 I + W_4 H + \frac{1}{2}\sum_{k=1}^{3} q_k u_k^2$$

In obtaining the minimum value of the Lagrange function, the Hamiltonian function in the system can be defined as follows:

$$\mathcal{H}(S, E, A, I, Q, H, R, D, u_k, \phi) = W_j + \frac{1}{2} q_k u_k^2 + \sum_{i=1}^{8} \Lambda_i f_i$$

Then the Hamiltonian equation is obtained as follows:

$$
\begin{aligned}
\mathcal{H} = {} & W_1E + W_2A + W_3I + W_4H + \frac{1}{2}(q_1u_1^2 + q_2u_2^2 + q_3u_3^2) \\
& + \Lambda_1(\lambda - \beta SA(1-u_1) - \beta SI(1-u_1) - \mu S - u_2S) \\
& + \Lambda_2(\beta SA(1-u_1) + \beta SI(1-u_1) - (\alpha_1 + \alpha_2 + \alpha_3)E - \mu E) \\
& + \Lambda_3(\alpha_1E - \eta A - (1-\eta)A - \mu A) \\
& + \Lambda_4(\alpha_2E + \eta A - (\gamma_1 + \gamma_2 + \gamma_3)I - u_3I - \mu I) \\
& + \Lambda_5(\alpha_3E + \gamma_1I - \delta Q - (1-\delta)Q - \mu Q) \\
& + \Lambda_6(\gamma_2I + \delta Q - \tau H - (1-\tau)H - u_3H - \mu H) \\
& + \Lambda_7((1-\eta)A + \gamma_3I + u_3I + (1-\delta)Q + (1-\tau)H + u_3H + u_2S - \mu R) \\
& + \Lambda_8(\tau H)
\end{aligned} \tag{9}
$$

with $\Lambda_i, i \in \{1,2,3,4,5,6,7,8\}$ is an adjoint variable (state variable). The system is obtained by taking the partial derivatives of the Hamiltonian equation (III.7) with respect to each state variable.

Theorem 3.3

There exist optimal controls u_1^*, u_2^*, u_3^* and solutions S, E, A, I, Q, H, R, D of the system of equations (III.3) and (III.4) that minimize $J(u_1, u_2, u_3)$ over U. Thus, there also exist adjoint variables (costate) Λ_1 until Λ_8 that satisfy the following :

$$\frac{d\Lambda_i}{dt} = -\frac{\partial \mathcal{H}}{\partial k} \tag{10}$$

with $i = 1,2,3,4,5,6,7,8, k = S, E, A, I, Q, H, R, D$, where the condition of transversality

$$\Lambda_1(T_f) = \Lambda_2(T_f) = \Lambda_3(T_f) = \Lambda_4(T_f) = \Lambda_5(T_f) = \Lambda_6(T_f) = \Lambda_7(T_f) = \Lambda_8(T_f) = 0 \quad \text{(III.9)}$$

and the optimal control variables u_1^*, u_2^*, u_3^* satisfy the following optimality conditions:

$$
\begin{aligned}
u_1^* &= max\left\{0, min\left(1, \frac{1}{q_1}(\beta SA(\Lambda_2 - \Lambda_1) + \beta SI(\Lambda_2 - \Lambda_1))\right)\right\} \\
u_2^* &= max\left\{0, min\left(1, \frac{1}{q_2}(S(\Lambda_1 - \Lambda_7))\right)\right\} \\
u_3^* &= max\left\{0, min\left(1, \frac{1}{q_3}(I(\Lambda_4 - \Lambda_7) + H(\Lambda_6 - \Lambda_7))\right)\right\}
\end{aligned} \tag{11}
$$

Proof :

The Hamiltonian function is useful for obtaining adjoint (costate) variables, so the adjoint (costate) equation can be written as:

$$
\begin{aligned}
\frac{d\Lambda_1}{dt} = -\frac{\partial \mathcal{H}}{\partial S} &= \Lambda_1(\beta A(1-u_1) + \beta I(1-u_1) + \mu + u_2) \\
&\quad - \Lambda_2(\beta A(1-u_1) + \beta I(1-u_1)) - \Lambda_7(u_2). \\
\frac{d\Lambda_2}{dt} = -\frac{\partial \mathcal{H}}{\partial E} &= -W_1 + \Lambda_2(\alpha_1 + \alpha_2 + \alpha_3 + \mu) - \Lambda_3(\alpha_1) - \Lambda_4(\alpha_2) - \Lambda_5(\alpha_3).
\end{aligned}
$$

Do the same thing until

$$\frac{d\Lambda_8}{dt} = -\frac{\partial \mathcal{H}}{\partial D} = 0 \quad (12)$$

Where the transversality condition $\Lambda_i(T_f) = 0$ with $i = 1,2,3,4,5,6,7,8$. The optimality condition is given by :

$$\frac{\partial \mathcal{H}}{\partial u_1} = 0$$

$$q_1 u_1 + \beta SA(\Lambda_1 - \Lambda_2) + \beta SI(\Lambda_1 - \Lambda_2) = 0 \leftrightarrow u_1 = \frac{1}{q_1}\big(\beta SA(\Lambda_2 - \Lambda_1) + \beta SI(\Lambda_2 - \Lambda_1)\big).$$

$$\frac{\partial \mathcal{H}}{\partial u_2} = 0,$$

$$q_2 u_2 - S(\Lambda_1 - \Lambda_7) = 0 \leftrightarrow u_2 = \frac{1}{q_2}\big(S(\Lambda_1 - \Lambda_7)\big).$$

$$\frac{\partial \mathcal{H}}{\partial u_3} = 0,$$

$$q_3 u_3 - I(\Lambda_4 - \Lambda_7) - H(\Lambda_6 - \Lambda_7) = 0 \leftrightarrow u_3 = \frac{1}{q_3}\big(I(\Lambda_4 - \Lambda_7) + H(\Lambda_6 - \Lambda_7)\big).$$

Then the optimal control u_1^*, u_2^*, u_3^* is obtained as follows:

$$u_1^* = \begin{cases} 0, if\ u_1 \le 0, \\ u_1, if\ 0 < u_1 < 1, \\ 1, if\ u_1 \ge 1, \end{cases} \leftrightarrow u_1^* = max\left\{0, min\left(1, \frac{1}{q_1}\big(\beta SA(\Lambda_2 - \Lambda_1) + \beta SI(\Lambda_2 - \Lambda_1)\big)\right)\right\}.$$

$$u_2^* = \begin{cases} 0, if\ u_2 \le 0, \\ u_2, if\ 0 < u_2 < 1, \\ 1, if\ u_2 \ge 1, \end{cases} \leftrightarrow u_2^* = max\left\{0, min\left(1, \frac{1}{q_2}\big(S(\Lambda_1 - \Lambda_7)\big)\right)\right\}.$$

$$u_3^* = \begin{cases} 0, if\ u_3 \le 0, \\ u_3, if\ 0 < u_3 < 1, \\ 1, if\ u_3 \ge 1, \end{cases} \leftrightarrow u_3^* = max\left\{0, min\left(1, \frac{1}{q_3}\big(I(\Lambda_4 - \Lambda_7) + H(\Lambda_6 - \Lambda_7)\big)\right)\right\}. \quad (13). \blacksquare$$

Building an Optimal System

The optimal system is obtained by substituting the feasible control results (Sadiq et al., 2013), namely equation (III.12) into equation (III.3) so that the optimal system is obtained as follows:

$$\begin{aligned} \frac{dS}{dt} &= \lambda - \beta SA(1 - u_1) - \beta SI(1 - u_1) - (\mu + u_2)S, \\ &= \lambda - \beta SA + u_1^*\beta SA - \beta SI + u_1^*\beta SI - \mu S - u_2^* S, \\ &= \lambda - \beta SA + \left(min\left\{u_{1max}, max\left(0, \frac{1}{q_1}(\beta SA(\Lambda_2 - \Lambda_1) + \beta SI(\Lambda_2 - \Lambda_1))\right)\right\}\right)\beta SA \\ &\qquad - \beta SI \\ &\qquad + \left(min\left\{u_{1max}, max\left(0, \frac{1}{q_1}(\beta SA(\Lambda_2 - \Lambda_1) \right.\right.\right. \\ &\qquad \left.\left.\left. + \beta SI(\Lambda_2 - \Lambda_1))\right)\right\}\right)\beta SI - \mu S \\ &\qquad - \left(min\left\{u_{2max}, max\left(0, \frac{1}{q_2}(S(\Lambda_1 - \Lambda_7))\right)\right\}\right)S. \end{aligned}$$

Do the same for $\frac{dE}{dt}, \frac{dA}{dt}, \frac{dI}{dt}, \frac{dQ}{dt}, \frac{dH}{dt}, \frac{dR}{dt}$ until

$$\frac{dD}{dt} = \tau H$$

with the following boundary conditions

$S(0) = S_0$ until $D(0) = D_0$

The adjoint equation formed is

$$\begin{aligned}
\frac{d\Lambda_1}{dt} &= \Lambda_1(\beta A(1-u_1) + \beta I(1-u_1) + \mu + u_2) \\
&\qquad - \Lambda_2(\beta A(1-u_1) + \beta I(1-u_1)) - \Lambda_7(u_2), \\
&= \Lambda_1(\beta A - u_1^*\beta A + \beta I - u_1^*\beta I + \mu + u_2^*) \\
&\qquad - \Lambda_2(\beta A - u_1^*\beta A + \beta I - u_1^*\beta I) - \Lambda_7(u_2^*), \\
&= \Lambda_1\Biggl(\beta A - \biggl(min\biggl\{u_{1max}, max\biggl(0, \frac{1}{q_1}(\beta SA(\Lambda_2-\Lambda_1) + \beta SI(\Lambda_2-\Lambda_1))\biggr)\biggr\}\biggr)\beta A + \beta I \\
&\qquad - \biggl(min\biggl\{u_{1max}, max\biggl(0, \frac{1}{q_1}(\beta SA(\Lambda_2-\Lambda_1) + \beta SI(\Lambda_2-\Lambda_1))\biggr)\biggr\}\biggr)\beta I + \mu \\
&\qquad + \biggl(min\biggl\{u_{2max}, max\biggl(0, \frac{1}{q_2}(S(\Lambda_1-\Lambda_7))\biggr)\biggr\}\biggr)\Biggr) \\
&\qquad - \Lambda_2\biggl(\beta A - \biggl(min\biggl\{u_{1max}, max\biggl(0, \frac{1}{q_1}(\beta SA(\Lambda_2-\Lambda_1) + \beta SI(\Lambda_2-\Lambda_1))\biggr)\biggr\}\biggr)\beta A \\
&\qquad + \beta I - \biggl(min\biggl\{u_{1max}, max\biggl(0, \frac{1}{q_1}(\beta SA(\Lambda_2-\Lambda_1) + \beta SI(\Lambda_2-\Lambda_1))\biggr)\biggr\}\biggr)\beta I\biggr) \\
&\qquad - \Lambda_7\biggl(min\biggl\{u_{2max}, max\biggl(0, \frac{1}{q_2}(S(\Lambda_1-\Lambda_7))\biggr)\biggr\}\biggr).
\end{aligned}$$

Do the same for $\frac{d\Lambda_2}{dt}, \frac{d\Lambda_3}{dt}, \frac{d\Lambda_4}{dt}, \frac{d\Lambda_5}{dt}, \frac{d\Lambda_6}{dt}, \frac{d\Lambda_7}{dt}$ until

$$\frac{d\Lambda_8}{dt} = 0$$

with the following boundary conditions:

$\Lambda_1(T_f) = 0$ until $\Lambda_8(T_f) = 0,$

This means that the final state is free, based on the Hamiltonian function $g(x(t)) \neq 0,$ resulting in $\Lambda_i(T_f) = 0, i = 1,2,3,4,5,6,7,8,$ where $\Lambda_1, \Lambda_2, \Lambda_3, \Lambda_4, \Lambda_5, \Lambda_6, \Lambda_7, \Lambda_8$ are adjoint/costate variables for each equation $\frac{dS}{dt}, \frac{dE}{dt}, \frac{dA}{dt}, \frac{dI}{dt}, \frac{dQ}{dt}, \frac{dH}{dt}, \frac{dR}{dt}, \frac{dD}{dt}.$

Numerical Simulation

A numerical simulation was conducted to model the transmission of COVID-19 within a resident by comparing the system before and after the enforcement of control steps. The COVID-19 disease spread simulation was conducted in MATLAB. The data were obtained through https://corona.jakarta.go.id/id/data-pemantauan from March 1 to August 31, 2022, with a total population in DKI Jakarta of 11,476,978 and a vulnerable population (ages 15–59) of 7,191,233 (https://jakarta.bps.go.id/), as well as from international journals.

The model parameter values were derived from COVID-19 data in Jakarta province using the Least Squares Method (Daniya et al., 2020; Nakamura et al., 2006; Cao et al., 2012). Based on the distribution of COVID-19 cases in the province, the initial variable values for the SEAIQHRD model were obtained, as presented in Table 1 below :

Table 1. Initial Value Data for Variables from the SEAIQHRD Model

Variable	Description	Initial value	Unit
N	Total population	11.476.978	Individual
$S(0)$	Initial values of susceptible individuals	7.191.233	Individual
$E(0)$	Initial values of exposed individuals	1.928.799	Individual
$A(0)$	Initial values of asymptomatic infected individuals	1.091.179	Individual
$I(0)$	Initial values of symptomatic infected individuals	87.294	Individual
$Q(0)$	Initial value of the quarantined individuals	32.698	Individual
$H(0)$	Initial values of hospitalized individuals	5.077	Individual
$R(0)$	Initial value of recovered individuals	1.126.008	Individual
$D(0)$	Initial value of deceased individuals	14.690	Individual

Table 2. Parameter Values from the SEAIQHRD Model

Parameter	Description	Unit	Value	Reference
λ	recruitment rate of susceptible subpopulations	People per day	$449,19679$	*Estimated*
μ	natural mortality rate	Per day	$3,91389 \times 10^{-5}$	*Estimated*
β	rate of effective interaction	Per person per day	$1,80142 \times 10^{-7}$	*Estimated*
α_1	rate at which the exposed subpopulation becomes the asymptomatic infected resident	Per day	$0,20441$	*Estimated*
α_2	rate at which an exposed resident becomes a symptomatic infected resident	Per day	$1,07550$	*Estimated*
α_3	rate at which exposed transitions to the quarantined resident	Per day	$0,00920715$	*Estimated*
η	rate at which asymptomatic infected people become symptomatic infected people	Per day	$0,32160$	*Estimated*
$1-\eta$	rate at which the asymptomatic infected transitions to the recovered subpopulation	Per day	$0,6784$	*Estimated*
γ_1	rate at which the symptomatic infected transitions to the quarantined resident	Per day	$0,000637620$	*Estimated*
γ_2	rate at which symptomatic infected individuals progress to	Per day	$0,00177894$	*Estimated*

	requiring hospitalization			
γ_3	recovery rate of the symptomatic infected subpopulation	Per day	1,01218	*Estimated*
δ	transmission rate from the quarantine to the hospitalized subpopulation	Per day	0,00371323	*Estimated*
$1-\delta$	The rate of transmission from the quarantined to the recovered subpopulation	Per day	0,99628677	*Estimated*
τ	mortality rate of the hospitalized subpopulation	Per day	0,35255	*Estimated*
$1-\tau$	transmission rate from the hospitalized to the recovered subpopulation	Per day	0,64745	*Estimated*

Based on Table 2, the parameter values λ until 1-τ were obtained from COVID-19 data in Jakarta province with the MATLAB software using the Least Squares Method. Next, by substituting the parameter values inside the basic reproduction number in equation (III.2), we obtain $\Re_0 = 2{,}1316$. This means that each infected individual can transmit the virus to two susceptible individuals. This indicates lest the model owns an EE point. By the theorem the EE point (Sundari et al., 2017; Marquez et al., 2003; Sastry et al., 1999; Goh et al., 1977) is globally asymptotically stable because $\Re_0 = 2{,}1316 > 1$. This suggests that infected subpopulations-whether asymptomatic or symptomatic-are capable of transmitting the virus to susceptible subpopulations, thereby allowing COVID-19 to persist within a population over an extended period of time. Table 3 below shows the differences in the number of people for each variable appeal to the initial condition, without control, and with control on day 7, as follows:

Table 3. Differences in the Number of Individuals for Each Variable Compared to the Initial Condition, Without Control, and With Control on Day 7.

Variable ($\times 10^3$)	**Initial Conditions** $t = 0$	**Without Control** $t = 7$	**With Control** $t = 7$	**Difference**	
				Individual	(%)
Susceptible (S)	7191233	10330000	5071000	↓ 5259000	↓ 50,91
Exposed (E)	1928799	25370	239	↓ 25131	↓ 99,06
Asymptomatic Infection (A)	1091179	2749	2077	↓ 672	↓ 24,45
Infected with Symptoms (I)	87294	9201	1354	↓ 7847	↓ 85,28
Quarantine(Q)	32,698	2147	84	↓ 2063	↓ 96,09
Hospitalized(H)	5077	74	3	↓ 70	↓ 95,63
Recovered(R)	1126008	4260000	9528000	↑ 5268000	↑ 123,66
Deceased (D)	14690	17840	15950	↓ 1890	↓ 10,60

Note: Units per individual.

Table 3 shows that administering the control reduced the number of individuals in the susceptible, exposed, asymptomatic, symptomatic, quarantined, hospitalized, and deceased categories, while increasing the number in the recovered category. The variables susceptible, exposed, asymptomatic, and symptomatic infected, quarantined, hospitalized, recovered, and deceased each decreased on the 7th day after the administration of the control, with the differences in the alleviation in the amount of people shown in the table.

The following graph depicts the effectiveness of control steps in the form of self-prevention through mask-wearing (u_1), vaccination (u_2), and cure (u_3) among the exposed, asymptomatic, and symptomatic infected, hospitalized, and recovered population, as shown in the figure below:

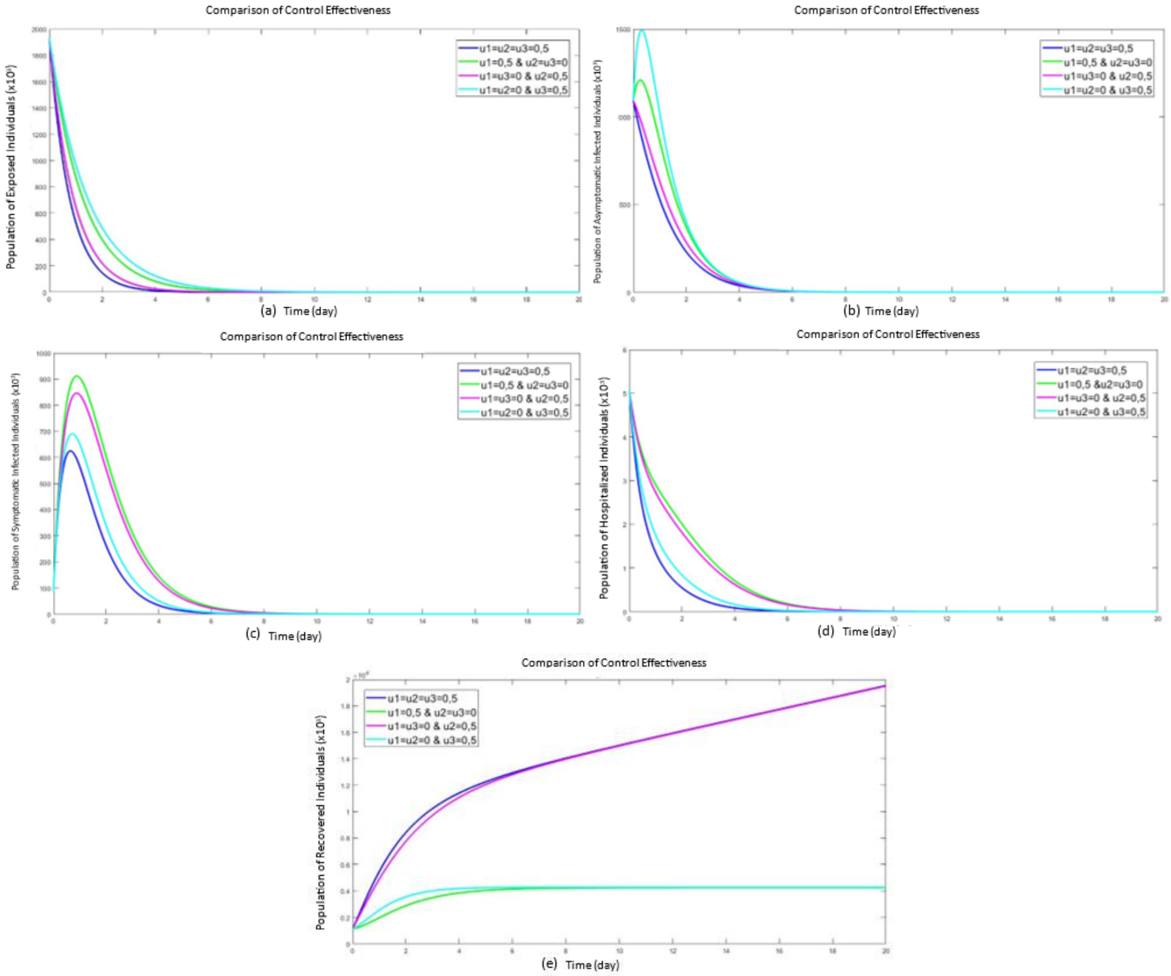

Figure 3. Comparison of Control Effectiveness in the (a) Exposed, (b) Asymptomatic Infected, (c) Symptomatic Infected, (d) Hospitalized, and (e) Recovered Population.

The graph above shows that implementing a combination of all three control measures is more effective than relying on just one of the self-prevention measures, vaccination, or cure. The implementation of self-prevention measures, specifically the use of masks (u_1), vaccination (u_2), and cure (u_3), effectively maximizes the reduction in the number of exposed, asymptomatic, and symptomatic subpopulations, while also accelerating recovery.

The following graph illustrates the optimal control measures for self-prevention, including mask use (u_1) when applied to susceptible and exposed populations, vaccination (u_2) when administered to susceptible populations, and cure (u_3) when provided to symptomatic and hospitalized infected individuals, as shown in the figure below:

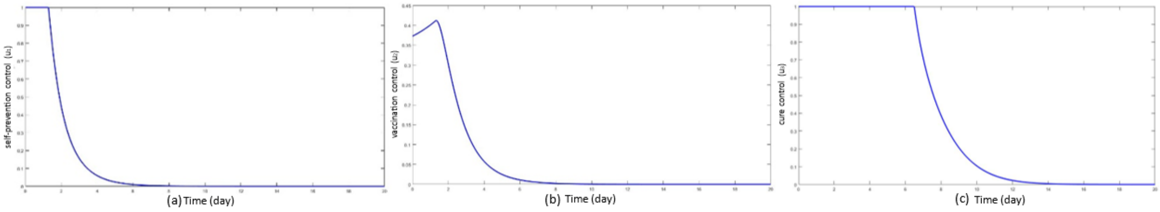

Figure 4. Rate of (a) Effectiveness of Self-Prevention Control (u_1), (b) Vaccination Control Effectiveness (u_2), and (c) Cure Control Effectiveness (u_3).

The graph above shows that self-prevention measures-such as mask-wearing, vaccination, and cure-are gradually declining over time. Even when these measures are not fully implemented, they still help reduce the number of people exposed to the virus, as well as those with asymptomatic and symptomatic infections; mitigate the effects of COVID-19 exposure; and accelerate recovery from COVID-19 infection. Based on all the outcomes achieved, in general, the implementation of control measures, such as self-prevention through mask-wearing, vaccination, and cure, can reduce the transmission of COVID-19 in accordance with their respective roles.

CONCLUSION

The model used to control the transmission of COVID-19 is SEAIQHRD. The SEAIQHRD model was developed by modifying its control variables, the study (Masud et al., 2021) used antiviral therapy for clinically infected and hospitalized cases, and isolation for hospitalized patients, meanwhile this study uses self-prevention, vaccination, and cure as control variables. Analytically, the transmission of disease depends on the basic reproduction number at the equilibrium point, which determines the model's stability. The calculation found that $\Re_0 = 2.1316$, indicating that the stability analysis at the EE point indicates global asymptotic stability. This implies that COVID-19 transmission is occurring within the population.

Based on numerical simulations, comparisons were made between the sizes of each subpopulation with and without control measures. Implementing self-prevention measures, vaccination, and cure can reduce the sizes of the susceptible, exposed, asymptomatic, symptomatic, quarantined, hospitalized, and deceased subpopulations, and increase the size of the recovered subpopulation. An optimal combination of control measures (self-prevention, vaccination, and cure) is the most effective and efficient tactic for reducing the transmission of COVID-19 in Jakarta province. The main contribution of this research is to provide specific optimal control solutions, offering policymakers a clear roadmap for formulating integrated, sustainable disease control strategies.

Build upon the outcome of this research, the constructed model may be refined by adding or removing parameters or variables and by modifying its control variables to suit the specific needs and conditions of the region. This is useful for optimising the model within the system and improving the accuracy of COVID-19 predictions.

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