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PROFILE OF DEEP LEARNING IMPLEMENTATION TO SUPPORT HIGH SCHOOL STUDENTS' CONCEPTUAL UNDERSTANDING OF THE FUTURE VALUE OF ANNUITY

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ABSTRACT

This study reviewed the implementation of deep learning through discovery learning on the future value of annuity topic in senior high school mathematics, addressing Indonesia's low PISA and OECD numeracy scores. Deep learning, grounded in the principles of meaningful, mindful, and joyful learning, is considered a promising approach to strengthen students' conceptual understanding. This study aims to (1) analyze the implementation profile of deep learning by a model teacher on the future value of annuity topic, and (2) examine how learning activities support students' conceptual understanding. A qualitative descriptive approach with a classroom-based study design was employed. The subject was a model teacher implementing on the topic of the Future Value of Annuity in a Grade XI class at a public senior high school in East Java. Data were collected through classroom video recordings and analyzed using lesson transcripts and student worksheets from three students representing high, moderate, and low ability levels, selected based on diagnostic assessment results. The analysis was conducted using the seven indicators of conceptual understanding proposed by the NCTM. Results show the teacher successfully integrated the three deep learning principles. In the opening activity, the teacher built readiness through contextual apperception and motivation. In the core activity, discovery learning enabled students to actively construct understanding of prannumerando and postnumerando through problem identification, data collection, processing, verification, and group presentation. Differentiated instruction based on diagnostic assessment and individual scaffolding supported students across ability levels. However, reflection and closing activities could not be completed due to time constraints from student mobilization between rooms. Overall, deep learning through discovery learning on the future value of annuity topic created active, contextual, and student-centered learning that strengthened students' conceptual understanding.

Keywords: Deep Learning, Conceptual Understanding, Future Value of Annuity, Discovery Learning, Differentiated Instruction

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PRELIMINARY

Teachers play a crucial role in determining the quality of mathematics learning, as their pedagogical competence influences students' academic achievement (Barut et al., 2021; Nugraheni & Jailani, 2020). Teachers not only convey content but also design learning experiences that foster students' ability to understand concepts in depth and relate knowledge to relevant contexts (Hino & Funahashi, 2022; Kooloos et al., 2022). Consequently, teachers' ability to implement deep learning approaches is a crucial factor in supporting students' conceptual understanding (Güç & Kollosche, 2022). One approach focused on deep conceptual understanding, where students do not merely memorise concepts, is the deep learning approach (Fawzia & Karim, 2024). The urgency of implementing this approach is further underscored by assessments such as PISA 2022, which indicate that Indonesian students' mathematical abilities still need to be improved (OECD, 2022).

PISA study results show that Indonesian students' numeracy scores reached only 366, far below the OECD average of 480. Furthermore, this achievement also lags behind several countries in the Southeast Asian region, such as Singapore with a score of 575, Vietnam 469, Malaysia 409, and Thailand 394 (OECD, 2022). This situation serves as both a driving force and a challenge for teachers to innovate in their teaching approaches, not merely to convey knowledge, but also to foster a deeper and more meaningful conceptual understanding in students. One approach considered capable of addressing this challenge is deep learning. This approach is grounded in three interrelated pillars: meaningful, mindful, and joyful, which together form a holistic learning experience (Azzahra & Jaya, 2025). Furthermore, compared to conventional approaches that focus on rote memorization, deep learning also emphasizes a contextual approach and encourages students to think critically and be able to solve various problems (Kemendikdasmen, 2025). Consequently, one of the skills that has the potential to develop through the implementation of deep learning is students' understanding of mathematical concepts

Understanding mathematical concepts refers to a student's ability to grasp the meaning of a concept, identify its characteristics, rephrase the concept in their own words, and relate it to other mathematical concepts (Kilpatrick et al., 2001; Komarudin et al., 2023). Mathematics learning uses conceptual understanding as the foundation for solving problems logically, meaningfully and systematically, rather than merely memorising formulae (Hermaya & Hendrayana, 2025; Kilpatrick et al., 2001). Therefore, in mathematics learning, planning is necessary to ensure that students not only obtain the final answer but also understand the reasoning behind the problem-solving process.

Deep learning and conceptual understanding are closely linked, as they enable students to explore concepts through questioning, discussion and reflection, and to apply them in everyday life (Hattie & Donoghue, 2016). Through meaningful, learner-centred inquiry activities, deep learning encourages students to explore ideas, form opinions, and construct understanding based on their learning experiences (Kemendikdasmen, 2025; Mystakidis, 2021). Furthermore, students are encouraged to link their existing knowledge with new concepts and real-life contexts, thereby forming stronger connections between concepts (Gianto et al., 2026; Kilpatrick et al., 2001). Thus, deep learning supports students in gaining a deeper understanding of concepts, not merely acquiring information, but also actively connecting and applying knowledge (Gianto et al., 2026; Mystakidis, 2021).

The topic of the final value of an annuity forms part of financial mathematics and relates to the accumulation of a series of periodic payments at the end of a specific period. An annuity is a series of fixed payments or capital receipts made at regular intervals over a specific period (Santoso & Noormandiri, 2023). This topic links several concepts previously studied by students, namely compound interest and geometric sequences, and is often considered difficult. Based on initial research observations, students find the topic of the final value of an annuity difficult because it is a new topic and is rarely studied at senior secondary school level; students often make mistakes in distinguishing between the final value of an annuity before and after the payment date.

The topic of the final value of an annuity is closely related to everyday life and therefore requires a deep understanding of the concepts, rather than merely being taught the formulae. Consequently, the topic of final interest value at senior secondary school level requires a deep learning approach to help students understand the concepts more thoroughly (OECD, 2022; Santoso & Noormandiri, 2023; Surven et al., 2022). The application of deep learning to the topic of interest can be implemented through real-life problem-solving scenarios. Furthermore, through discussions, video simulations and group activities, learning becomes more active and engaging. Consequently, through deep learning, students are not only able to perform calculations but also gain a deeper understanding of the concepts (Azzahra & Jaya, 2025; Hattie & Donoghue, 2016).

Numerous studies on deep learning have been conducted, particularly regarding the enhancement of learning engagement, critical thinking skills, and conceptual understanding. However, research specifically examining the implementation profile of deep learning in the topic of Future Value of Annuities at the high school level remains

limited. Usman & Husniati, (2025) and Firdausy et al. (2023) noted that while research on student's profile and teachers' classroom management has advanced, it has not yet addressed the profile of deep learning in financial mathematics, specifically the future value of an annuity, making further investigation necessary. This research gap provides an opportunity for researchers to describe how teachers design, implement, and reflect on deep learning to support students' conceptual understanding. Based on this background, it is important to conduct research to examine the profile of deep learning implementation in the topic of the Future Value of Annuities. The research focuses not only on how teachers implement learning but also on how learning activities support high school students' conceptual understanding. This study is expected to contribute to the development of mathematics learning that is more meaningful, reflective, contextual, and student-centered.

A number of researchers have conducted studies on teacher profiles. Research on classroom management profiles in mathematics has been carried out by (Aprilia et al., 2024; Azizah et al., 2024; Firdausy et al., 2023). Furthermore, research on teachers' readiness profiles for the implementation of the Merdeka Curriculum has been carried out by (Nisak & Yulastuti, 2022). Whilst studies on teachers' statistical literacy profiles, mathematical reasoning profiles, and difficulty level profiles have been conducted by (Andriatna et al., 2021; Nisak & Yulastuti, 2022; Sufa, 2022; Zuhri & Purwosetiyono, 2019). Furthermore, research analysing the needs for deep learning has been carried out by (Dalia et al., 2025) (Dalia et al., 2025). Furthermore, research on deep learning to support mathematics learning has been carried out by (Dalia et al., 2025; Maulidya et al., 2025). However, studies relating to the implementation profile of deep learning, particularly regarding the topic of the final value of an annuity, have not been extensively conducted. Therefore, based on this background, this study aims to:. Therefore, based on this background, this study aims to:

1. Analyze the profile of deep learning implementation carried out by model teachers on the topic of the Future Value of an annuity
2. Examine how the learning activities carried out can support high school students' conceptual understanding of the Future Value of Annuities material.

METHODS

This study uses a descriptive qualitative approach with a classroom-based study design focused on the learning process, student activities, and forms of conceptual understanding that emerge naturally and contextually during instruction (Huda & Marzal,

2026). This design was chosen because the research questions require in-depth observation of natural classroom phenomena rather than experimental manipulation. The research procedure consisted of three stages: (1) planning, which included developing observation instruments; (2) data collection, conducted during one learning session for 84 minutes recorded on video; and (3) data analysis, which involved transcripts and student worksheets against the seven NCTM indicators. The researcher acted as a non-participant observer throughout the session. Data analysis followed the Miles et al. (2014) interactive model, comprising data reduction, data display, and conclusion drawing. Trustworthiness was established through data triangulation by cross-checking three data sources: video transcripts, student activity sheets (SAS), and field observation. The subject was a model teacher (referred to as AT) who implemented deep learning on the topic of the Future Value of Annuity in a Grade XI class at an urban public senior high school in East Java. AT was selected based on the criteria of having implemented deep learning in mathematics instruction and having completed a structured lesson plan aligned with deep learning principles. The class consisted of 36 students whose learning activity sheets (SAS) served as supporting data to examine how the implemented learning activities supported students' conceptual understanding. For in-depth analysis of conceptual understanding, three student representatives were selected based on diagnostic assessment results, representing high, moderate, and low ability levels, who had studied prerequisite material (sequences and series, simple interest, and compound interest). Data collection techniques included observation using a structured observation, analysis of video transcripts of learning sessions, and a review of student activity sheets (SAS) as instructional materials. Students' conceptual understanding was assessed based on the National Council of Teachers of Mathematics (NCTM) indicators, namely: 1) defining concepts verbally and in writing; 2) providing examples and non-examples; 3) using various diagrams, models, and symbols to represent concepts; 4) transforming one form of representation into another; 5) understanding the meaning of the concept; 6) identifying the properties and conditions of the concept; and 7) distinguishing between different types of concepts (Khusna et al., 2021).

RESULT AND DISCUSSION

During the planning stage, the teacher established three learning objectives: students can accurately explain the difference between the concepts of the future value of pre-numerando and post-numerando interest accurately, students can derive the formulas

for the future value of an annuity due and an annuity payable using the concepts of compound interest and geometric sequences, and students can apply the concepts of the future value of an annuity due and an annuity payable to solve contextual problems. The problems addressed were selected because they are relevant to students' daily lives, as mathematics learning needs to develop the ability to understand concepts, construct knowledge, and apply it in contextual situations (Marwiyah & Wardani, 2024).

Assessments are comprehensively designed, including a pre-instructional diagnostic assessment to determine students' readiness for prerequisite material such as sequences and series, simple interest, and compound interest, as well as a formative assessment at the end of the lesson in the form of an essay-style quiz and reflection. The results of the diagnostic assessment serve as the basis for grouping students and adjusting the level of scaffolding in the Student Activity Sheet (SAS).

The SAS is structured based on the discovery learning syntax with content differentiation through the provision of scaffolding according to student ability. As stated by (Hadiyani et al., 2025), "students with low ability will receive more guidance, students of average ability will receive less guidance, and high-ability students will be given the least amount of guidance." Student grouping is also based on the results of the diagnostic assessment so that group discussions proceed according to each student's capacity. During the discussion, students collaboratively work on the Student Worksheet (Lembar Aktivitas Murid (LAM)), starting from analyzing the differences between pre-numerando and post-numerando interest rates, constructing interest rate table patterns, to deriving the final interest value formula using a geometric sequence. The presentation session is also designed to allow students to communicate their work results and receive feedback.

The entire lesson plan integrates three principles of deep learning: meaningful learning through relevant problem contexts and diagnostic-based LAM; mindful learning through prompting questions that encourage students' active engagement in the thinking process; and joyful learning through collaborative discussions, instructional videos, and expressive presentation sessions.

Implementation of Deep Learning in the Classroom

Based on the results of observations and analyses of transcripts and teaching modules, the implementation of deep learning by model teachers in the topic of "Future Value of Annuity" can be broken down into the following three stages.

1. Introductory Activities

The teacher presents an aperseption: *“Before we dive into the interest material, let’s recall what we learned yesterday?”*; *“What is the difference between simple interest and compound interest?”*; *“Yesterday we learned to derive the formula for the future value of compound interest, what is the formula?”*; *“To determine the compound interest formula, what series do we use? Geometric, right? Arithmetic is different”*; *“Then, what is the formula for present value?”*; *“In the previous session, we also learned about the parameters of compound interest: interest rate and period. Now, let me ask: suppose I want to save money and have two bank options with the same interest rate and the same principal, but the difference is that Bank A pays interest monthly while Bank B pays annually. Which one should I choose?”* To introduce the material, the teacher provided an overview of interest: *“Earlier, the initial principal was paid just once upfront. What if, for example, the principal is paid multiple times over the same period with the same amount?”*; *“So, if the principal is provided regularly, with the same amount, the calculation works like this: the initial principal is used, then in the second period, the initial principal is added again, and the total is compounded; after that, the initial principal is added again and compounded, and so on. That is what we will learn, compound interest.”*

The next activity involved the teacher providing motivation: *“Why do we need to learn about interest? You’ve probably heard of things like phone loans or motorcycle loans. If we use an interest-based system where we pay in installments, we’ll eventually realize that the total amount we end up paying is much higher. Yet if we choose to pay it off upfront, we wouldn’t have to pay nearly as much. So, the lesson here is that if we want to buy something, we should save up first and pay it off upfront.”*

At the end of the opening activity, the teacher explains the learning objectives: *“So, our learning objectives for today are for you to be able to accurately distinguish between the concepts of the future value of an annuity using the ‘pranumerando’ and ‘postnumerando’ methods. Then, to derive the formulas for the future value of an annuity using the ‘pranumerando’ and ‘postnumerando’ methods by applying the concepts of compound interest and geometric sequences. And to apply the concepts of the future value of an annuity using the ‘pranumerando’ and ‘postnumerando’ methods to solve contextual problems.”*

2. Main Activity

This phase is the core of Deep learning combined with Discovery Learning, where students construct their own understanding of the concepts regarding the difference between the future values of and derive the formula for the future value of an annuity using geometric series patterns.

Learning Experiences: Understanding

The core activity begins with providing a learning experience focused on understanding through several activities, the first being the presentation of a stimulus. In this activity, the teacher presents a problem as a stimulus. In addition to asking students to understand the problem, the teacher also asks them to identify the differences in the problems presented in the cases, how to calculate the total savings, and to make a tentative guess regarding which savings account will have the larger total. *“What is the main difference between the two interest systems in the cases above? How do you determine the total amount of money at the end of the period (future value) for the older sibling’s and younger sibling’s savings? In your estimation, which savings account yields a larger future value? Explain your reasoning!”* The next activity is data collection, where the teacher shows a video to help students construct their understanding through LAM. After the video finishes playing, the teacher ensures students’ understanding of the video: *“Do you understand?”* A student asked why the formula wasn’t shown in the video: *“Teacher, the formula isn’t there.”* The teacher explained that the students would discover the formula themselves: *“...please open the formula on page four. You’ll find the formula there... I didn’t give you the formula right away so you’d understand where it comes from.”* A student asked based on the video shown, *“Why can’t the postnumerando interest, the very last payment (the final deposit doesn’t earn interest), be calculated?”* The teacher didn’t answer immediately but instead tried to pose a question to the students: *“Does anyone know the answer?”* A student replied, *“Because it’s like closing the book, Ms. At the end.”* The teacher then provided feedback on the student’s answer: *“Correct... suppose I save at the end of each period for five months... in the fifth month, I’ve already withdrawn the savings. Is there a chance for interest No, right?”* The teacher did not provide the answer immediately but gave the student an opportunity to think, connect with their own experiences, and build their own understanding through a prompting question. The teacher uses *scaffolding* by posing questions to the students, thereby fostering a process of reflection and active engagement in learning, where knowledge is constructed by the students themselves through experience and learning interactions within a social context (Vygotsky, 1978).

Learning Experience: Application

The next stage of deep learning begins with data processing activities that include several tasks. The first activity involves students constructing the difference in final interest values between the pre-numerando and post-numerando terms using a table. During the data processing activity, the teacher acts as a facilitator, providing individualized *scaffolding* to each student. The Student Activity Sheets distributed and the discussion groups formed during the lesson have been differentiated based on students' readiness levels, as determined by diagnostic assessments, to facilitate the provision of scaffolding (Insani et al., 2023). Diagnostic assessments used as the basis for designing differentiated instruction can help teachers adapt responsive and adaptive teaching approaches to meet students' needs (Febrianti et al., 2025).

Reflecting on the Learning Experience

The summary activity was not carried out because the learning time had run out. This activity should have involved summarizing all the concepts learned that day and reinforcing them by the teacher. The reflection and summary stages in learning play a crucial role in strengthening students' conceptual understanding, as this process encourages students to integrate the various knowledge gained during the lesson into a cohesive understanding (Afidati & Malasari, 2023). Without this stage, there is a risk that the knowledge acquired by students may not be connected to one another (Azzahra & Jaya, 2025; Pujawati et al., 2025).

3. Closing Activities

In the written teaching module, the closing activities included a formative assessment quiz in the form of essay questions, personal and peer reflections, and the provision of a plan for what students would study next. However, the closing activities could not be carried out due to time constraints and student mobilization issues.

Students' Conceptual Understanding of the Future Value of Annuity

1. Concept Understanding of Students with High Concept Understanding Abilities

a. Defining concepts verbally and in writing

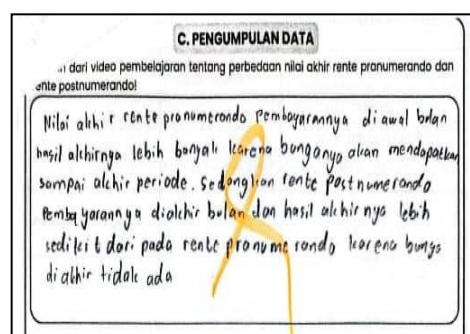


Figure 1. Students with High Conceptual Understanding in Defining Concepts

Based on Figure 1, student with high conceptual understanding ability explained “*With a pre-numerando interest, payments are made at the beginning of the month, resulting in a higher total amount because interest accumulates until the end of the period. In the case of a post-numerando interest, payments are made at the end of the month, and the total amount is lower than that of a pre-numerando interest because no interest accumulates at the end.*” Student only mentioned the general differences between pre-numerand interest and post-numerand interest and did not include mathematical notation, symbols, or formulas. This finding is consistent with Latifa et al. (2025), who argued that conceptual understanding and mathematical representation are closely related but do not develop automatically. Students may demonstrate conceptual understanding through verbal or written explanations even when they have not yet expressed the concept using mathematical symbols, diagrams, or other representations.

b. Providing examples and non-examples

TABUNGAN KAKAK			
Bulan Setor	Waktu Setor	Frekuensi Berbunga (berapa kali tiap setoran berbunga dari awal setor hingga setoran terakhir)	Nilai Akhir tiap Setoran (gunakan rumus nilai akhir pada bunga majemuk)
Bulan 1	Awal bulan ke-1	3 bulan (bulan ke 1, bulan ke 2, dan bulan ke 3)	$M_3 = 100.000(1,025)^3$ $M_3 = 107.689$
Bulan 2	Awal bulan ke-2	2 bulan (bulan ke 2, dan bulan ke 3)	$M_2 = 100.000(1,025)^2$ $M_2 = 105.063$
Bulan 3	Awal bulan ke-3	1 bulan (bulan ke 3)	$M_1 = 100.000(1,025)^1$ $M_1 = 102.500$
Total uang yang diperoleh selama 3 bulan (N_3)			$N_3 = M_1 + M_2 + M_3$ $N_3 = 102.500 + 105.063 + 107.689$ $N_3 = 315.252$

TABUNGAN KAKAK			
Bulan Setor	Waktu Setor	Frekuensi Berbunga (berapa kali tiap setoran berbunga dari awal setor hingga setoran terakhir)	Nilai Akhir tiap Setoran (gunakan rumus nilai akhir pada bunga majemuk)
Bulan 1	Akhir bulan ke-1	2 bulan (bulan ke 2 dan bulan ke 3)	$M_2 = 100.000(1,025)^2$ $M_2 = 105.063$
Bulan 2	Akhir bulan ke-2	1 bulan (bulan ke 3)	$M_1 = 100.000(1,025)^1$ $M_1 = 102.500$
Bulan 3	Akhir bulan ke-3	0 bulan (tidak berbunga)	$M_0 = 100.000(1,025)^0$ $M_0 = 100.000$
Total uang yang diperoleh selama 3 bulan (N_3)			$N_3 = M_0 + M_1 + M_2$ $N_3 = 100.000 + 102.500 + 105.063$ $N_3 = 307.563$

Figure 2. Students Excel at Providing Examples and Counterexamples

Based on Figure 2, the student has provided a calculation showing that the numerator yields a larger future value than the denominator. This indirectly positions the numerator as an example of a profitable interest rate and the denominator as a non-example (a comparator with a smaller value). Therefore, the student meets the indicator for providing examples and non-examples. This finding is in accordance with Yani et al. (2019) that in the indicator of providing examples and non-examples of a concept, high-ability students have been able to do it well.

c. Using various diagrams, models, and symbols to present concepts

Based on Figure 2, it is shown that the student wrote down a representation for each deposit in both cases of the problem and that the calculations are correct. This indicates that the student meets the indicator of using various diagrams, models, and symbols to present concepts. These findings are consistent with those of Setyo et al. (2025), that students with higher mathematical ability have more effective reasoning in solving diagrams, models, and symbols.

d. Converting one form of representation into another.

Figure 3. Student's Representation of a Geometric Series in Another Form

Figure 3 shows that the student was able to represent the sum of each installment in another form by relating it to the concept of a geometric sequence. The student successfully transformed the representation from a form of repeated addition into a general mathematical formula that can be used to calculate the final interest value at any period. This finding is in accordance with Nurilla, (2022) that high-school students with high logical-mathematical intelligence can identify patterns, use them to determine the next term, and formulate general formulas to solve problems. Therefore, the student met the indicator of converting one form of representation into another.

e. Understanding the meaning of the concept

Figure 4. High-Achieving Students in Understanding the Meaning of the Concept

Based on Figure 4, the student has demonstrated an understanding of the meaning of the concept. In question number 3, the student only wrote, "because the money was deposited earlier, so it earns interest for a longer period," without explaining mathematically why this affects the future value. This finding is in accordance with Kurnia et al. (2024) that high ability students understanding mathematical concepts. Therefore, this sufficiently meets the indicator of understanding the meaning of the concept of the future value of interest, both the numerator and denominator.

f. Identifying the properties and conditions of a concept

Based on Figure 4, the student was able to state the properties of the future values of the pre-numerando and post-numerando interest. Therefore, the student meets the indicator of stating the properties and conditions of a concept.

g. Distinguishing between different types of concepts

Based on Figure 2 and Figure 3, students are able to distinguish between the future values of an annuity due and an annuity payable. Therefore, students meet the indicator of distinguishing various types of concepts

2. Concept Understanding of Students with Moderate Concept Understanding Abilities

a. Defining concepts verbally and in writing

Prenumerando	Postnumerando
• Pembayaran diawal periode • Angsuran terakhir mengalami pembungaan 1 periode • Hasil akhir lebih besar • Setelah terakhir masih dapat bunga 1 periode • Sederupan dengan waktu deposito	• pembayaran dibayarkan di akhir periode • Angsuran terakhir tidak mengalami pembungaan • Hasil akhir lebih kecil • Porsi pertumbuhan berkurang

Figure 5. Students with Moderate Conceptual Understanding in Defining Concepts

Based on Figure 5, student with moderate conceptual understanding explained, “*Prenumerando refers to payments made at the beginning of each period. Because each deposit starts earning interest immediately, the final accumulated value is higher. The last deposit still earns interest for one full period, allowing every contribution to have the maximum time to grow. Postnumerando refers to payments made at the end of each period. Since deposits begin earning interest later, the final accumulated value is lower. The last deposit does not receive any additional interest, resulting in reduced growth potential compared to prenumerando.*” Student only mention the differences and have not yet provided definitions of the concepts of the future value of an annuity due and an annuity payable. This finding is in accordance with (Kurnia et al., 2024) that that students with average ability have not yet reached their full potential in explaining a concept in depth. Therefore, students have not met the indicator of conceptual understanding regarding defining concepts verbally and in writing

b. Providing examples and non-examples

TABUNGAN KAKAK			
Bulan Setor	Waktu Setor	Frekuensi Berbunga (Berapa kali tiap setoran berbunga dari awal setoran hingga setoran terakhir)	Nilai Akhir tiap Setoran (gunakan rumus nilai akhir pada bunga majemuk)
Bulan 1	Awal bulan ke-1	3 bulan (bulan ke 1, bulan ke 2, dan bulan ke 3)	$M_3 = 100.000(1,025)^3$ $M_3 = 107.689$
Bulan 2	Awal bulan ke-2	2 Bulan (bulan ke 2 dan bulan ke 3)	$M_2 = 100.000(1,025)^2$ $M_2 = 105.063$
Bulan 3	Awal bulan ke-3	1 Bulan (bulan ke 3)	$M_1 = 100.000(1,025)^1$ $M_1 = 102.500$
Total uang yang diperoleh selama 3 bulan (N_3)			$N_3 = M_1 + M_2 + M_3$ $N_3 = 102.500 + 105.063 + 107.689$

TABUNGAN ADIK			
Bulan Setor	Waktu Setor	Frekuensi Berbunga (Berapa kali tiap setoran berbunga dari awal setoran hingga setoran terakhir)	Nilai Akhir tiap Setoran (gunakan rumus nilai akhir pada bunga majemuk)
Bulan 1	Akhir bulan ke-1	2 bulan (bulan ke 1 dan bulan ke 2)	$M_2 = 100.000(1,025)^2$ $M_2 = 105.063$
Bulan 2	Akhir bulan ke-2	1 Bulan (bulan ke 2)	$M_1 = 100.000(1,025)^1$ $M_1 = 102.500$
Bulan 3	Akhir bulan ke-3	0 Bulan (bulan ke 3)	$M_0 = 100.000(1,025)^0$ $M_0 = 100.000$
Total uang yang diperoleh selama 3 bulan (N_3)			$N_3 = M_0 + M_1 + M_2$ $N_3 = 100.000 + 102.500 + 105.063$

Figure 6. Students with Moderate Ability in Providing Examples and Non-examples

Based on the figure 6, the student has not yet been able to provide examples and non-examples of the future values of pre-numbered and post-numbered interest. This is evident in the error made by the student when entering the months in the compounding frequency for the younger and older siblings' savings accounts. The error indicates that the student has not yet been able to distinguish between the future values of pre-numbered and post-numbered interest. These findings are consistent with Ratnayanti et al. (2021), which showed that students still make errors regarding the ability to provide examples and non-examples of a concept, as evidenced by their provision of inappropriate or incomplete examples, or even their inability to distinguish between examples and non-examples of a concept. Therefore, the student has not met the indicator for providing examples and non-examples

c. Using various diagrams, models, and symbols to present concepts

Based on the figure 6, it is shown that the student wrote down the representation for each deposit in both cases of the problem and the calculations are correct. This is consistent with Budiarty & Utomo, (2024), who found that students with moderate ability are capable of creating appropriate representations through both visual and symbolic means. This indicates that the student meets the indicator of using various diagrams, models, and symbols to present concepts.

d. Converting one form of representation into another

Maka untuk:

$$N_t = M_0(1+i)^t + M_0(1+i)^{t-1} + \dots + M_0(1+i)^2 + M_0(1+i)^1 + M_0(1+i)^0$$

Berdasarkan pola di atas, ketiganya membentuk deret geometri dengan suku pertama $(a) M_0(1+i)^0$ dan rasio (r) antar suku adalah $(1+i)$. Selanjutnya kita akan menentukan rumus menghitung nilai akhir rente pronomerando untuk suku ke t periode.

Rumus dari deret diatas adalah:

$$S_n = \frac{a(r^n - 1)}{r - 1}, \text{ jika } r > 1$$

Jika $N_t = S_n$, $a = M_0(1+i)^0$, $r = (1+i)$, maka diperoleh:

$$N_t = \frac{M_0[(1+i)^t - 1]}{(1+i) - 1}$$

$$N_t = \frac{M_0[(1+i)^t - 1]}{i}$$

Jika untuk:

$$N_t = M_0(1+i)^t + M_0(1+i)^{t-1} + \dots + M_0(1+i)^2 + M_0(1+i)^1 + M_0(1+i)^0$$

Berdasarkan pola di atas, ketiganya membentuk deret geometri dengan suku pertama $(a) M_0(1+i)^0$ dan rasio (r) antar suku adalah $(1+i)$. Selanjutnya kita akan menentukan rumus menghitung nilai akhir rente postnumerando untuk suku ke t periode.

Rumus dari deret diatas adalah:

$$S_n = \frac{a(r^n - 1)}{r - 1}, \text{ jika } r > 1$$

Jika $N_t = S_n$, $a = M_0(1+i)^0$, $r = (1+i)$, maka diperoleh:

$$N_t = \frac{M_0[(1+i)^{t+1} - 1]}{(1+i) - 1}$$

$$N_t = \frac{M_0[(1+i)^{t+1} - 1]}{i}$$

Figure 7. Student's Representation Converted into Another Form

Based on Figure 7, the student is able to represent the sum of each installment in another form by relating it to the concept geometric series. According to a study by Sri Wahyuning and Utami, (2024), when studying sequences and series, students with moderate ability can use mathematical symbols to construct formulas. Therefore, the student meets the indicator of converting one form of representation into another.

e. Understanding the meaning of the concept

F. GENERALISASI

1. Nilai akhir rente pronomerando memiliki waktu setoran di awal periode.
2. Nilai akhir rente postnumerando memiliki waktu setoran di akhir periode.
3. Dengan besar setoran yang sama dan besar suku bunga yang sama nilai akhir manakah yang memiliki nilai akhir lebih besar? Apa yang memengaruhi? Jelaskan jawabanmu!
Nilai akhir pronomerando lebih besar, yang memengaruhi adalah frekuensi bunga di awal periode sudah jalan bunganya.
4. Bagaimana rumus mencari nilai akhir rente pronomerando?
 $Na = M_0((1+i)^t - 1)$
5. Bagaimana rumus mencari nilai akhir rente postnumerando?
 $Na = (M_0(1+i))((1+i)^t - 1)$

Figure 8. Students Understanding the Meaning of the Concept

Based on Figure 8, the student has demonstrated an understanding of the meaning of the concepts of the future value of the pronomerando and postnumerando interest, as seen in the answer to question number 3: "The larger future value of the pre-numerand is influenced by the frequency of interest at the beginning of the period when the interest has already started accruing." Additionally, the formulas written by the student are reversed; the formula written for the pre-numerando is actually the post-numerando formula, and vice versa. Students with moderate conceptual understanding have not met the indicator because, whilst they are often able to follow specific procedures, they still fail to meet the indicator requiring an understanding of conceptual relationships and their correct application (Lefrida, 2024). Therefore, the student has not met the indicator of understanding the meaning of the concept

f. Identifying the properties and conditions of a concept

Based on Figure 5, the students were able to identify the properties of the respective final values of the pre-numerando and post-numerando annuities. Therefore, the students met the indicator of identifying the properties and conditions of a concept. This is in line with the research by Lefrida, (2024), which found that students with moderate conceptual understanding are generally able to identify the attributes and characteristics of a concept, although they are not yet fully able to explain the relationships between concepts in depth..

g. Distinguishing between different types of concepts

Based on Figure 5 and Figure 8, students with moderate conceptual understanding are able to distinguish between the final values of pre-numerando and post-numerando annuities. Therefore, the students meet the indicator of distinguishing between different types of concepts. This is in line with the research by Mayasari and Habeahan (2021), Mumtazah and Triyana (2025), Nuraini and Imami (2025), which found that students in the moderate ability category are generally able to meet most of the indicators of conceptual understanding, including those relating to classifying or distinguishing concepts. Students in the moderate ability category were reported to have mastered between three and five of the six indicators of conceptual understanding.

3. Concept Understanding of Students with Low Concept Understanding Abilities

a. Defining concepts verbally and in writing

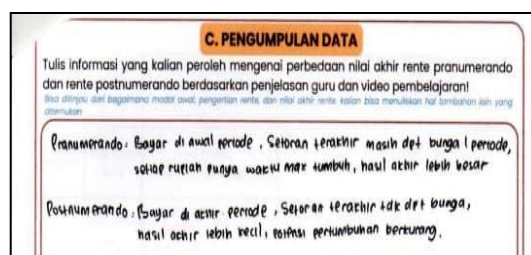


Figure 9. Students with Low Concept Understanding in Defining Concepts

Based on Figure 9, students with low conceptual understanding ability explained “Pranumerando: Pay at the beginning; the final deposit still earns interest for one period; every rupiah has a maximum growth period; the final return is higher Postnumerando: Pay at the end of the period; the final deposit does not earn interest; the final return is lower; growth potential”. Student only mention the differences and have not provided definitions of the concepts of the future value of

an annuity due and an annuity payable. This is consistent with the findings of Surven et al. (2022), which show that low conceptual understanding is characterised by students' inability to restate the definition of a concept. This situation arises because students have understood some of the ideas, but not sufficiently to arrive at the correct definition or result (Burfitt, 2021). Therefore, students have not met the indicator of conceptual understanding regarding defining concepts verbally and in writing.

b. Providing examples and non-examples

TABUNGAN KAKAK				TABUNGAN ADIK			
Bulan Setor	Waktu Setor	Frekuensi Berbunga (Berapa kali tiap setoran berbunga dari awal setor hingga setoran terakhir)	Nilai Akhir tiap Setoran (gunakan rumus nilai akhir pada bunga majemuk)	Bulan Setor	Waktu Setor	Frekuensi Berbunga (Berapa kali tiap setoran berbunga dari awal setor hingga setoran terakhir)	Nilai Akhir tiap Setoran (gunakan rumus nilai akhir pada bunga majemuk)
Bulan 1	Awal bulan ke-1	3 bulan (bulan ke-1, bulan ke-2, dan bulan ke-3)	$M_3 = 100.000(1,025)^3$ $M_3 = 107.689$	Bulan 1	Akhir bulan ke-1	2 bulan (bulan ke-2 dan bulan ke-3)	$M_2 = 100.000(1,025)^2$ $M_2 = 105.063$
Bulan 2	Awal bulan ke-2	2 Bulan (bulan ke-2 dan bulan ke-3)	$M_2 = 100.000(1,025)^2$ $M_2 = 105.063$	Bulan 2	Akhir bulan ke-2	1 Bulan (bulan ke-3)	$M_1 = 100.000(1,025)^1$ $M_1 = 102.500$
Bulan 3	Awal bulan ke-3	1 Bulan (bulan ke-3)	$M_1 = 100.000(1,025)^1$ $M_1 = 102.500$	Bulan 3	Akhir bulan ke-3	0 Bulan	$M_0 = 100.000(1,025)^0$ $M_0 = 100.000$
Total uang yang diperoleh selama 3 bulan (N_3)			$N_3 = M_3 + M_2 + M_1$ $N_3 = 107.689 + 105.063 + 102.500$ $N_3 = 315.252$	Total uang yang diperoleh selama 3 bulan (N_3)			$N_3 = M_2 + M_1 + M_0$ $N_3 = 105.063 + 102.500 + 100.000$ $N_3 = 307.563$

Figure 10. Students Low Performance in Providing Examples and Non-examples

Figure 10 illustrates the errors made by students when entering the number of months for the compounding frequency in the younger and older siblings' savings accounts. These errors indicate that the students are not yet able to distinguish between the final values of the pre-numerando and post-numerando interest. Consequently, the students have not yet met the indicator of being able to identify examples and non-examples. This may occur because students are accustomed to teacher-centred learning, which focuses on formulas and routine examples without emphasising the origins and conditions of the concepts. Furthermore, due to limited teaching time, the transition to symbolic operations is too rapid before the definitions, characteristics, and comparisons of examples and non-examples have been firmly established (Guler & Tarim, 2024).

c. Using various diagrams, models, and symbols to present concepts

Figure 10 shows that the student did not write the exponent for the third term in either case, even though the calculation was correct. Consequently, students with a low level of conceptual understanding have not yet met the indicator of using various diagrams, models and symbols to represent concepts. This may be because students are accustomed to quick, routine problem-solving rather than exploring the meaning of symbols, the relationships between forms of

representation, and the context of the problem (Johar & Lubis, 2018; Sari et al., 2025).

d. Converting one form of representation into another

Maka untuk

$$N_t = M_0(1+i)^1 + M_0(1+i)^2 + \dots + M_0(1+i)^{t-1} + M_0(1+i)^{t-2} + M_0(1+i)^{t-1} + M_0(1+i)^t$$

Berdasarkan pola di atas, ketiganya membentuk deret geometri dengan suku pertama $(a) M_0(1+i)^1$ dan rasio (r) antar suku adalah $\frac{(1+i)^2}{(1+i)^1}$. Selanjutnya kita akan menentukan rumus menghitung nilai akhir rente pranonumerado untuk suku ke t periode.

Rumus dari deret diatas adalah

$$S_n = \frac{a(r^n - 1)}{r - 1}, \text{ jika } r > 1$$

Jika $N_t = S_n$, $a = M_0(1+i)^1$, $r = (1+i)$, maka diperoleh:

$$N_t = \frac{M_0(1+i)[(1+i)^t - 1]}{(1+i) - 1}$$

$$N_t = \frac{M_0(1+i)[(1+i)^t - 1]}{1}$$

Maka untuk

$$N_t = M_0(1+i)^0 + M_0(1+i)^1 + \dots + M_0(1+i)^{t-1} + M_0(1+i)^{t-2} + M_0(1+i)^{t-1} + M_0(1+i)^{t-1}$$

Berdasarkan pola di atas, ketiganya membentuk deret geometri dengan suku pertama $(a) M_0(1+i)^0$ dan rasio (r) antar suku adalah $\frac{(1+i)^1}{(1+i)^0}$. Selanjutnya kita akan menentukan rumus menghitung nilai akhir rente postnumerado untuk suku ke t periode.

Rumus dari deret diatas adalah

$$S_n = \frac{a(r^n - 1)}{r - 1}, \text{ jika } r > 1$$

Jika $N_t = S_n$, $a = M_0(1+i)^0 = M_0$, $r = (1+i)$, maka diperoleh:

$$N_t = \frac{M_0[(1+i)^t - 1]}{(1+i) - 1}$$

$$N_t = \frac{M_0[(1+i)^t - 1]}{1}$$

Figure 11. Student's Representation Converted into Another Form

Figure 11 shows that students are able to represent the sum of each term in a different form by relating it to the concept of a geometric sequence. Therefore, they meet the indicator of converting one form of representation into another. Students with a low level of conceptual understanding are able to meet the indicator because there are 'partially correct constructs', knowledge that is partly correct but gives a misleading picture of the concept and leads to inconsistent answers, so that sometimes they are able to do so and sometimes they are not (Ron et al., 2017).

e. Understanding the meaning of the concept

F. GENERALISASI

1. Nilai akhir rente pranonumerado memiliki waktu setoran di Awal

2. Nilai akhir rente postnumerado memiliki waktu setoran di Akhir

3. Dengan besar setoran yang sama dan besar suku bunga yang sama nilai akhir manakah yang memiliki nilai akhir lebih besar? Apa yang memengaruhi? Jelaskan jawabanmu!

Pranonumerado, karena bayar & di terima diawal dgn angsuran bunga lebih bgyr

4. Bagaimana rumus mencari nilai akhir rente pranonumerado?

$M_t = M(1+i)(1+i)^t - 1$

5. Bagaimana rumus mencari nilai akhir rente postnumerado?

$M_t = M((1+i)^t - 1)$

Figure 12. Students with Low Performance in Understanding the Meaning of the Concept

As shown in Figure 12, students with low conceptual understanding ability explained "Prepayment, because payment is made and received upfront with higher interest installments". The students have not provided a complete explanation for question three. They should have explained their reasoning in greater detail. Furthermore, the reason the final value of the annuity is greater is because it is compounded more frequently, not because it runs for a longer period. Consequently, the students have not met the indicator of understanding the meaning of a concept. This is consistent with the research by May (2024) and Novita et al.

(2025), which found that students with low conceptual understanding tend to be able to provide answers or state certain facts, but struggle to explain the reasoning behind those answers because they do not yet understand the relationships between ideas and the principles that form a concept. Consequently, their knowledge remains procedural in nature and has not yet reached a deep conceptual understanding. Therefore, students have not yet met the indicator of conceptual understanding, which is knowing the meaning of a concept.

f. Identifying the properties and conditions of a concept

Based on Figure 9, the students were able to state the properties of the respective final values of the pre-numerando and post-numerando interest rates. Therefore, the students met the indicator of stating the properties and conditions of a concept. Students with low conceptual understanding are able to meet the indicators because there are both light and heavy indicators; students (including those with weaker abilities) can still restate the concept or explain its basic properties, but fail on other indicators such as classifying objects, using procedures, or developing the conditions of the concept (Rahmawati & Roesdiana, 2022).

g. Distinguishing between different types of concepts

Based on Figure 9 and Figure 12, students are able to distinguish between the final values of the 'pranumerando' and 'postnumerando' annuities. Students with low conceptual understanding are able to meet the indicators because students with low ability may only meet the classification indicator, whilst other indicators (representation, conceptual relationships, necessary and sufficient conditions) have not yet been achieved (Kurnia et al., 2024). Therefore, students meet the indicator of distinguishing between various types of concepts.

Implementation of the Principles of Meaningful, Mindful, and Joyful Learning.

Meaningful learning is evident in the third opening activity where the teacher provides an apersepsi. The apersepsi activity aligns with the principle of meaningful learning because it connects previously learned material with new material to be studied, making the learning more meaningful and easier for students to grasp (Azzahra & Jaya, 2025; Hattan et al., 2023). Meaningful learning is evident in the problem example used by the teachers, namely the savings between an older sibling and a younger sibling. The savings problem is closely related to the students' daily lives, thereby encouraging them to apply their

knowledge in real-life situations (Kemendikdasmen, 2025). Additionally, the teachers designed learning materials and organized discussion groups based on the initial assessment, demonstrating the application of the meaningful learning principle. Learning activities designed based on initial learning assessments constitute the application of the meaningful learning principle (Kemendikdasmen, 2025).

The presentation of learning objectives aligns with the principle of deep learning, specifically "*mindful*" (conscious), as students become aware of what they will achieve (Albina & Pratama, 2025; Azzahra & Jaya, 2025; Bordunos et al., 2024; Lestariningsih, 2022; Nurrahmawati et al., 2024; Rocane & Samusevica, 2023; Sewagegn, 2020). According to the Kemendikdasmen (2025), students knowing what they will achieve is an application of the mindful learning principle. Conscious learning is applied when a model teacher motivates learning by relating it to real-life examples, such as cell phone and motorcycle loans, and conveying the lesson that saving is better than paying off loans. Learning motivation aligns with the principle of meaningfulness because it connects the concepts to be learned with students' lives, thereby sparking their interest in learning (Polman et al., 2021; Zebua, 2025). According to the Kemendikdasmen (2025), activities that demonstrate the benefits of students' learning outcomes constitute mindful learning. The principle of engagement is evident in the teacher's activities of providing stimuli, posing probing questions, and identifying problems related to the future values of pre-numerando and post-numerando interest. These activities constitute the implementation of the mindful learning principle, as they guide students to engage in discussion and exploration (Kemendikdasmen, 2025). Additionally, the process of independently identifying problems is a key characteristic of *mindful learning* that encourages students to actively and consciously engage in the thinking process, rather than merely passively receiving information (Pujawati et al., 2025; Ricita et al., 2025).

The principle of joyful learning is evident in group discussion activities and presentations. These activities demonstrate that teachers provide opportunities for students to interact with one another, stimulate students' personal thinking, and engage in discussions. Additionally, joyful learning is evident in the instructional videos created by teachers. Through these videos, interaction occurs between students and the learning material; furthermore, the act of watching the videos sparks students' interest in the subject matter being studied (Kemendikdasmen, 2025).

The principle of conscious learning occurs when students are aware of their role as active learners and know what they are about to learn. This supports the process of

conceptual understanding, making it more effective (Azzahra & Jaya, 2025; Pujawati et al., 2025). Furthermore, meaningful learning supports students in addressing contextual problems and discovering formulas, thereby enhancing their conceptual understanding, as understanding built through independent discovery is deeper and more enduring (Mawaddah & Maryanti, 2016). Additionally, the process of meaningful learning, where students construct their own conceptual understanding, effectively supports the improvement of students' conceptual understanding. Problem-Based Learning, Discovery Learning, and constructivist-based learning strategies have consistently been shown to improve students' mathematical conceptual understanding, critical thinking, and active engagement (Saleh, 2025). Furthermore, the principle of enjoyment reflected in presentation activities also supports students' conceptual understanding. Presenting discussion outcomes in a discovery learning setting has been shown to enhance mathematical communication skills and strengthen students' conceptual understanding because, through the process of verbalization and justification, students are compelled to restructure their knowledge in a more systematic and coherent manner (Pujawati et al., 2025; Mawaddah & Maryanti, 2016).

Challenges and Strategies in Implementing Deep Learning

The main challenge faced by the teacher was limited instructional time, which resulted in the closing activity not being implemented as planned. This issue was primarily caused by the transition of students from the computer laboratory to the classroom, which took longer than anticipated. As the teacher reflected, "...the planning should have been sufficient since it was estimated from the core opening to the closing, but there was an internal factor: moving students from the lab to the classroom. Perhaps because of the mobilization and preparation, it ended up taking longer..." This aligns with findings from various studies that the implementation of the deep learning approach in the field still faces several challenges, including limitations in planning and implementation time, where the differentiation of content, process, and products required for quality learning takes longer than conventional learning (Mukhoyaroh et al., 2025). Additionally, the teacher encountered challenges with the characteristics of students in the classroom, who were quite active, talking and moving around a lot, thus requiring more time during the conditioning process. The strategy employed by the model teacher to overcome these challenges is to have students engage in self-reflection at home. The teacher provides a QR code that students can scan, which contains reflection questions related to the learning that has taken place. Thus, the function of reflection as an important part of the learning process remains sufficiently fulfilled.

CONCLUSION

This study found that the implementation of deep learning through the discovery learning model on the topic of the future value of an annuity supported students' conceptual understanding of mathematics. The results revealed differences in conceptual understanding across student ability levels. Students with high conceptual understanding achieved all seven NCTM indicators of conceptual understanding. Students with moderate conceptual understanding achieved five indicators but experienced difficulties in defining concepts verbally and in writing, providing examples and non-examples, and understanding the meaning of concepts. Meanwhile, students with low conceptual understanding achieved only three indicators and faced difficulties in defining concepts verbally and in writing, providing examples and non-examples, using diagrams, models, and symbols to represent concepts, and identifying the properties and conditions of concepts.

These findings suggest that deep learning can facilitate students' conceptual understanding through contextual problem-solving, discussion, and active knowledge construction. However, students with lower levels of conceptual understanding still required additional support, particularly in developing a deeper understanding of mathematical concepts and their representations. Furthermore, the implementation was not fully optimized because reflection and concluding activities could not be completed due to time constraints, highlighting the importance of effective time management in supporting the complete deep learning process.

This study was limited to one class, and one mathematics topic, which may limit the transferability of the findings. Therefore, future research should involve a larger number of participants and various mathematics topics to examine the effectiveness of deep learning across different learning contexts and student characteristics.

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REFERENCES

- Afidati, M., & Malasari, P. N. (2023). Pembelajaran matematika yang bermakna menggunakan pendekatan teori kognitivisme. *Journal of Mathematics Educations*, 2(2), 67–77. <https://doi.org/https://doi.org/10.58917/ijme.v2i2.67>
- Albina, M., & Pratama, K. B. (2025). Peran tujuan pembelajaran dalam perencanaan pembelajaran: dasar untuk pembelajaran yang efektif. *Harmoni Pendidikan: Jurnal Ilmu Pendidikan*, 2(2), 55–61. <https://doi.org/DOI:https://doi.org/10.62383/hardik.v2i2.1233>
- Andriatna, R., Kurniawati, I., Wulandari, A. N., Studi, P., Matematika, P., & Maret, U. S. (2021). Profil kemampuan literasi statistik mahasiswa calon guru matematika. *FIBONACCI:Jurnal Pendidikan Matematika*, 7(1), 19–28. <https://doi.org/https://dx.doi.org/10.24853/fbc.7.1.19-28>
- Aprilia, E. D., Nurfitriya, L., Ningrum, S. C., & Rofiki, I. (2024). Profil pengajaran guru matematika terhadap pengelolaan kelas untuk siswa smp pada materi bilangan berpangkat. *Juring (Journal for Research in Mathematics Learning)*, 7(2), 171–184. <https://doi.org/http://dx.doi.org/10.24014/juring.v7i2.29140>
- Azizah, A. N., Arrindha, N., Suwana, C. A., & Rofiki, I. (2024). Profil pengelolaan kelas guru matematika smp dengan sistem blok pada materi teorema pythagoras. *Suska Journal of Mathematic Education*, 10(1), 25–38. <https://doi.org/http://dx.doi.org/10.24014/sjme.v10i1.29146>
- Azzahra, Y., & Jaya, C. A. (2025). Pendekatan deep learning: transformasi mindful, meaningful, dan joyful dalam pembelajaran holistik. *EduInovasi: Journal of Basic Educational Studies*, 5(3), 769–776. <https://doi.org/47467/eduinovasi.v5i3.9245>
- Barut, M. E. O. B., Wijaya, A., & Retnawati, H. (2021). Hubungan pedagogical content knowledge guru matematika dan prestasi belajar siswa sekolah menengah pertama. *Pythagoras: Jurnal Pendidikan Matematika*, 15(2), 178–189. <https://doi.org/10.21831/pg.v15i2.35375>
- Bordunos, A. K., Miletich, M. P., & Volkova, N. V. (2024). Mindful learning: Principles and prospect of use in higher education. *Psychological Science and Education*, 29(4), 16–30. <https://doi.org/https://doi.org/10.17759/pse.2024290402>
- Budiarty, P. D., & Utomo, R. B. (2024). Kemampuan representasi matematis siswa ditinjau dari kemampuan awal matematika. *Cendekia*, 2(3), 420–427. <https://doi.org/https://doi.org/10.572349/cendikia.v2i3.1150>
- Burfitt. (2021). Using partial knowledge to inform the creation of learning progressions. *Australian Journal of Education*, 65(3), 299–312. <https://doi.org/https://doi.org/10.1177/00049441211037409>
- Dalia, A., Muslihin, H. Y., & Nur, L. (2025). Analisis kebutuhan desain pembelajaran mendalam (deep learning) matematika berbasis permainan congklak di sekolah dasar. *JPP*, 22(2), 202–210. <https://doi.org/https://doi.org/10.17509/jpp.v25i2.83393>
- Fawzia, S., & Karim, A. (2024). Exploring the connection between deep learning and learning assessments: a cross-disciplinary engineering education perspective. *Humanit Soc Sci Commun Icatations*, 11(29), 1–10. <https://doi.org/10.1057/s41599-023-02542-9>
- Febrianti, T. S., Nisa, R., & Indonesia, U. P. (2025). Asesmen diagnostik matematika: Mendesain pembelajaran efektif berdasarkan kesiapan belajar siswa SMK. *Jurnal Penelitian Kebijakan Pendidikan*, 17(2), 81–98. <https://doi.org/https://doi.org/10.24832/jpkp.v17i2.900>

- Firdausy, F. H., Rofiki, I., Nova, S., & Cesar, R. (2023). Profil pengelolaan kelas guru matematika SMA pada materi lingkaran. *Juring (Journal for Research in Mathematics Learning)* p-ISSN:, 6(4), 373–390. <https://doi.org/http://dx.doi.org/10.24014/juring.v6i4.26335>
- Gianto, Mariana, N., & Umah, U. (2026). Enhancing learning through deep learning : Does it foster elementary s tudents ' conceptual understanding and motivation in mathematics ? *JMPM: Jurnal Matematika Dan Pendidikan Matematika*, 11(1), 23–35. <https://doi.org/https://journal.unipdu.ac.id/index.php/jmpm/article/view/6373>
- Güç, A., & Kollosche, D. (2022). Mathematics learner identity in student-centred teaching. *J Math Didakt*, 43(6), 231–254. <https://doi.org/10.1007/s13138-021-00187-2>
- Guler, N. K., & Tarim, K. (2024). An analysis of classification skills of the 6 th grade students on fractions. *Pedagogical Research*, 9(2), 1–16. <https://doi.org/https://doi.org/10.29333/pr/14278>
- Hadiyani, V. P., Sulalah, S., & Widodo, B. (2025). Pembelajaran matematika menggunakan metode scaffolding berbasis diferensiasi terhadap kemandirian belajar siswa. *Biormatika : Jurnal Ilmiah Fakultas Keguruan Dan Ilmu Pendidikan*, 11(1), 63–75. <https://doi.org/10.35569/biormatika.v11i1.2240>
- Hattan, C., Alexander, P. A., & Lupo, S. M. (2023). Leveraging what students know to make sense of sexts: What the research says about prior knowledge activation. *Sage Journal*, 94(1), 71–111. <https://doi.org/https://doi.org/10.3102/00346543221148478>
- Hattie, J. A. C., & Donoghue, G. M. (2016). Learning strategies : a synthesis and conceptual model. *Nature Publishing Group*, 1(1), 1–13. <https://doi.org/https://doi.org/10.1038/npjscilearn.2016.13>
- Hermaya, A., & Hendrayana, A. (2025). The effect of conceptual understanding on mathematical problem-solving skills in gamification learning. *EMTEKA: Jurnal Pendidikan Matematika*, 6(2), 896–908. <https://doi.org/https://doi.org/10.24127/emteka.v6i1.7874>
- Hino, K., & Funahashi, Y. (2022). Teachers ' guidance of students ' focus toward lesson objectives : how does a competent teacher make decisions in the key interactions ? *ZDM – Mathematics Education*, 54(2), 343–357. <https://doi.org/10.1007/s11858-022-01345-7>
- Huda, N., & Marzal, J. (2026). Analyzing students' thinking processes in solving exponential problems based on APOS theory : A qualitative study at SMA negeri 7 tanjung jabung timur. *EDUMATSAINS:Jurnal Pendidikan Matematika Dan Sains*, 10(April), 69–81. <https://doi.org/https://doi.org/10.33541/edumatsains.v10i4.8076>
- Insani, F., Nuroso, H., Purnamasari, I., Sarjana, F. P., & Semarang, U. P. (2023). Analisis hasil asesmen diagnostik sebagai dasar pelaksanaan pembelajaran berdiferensiasi di sekolah dasar. *Didaktik : Jurnal Ilmiah PGSD FKIP Universitas Mandiri*, 09(2), 4450–4458. <https://doi.org/https://doi.org/10.36989/didaktik.v9i2.1154>
- Johar, R., & Lubis, K. R. (2018). The analysis of students ' mathematical representation errors in solving word problem related to graph. *Jurnal Riset Pendidikan Matematika* 5, 5(1), 96–107. <https://doi.org/http://dx.doi.org/10.21831/jrpm.v5i1.17277>
- Kemendikdasmen. (2025). *Pembelajaran mendalam*. Pusat Kurikulum dan Pembelajaran Badan Standar, Kurikulum, dan Asesmen Kementrian Pendidikan Dasar dan Menengah Republik Indonesia.
- Khusna, H., Purnomo, B. A., & Awalludin, S. A. (2021). Perspektif gender dalam merancang model matematika pada siswa smp. *Jurnal Equation*, 4(1), 60–68. <https://doi.org/10.29300/equation.v4i1.4104>

- Kilpatrick, J., Swafford, J., & Findell, B. (2001). Adding it up: Helping children learn mathematics. *National Academic Press*.
<https://doi.org/https://doi.org/10.17226/9822%E2%81%A0%EF%BF%BD>
- Komarudin, Rinaldi, A., Yulina, R., Suherman, & Puspita, L. (2023). Kemampuan berpikir kreatif dan pemahaman konsep matematis siswa : Analisis jalur dengan structural equation model. *PYTHAGORAS: Jurnal Matematika Dan Pendidikan Matematika*, 18(1), 67–76. <https://doi.org/https://doi.org/10.21831/pythagoras.v18i1.57048>
- Kooloos, C., Oolbekkink, H., & Boven, S. Van. (2022). Building on student mathematical thinking in whole-class discourse: exploring teachers' in-the-moment decision-making, interpretation, and underlying conceptions. *Journal of Mathematics Teacher Education*, 25(4), 453–477. <https://doi.org/10.1007/s10857-021-09499-z>
- Kurnia, W. H., Imswatama, A., & Setiani, A. (2024). Analysis of ability to understand mathematical concepts class students XI in geometric transformation materials. *PRISMA*, 13(1), 52–59. <https://doi.org/10.35194/jp.v13i1.4005>
- Latifa, U., Indrasari, P., Pramuditya, S. A., & Asnawati, S. (2025). Connecting concepts and representations in mathematics: A sistematic literature review. *International Journal of Contemporary Studies in Education*, 4(2), 100–113. <https://doi.org/https://doi.org/10.56855/ijcse.v4i2.1483>
- Lefrida, R. (2024). Concept understanding students on the material of two-variables linear equation system in terms of mathematics ability. *Prisma Sains: Jurnal Pengkajian Ilmu Dan Pembelajaran Matematika Dan IPA IKIP Mataram*, 12(1), 160–171. <https://doi.org/https://doi.org/10.33394/j-ps.v12i1.10532>
- Lestariningsih, S. (2022). Meaningful learning model in the field of Indonesian language studies. *Lestariningsih LINGUISTICS AND EDUCATION JOURNAL*, 2(2), 104–110. <https://doi.org/https://doi.org/10.26877/lej.v2i2.22028>
- Marwiyah, & Wardani, K. W. (2024). Implementasi model discovery learning melalui pembelajaran berdiferensiasi untuk meningkatkan kolaborasi pelajaran matematika kelas 1 SD. *FARABI Jurnal Matematika Dan Pendidikan Matematika*, 7(2023), 140–151. <https://doi.org/https://doi.org/10.47662/farabi.v7i2.748>
- Maulidya, S. R., Insani, S. U., Studi, P., Matematika, P., Pendidikan, F. I., Pahlawan, U., & Tambusai, T. (2025). Deep learning untuk mendukung pemahaman mendalam dalam pembelajaran matematika. *Jurnal Pengabdian Masyarakat Dan Riset Pendidikan*, 4(1), 1274–1278. <https://doi.org/https://doi.org/10.31004/jerkin.v4i1.1729>
- Mawaddah, S., & Maryanti, R. (2016). Kemampuan pemahaman konsep matematis siswa smp dalam pembelajaran menggunakan model penemuan terbimbing (discovery learning)le. *EDU-MAT Jurnal Pendidikan Matematika*, 4(April), 76–85. <https://doi.org/https://dx.doi.org/10.20527/edumat.v4i1.2292>
- May, B. (2024). Retrieval practice and test-potentiated learning : A comparison of high and low prior topic knowledge students in terms of procedural fluency and conceptual understanding. *Pythagoras - Journal of the Association for Mathematics Education of South Africa*, 45(1), 1–14. <https://doi.org/https://doi.org/10.4102/pythagoras.v45i1.755>
- Mayasari, D., & Habeahan, N. L. S. (2021). The ability of students' conceptual understanding in completing story problems on mathematics. *JURNAL PENDIDIKAN MATEMATIKA DAN IPA*, 12(2), 123–136.
<https://doi.org/http://dx.doi.org/10.26418/jpmipa.v12i2.43354>
- Miles, M. B., Hubberman, A. M., & Saldaña, J. (2014). *Qualitative Data Analysis: A Method Sourcebook* (3rd ed.). Sage Publication, Inc.
- Mukhoyaroh, Sodikin, A., & Waluyo. (2025). Implementation of deep learning approaches : Challenges and solutions for teachers. *Implementation of Deep Learning Approaches*:

- Challenges and Solutions for Teachers*, 6(2), 134–146.
<https://doi.org/https://doi.org/10.52187/rdt.v6i2.335>
- Mumtazah, M. R., & Triyana, I. W. (2025). Kemampuan pemahaman konsep bangun ruang ditinjau dari motivasi belajar. *SCIENCE : Jurnal Inovasi Pendidikan Matematika Dan IPA Vol.*, 5(3), 1189–1200. <https://doi.org/https://doi.org/10.51878/science.v5i3.6666>
- Mystakidis, S. (2021). Deep Meaningful Learning. In *Encyclopedia 2021* (pp. 988–997). <https://doi.org/https://doi.org/10.3390/encyclopedia1030075> Academic
- Nisak, A., & Yuliasuti, R. (2022). Profil kesiapan guru dalam mengimplementasikan kurikulum merdeka di smp negeri 1 palang. *Jurnal Riset Pembelajaran Matematika*, 4(2), 61–66. <https://doi.org/https://doi.org/10.55719/jrpm.v4i2.527>
- Novita, R., Putra, M., Usman, & Junita, S. (2025). Conceptual and procedural knowledge in students' fraction division thinking : Insights from a hermeneutic phenomenological study. *Journal on Mathematics Education*, 16(4), 1233–1256. <https://doi.org/https://doi.org/10.22342/jme.v16i4.pp1233-1256>
- Nugraheni, T. V. T., & Jailani, J. (2020). Pengembangan keprofesian berkelanjutan (PKB) dalam kaitannya dengan kompetensi dan praktik pembelajaran guru matematika SMA. *Pythagoras: Jurnal Pendidikan Matematika*, 15(1), 48–60. <https://doi.org/10.21831/pg.v15i1.34601>
- Nuraini, I., & Imami, A. I. (2025). Analysis of the students' mathematical conceptual understanding ability on sequences and series material in senior high school. *EMTEKA: Jurnal Pendidikan Matematika*, 6(1), 226–238. <https://doi.org/https://doi.org/10.24127/emteka.v6i1.7661> e-ISSN
- Nurilla, J. (2022). High school students' generalization viewed from logical mathematical intelligence. *MATHEdunesa Jurnal Ilmiah Pendidikan Matematika*, 11(1), 111–120. <https://doi.org/https://doi.org/10.26740/mathedunesa.v11n1.p111-127>
- Nurrahmawati, D., Magdalena, S. E., Vidiastuti, Y., Suryanda, A., Safitri, D., & Isfaeni, H. (2024). Analisis pengaruh tujuan pembelajaran yang jelas terhadap ketercapaian pembelajaran biologi siswa SMA ketercapaian pembelajaran biologi siswa SMA. *JB&P : Jurnal Biologi Dan Pembelajarannya*, 11(2), 189–199. <https://doi.org/https://doi.org/10.29407/jbp.v11i2.22838>
- OECD. (2022). *PISA 2022 results comparing countries' and economies' performance in reading PISA 2022 results*. OECD Publishing.
- Polman, J., Hornstra, L., & Volman, M. (2021). The meaning of meaningful learning in mathematics in upper - primary education. *Learning Environments Research*, 24(3), 469–486. <https://doi.org/10.1007/s10984-020-09337-8>
- Pujawati, F., Azkia, M. N., & Susilawati, W. (2025). Exploration of the implementation of deep learning approach in teaching mathematics in secondary schools. *Unnes Journal of Mathematics Education*, 6927(2), 98–105. <https://doi.org/https://doi.org/10.15294/ujme.v14i2.27374>
- Rahmawati, N. D., & Roesdiana, L. (2022). Analisis kemampuan pemahaman konsep matematis siswa sma pada materi turunan fungsi aljabar. *Jurnal Edukasi Dan Sains Matematika (JES-MAT)*, 8(1), 17–32. <https://doi.org/https://doi.org/10.25134/jes-mat.v8i1.5579>
- Ratnayanti, N., Suwanti, V., Matematika, P., & Kanjuruhan, U. P. (2021). Analisis kesalahan konsep matematika siswa dalam menyelesaikan soal berdasarkan kaksonomi SOLO. *Buana: Jurnal Ilmiah Matematika Dan Pendidikan Matematika*, 11(1), 95–110. <https://doi.org/https://doi.org/10.36456/buanamatematika.v11i1.3714>
- Ricita, D. A., Halim, A., Muhammadiyah, U., & Timur, K. (2025). Exploring teachers' and students' perceptions of deep learning : Integrating meaningful, mindful, and joyful

- learning in ELT classrooms. *Broju: Borneo Education Journal*, 7(2), 323–347. <https://doi.org/https://doi.org/10.24903/bej.v7i2.2078>
- Rocane, M., & Samusevica, A. (2023). Developing a culture of mindful learning: Challenges, benefit, and opportunities. *Proceedings of the International Scientific Conference, I*, 644–654. <https://doi.org/https://doi.org/10.17770/sie2023vol1.7111>
- Ron, G., Dreyfus, T., & Hershkowitz, R. (2017). Looking back to the roots of partially correct constructs: The case of the area model in probability. *The Journal of Mathematical Behavior*, 45, 15–34. <https://doi.org/10.1016/j.jmathb.2016.10.004>
- Saleh, R. (2025). Tinjauan sistematis terhadap strategi pembelajaran matematika yang efektif di Sekolah dasar: Perspektif teori konstruktivisme. *Jurnal Pendidikan Tambusai*, 9(2), 19092–19100. <https://doi.org/https://doi.org/10.31004/jptam.v9i2.29204>
- Santoso, I., & Noormandiri, B. . (2023). *Matematika untuk SMA/MA kelas XII*. Erlangga.
- Sari, N. P., Caswita, & Wijaya, A. P. (2025). Analisis kesalahan kemampuan representasi matematis dalam mengerjakan soal matematika materi statistik. *Mandalika Mathematics and Education Journal*, 7(4), 1774–1788. <https://doi.org/http://dx.doi.org/10.29303/jm.v3i1.10577> Analisis
- Setyo, A., Lestari, B., & Arifah, S. (2025). Menyingkap karakteristik penalaran matematis siswa dalam pemecahan masalah geometri ruang : Tinjauan berdasarkan kategori kemampuan matematika. *Internasional Journal of Progressive Mathematics Education ISSN*, 5(1), 256–269. <https://doi.org/https://doi.org/10.22236/ijopme.v5i1.18643>
- Sewagegn, A. A. (2020). Learning objective and assessment linkage : Its contribution to meaningful student learning. *Universal Journal of Educational Research*, 8(11), 5044–5052. <https://doi.org/https://doi.org/10.13189/ujer.2020.081104>
- Sri Wahyuning, L. D., & Utami, N. S. (2024). Mathematical representation ability-based mathematical contextual problems of sequences and progression material. *Vygotsky: Jurnal Pendidikan Matematika Dan Matematika*, 6(1), 61–70. <https://doi.org/https://garuda.kemdiktisaintek.go.id/documents/detail/3920393>
- Sufa, F. F. (2022). Profil tingkat kesulitan guru matematika dalam penerapan pembelajaran berbasis hots dilihat dari dimensi penilaian. *Jurnal Riset Pembelajaran*, 4(2), 37–42. <https://doi.org/https://doi.org/10.55719/jrpm.v4i2.512>
- Surven, Sarumaha, A., & La'ia, H. T. (2022). Analisis kemampuan pemahaman konsep matematika siswa pada materi spltv kelas x smk negeri 1 toma. *AFORE: Jurnal Pendidikan Matematika*, 1(1), 26–40. <https://doi.org/https://doi.org/10.57094/afore.v1i1.434>
- Usman, M. R., & Husniati, A. (2025). The profile of secondary school students' written mathematical communication through problem solving assessment viewed from differences in mathematical ability. *M A T H L I N E JURNAL MATEMATIKA DAN PENDIDIKAN MATEMATIKA ISSN*, 9(1), 97–110. <https://doi.org/https://doi.org/10.31943/mathline.v10i1.729>
- Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes*. Harvard University Press.
- Yani, C. F., Roza, Y., Murni, A., & Daim, Z. (2019). Analisis kemampuan pemahaman matematis siswa pada materi bangun ruang sisi lengkung. *Mosharafa : Jurnal Pendidikan Matematika*, 8(2), 203–214. <https://doi.org/https://doi.org/10.31980/mosharafa.v8i2.553>
- Zebua, N. (2025). Education transformation : Implementation of deep learning in 21st-century learning. *Harmoni Pendidikan: Jurnal Ilmu Pendidikan*, 2(2), 146–152. <https://doi.org/https://doi.org/10.62383/hardik.v2i2.1405>

- Zuhri, M. S., & Purwosetiyono, F. D. (2019). Profil kemampuan penalaran matematis dalam pemecahan masalah pada mahasiswa calon guru matematika. *JIPMat*, 4(1), 38–43. <https://doi.org/https://scholar.archive.org/work/cngpzchynvbozenjibghuutgjm/access/wayback/http://journal.upgris.ac.id/index.php/JIPMat/article/download/3548/pdfhttps://scholar.archive.org/work/cngpzchynvbozenjibghuutgjm/access/wayback/http://journal.upgris.ac.id/index.php/JIPMat/article/download/3548/pdf>
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