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AUGMENTED AND VIRTUAL REALITY FOR ABSTRACT MATHEMATICAL VISUALIZATION: A SYSTEMATIC REVIEW OF COGNITIVE OUTCOMES

Hasby Assidiqi[1]*, Sugiman[2]
[1]Doctoral Student of Mathematics Education, Universitas Negeri Semarang, Semarang, Indonesia
[2]Department of Mathematics Education, Universitas Negeri Semarang, Semarang, Indonesia

*Correspondence: hasbymath19@students.unnes.ac.id

ABSTRACT

Augmented reality and virtual reality provide interactive visual and spatial environments for representing mathematical objects that are difficult to explore through static two dimensional media. This systematic review synthesizes how AR and VR support abstract mathematical visualization in mathematics education and examines the cognitive outcomes associated with these immersive learning experiences. Following the PRISMA 2020 framework, studies indexed in Scopus and published between 2015 and 2025 were identified, screened, and assessed for eligibility. Studies were included when they involved AR, VR, mixed reality, extended reality, or similar immersive environments in mathematics education; addressed mathematical objects, concepts, structures, or representations; and reported pedagogical applications, technological affordances, learning processes, or cognitive outcomes. Studies unrelated to mathematics education, purely technical papers without educational implementation, broad STEM studies without separable mathematics data, and reports with insufficient full text information were excluded. From 297 initial records, 42 studies were included in the final thematic synthesis. AR/VR was most frequently applied to domains with strong visual and spatial demands, especially geometry and solid geometry, while applications in calculus, functions, vectors, transformations, algebra, and trigonometry reflected broader efforts to connect symbolic, graphical, spatial, and embodied representations. Spatial ability, spatial visualization, and conceptual understanding were the most consistently reported outcomes and were associated with the externalization of abstract objects, interactive manipulation, and coordination among multiple representations. Evidence for representational fluency, problem solving, memory retention, and cognitive load was more variable. The review concludes that AR/VR can support mathematical learning when integrated with theoretically grounded task design, scaffolding, teacher orchestration, and rigorous empirical evaluation.

Keywords: Spatial Visualization, Conceptual Understanding, Immersive Technologies, Representational Mediation, Mathematics Learning.

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PRELIMINARY

Mathematical objects such as functions, vectors, transformations, derivatives, surfaces, geometric solids, and systems of equations are commonly represented and understood through symbolic, graphical, visual, verbal, spatial, and mental representations.

External visualization is therefore not merely an auxiliary instructional medium but a component of mathematical thinking and meaning making, particularly when learners must coordinate representations to develop conceptual understanding and reasoning (Schenck & Nathan, 2024; Schoenherr & Schukajlow, 2024).

These representational demands are especially pronounced in solid geometry, vectors, multivariable calculus, trigonometry, transformations, three dimensional graphs, and systems of linear equations, where learners engage in mental rotation, spatial visualization, orientation, perspective taking, and structural interpretation. Static textbooks and two dimensional images can display such objects but provide limited support for manipulating, comparing, rotating, or connecting dynamic and multidimensional relations across symbolic, graphical, and spatial forms. Learning environments therefore require stronger theoretical and pedagogical alignment with spatial and representational processes (Schenck & Nathan, 2024; Schoenherr & Schukajlow, 2024).

This limitation creates a gap between seeing a mathematical object and interpreting its structure. Static representations rarely allow learners to explore how an object changes, how its components are related, or how visual and spatial experience connects with formal mathematics. Accordingly, this review focuses on spatial visualization, conceptual understanding, representational fluency, problem solving, memory retention, and cognitive load regulation. These outcomes are central because immersive technology is pedagogically meaningful only when perceptual interaction contributes to mathematical interpretation and reasoning.

AR and VR can address this representational gap in different ways. AR overlays digital mathematical objects on physical space, whereas VR provides a fully simulated environment for exploration from multiple perspectives and for spatial navigation. This review therefore treats AR and VR as related immersive representational technologies while retaining their distinct affordances. AR supports connections between physical and digital space, whereas VR supports exploration within a fully immersive spatial environment. Previous reviews report benefits for engagement, interaction, conceptual understanding, and immersive learning, together with challenges involving cognitive load, usability, accessibility, technological readiness, and pedagogical integration (Akçayır & Akçayır, 2017; Makransky & Petersen, 2021; Radianti et al., 2020).

From a cognitive perspective, the value of AR/VR lies not in visual richness itself but in how immersion and interactivity support representational processing within the limits of working memory. Cognitive load theory emphasizes the management of unnecessary demands so that visual and interactive information can support schema construction (Sweller et al., 2019). Multimedia learning theory explains learning through the selection, organization, and integration of verbal and visual information (Mayer, 2021). CAMIL further relates immersive learning to presence, agency, embodiment, motivation, self regulation, and cognitive load (Makransky & Petersen, 2021).

Despite growing research, the literature remains fragmented across mathematical domains, technological designs, and forms of evidence. Patterns across domains have not been sufficiently synthesized, many studies emphasize usability or engagement rather than the relationship between AR/VR affordances and cognitive outcomes, and methodological strength ranges from quasi experiments to prototypes, pilot studies, usability tests, and perception studies.

To address these gaps, this systematic review synthesizes studies indexed in Scopus and published from 2015 to 2025 through four analytical dimensions: represented mathematical objects, pedagogical and technological affordances, cognitive outcomes, and evidence strength. The review addresses four questions: (1) What abstract mathematical objects have been visualized using AR/VR? (2) How are AR and VR applied pedagogically, and what technological affordances are used? (3) What cognitive outcomes are associated with visualization using AR/VR? (4) What research designs, evidence bases, methodological gaps, and future directions are evident? By relating what is represented, how learners interact with representations, and what cognitive evidence is reported, the review moves beyond mapping applications toward identifying conditions for meaningful mathematical mediation.

METHODS

This study used a systematic literature review design and followed PRISMA 2020 to support transparent reporting of identification, screening, eligibility, and inclusion (Page et al., 2021). Because the included studies varied in research design, participants, educational level, technology, mathematical objects, pedagogical applications, and outcome measures, an interpretive thematic synthesis was used rather than simple frequency aggregation. The analysis involved extracting study information, coding mathematical objects and AR/VR affordances, grouping codes into themes, comparing patterns across studies, and interpreting the findings in relation to the research questions.

Scopus was selected because its multidisciplinary coverage spans education, mathematics, computer science, learning technology, and social sciences, the areas across which research on AR/VR in mathematics education is distributed.

The search covered studies published from 2015 to 2025 and was conducted in the Scopus TITLE-ABS-KEY field on 16 March 2026 using the following query: TITLE-ABS-KEY (("augmented reality" OR "virtual reality" OR "mixed reality" OR "extended reality" OR "immersive technology" OR "immersive environment" OR "3D visualization" OR "three-dimensional visualization") AND ("mathematics education" OR "mathematics learning" OR "mathematics teaching" OR "learning mathematics" OR "teaching mathematics") AND (visualization OR visualisation OR representation OR "mathematical representation" OR "abstract mathematical object" OR "mathematical object" OR geometry OR calculus OR function OR graph OR vector OR algebra OR trigonometry)). The terms were specified explicitly without wildcard operators to preserve conceptual precision.

The records were limited to English language articles, conference papers, and reviews in Computer Science, Mathematics, and Social Sciences. The initial search returned 297 records, and database filtering left 142 records for title screening. AR was operationally defined as an environment that overlays digital mathematical objects on physical space. VR was defined as a simulated environment that supports immersive navigation or spatial interaction. MR and XR were included when they provided comparable immersive or spatially interactive representations of mathematical objects.

Studies were included when they involved AR, VR, MR, XR, or similar immersive visual and spatial environments in mathematics education; addressed mathematical objects, concepts, structures, or representations; reported pedagogical applications, technological affordances, learning processes, or cognitive outcomes; met the language and publication year limits; and provided sufficient full text information for systematic extraction. Studies were excluded when they were unrelated to mathematics education, were purely technical without educational implementation, addressed broad STEM learning without separable mathematics data, did not involve actual AR/VR implementation, duplicated or overlapped with another report, or lacked sufficient information for coding.

Table 1. Data Extraction and Coding Matrix

Category	Operational description	RQ
Technology type	AR, VR, MR, XR, WebXR, mobile AR, or immersive 3D environment	RQ2
Mathematical focus	Domain and object, such as geometry, calculus, functions, vectors, transformations, solids, surfaces, graphs, systems of equations, or trigonometry	RQ1
Pedagogical use and affordance	Learning tasks, visualization, manipulation, immersion, embodiment, interaction, collaboration, accessibility, or scaffolding	RQ2

Category	Operational description	RQ
Learning process	Observing, manipulating, comparing, navigating, explaining, validating, or coordinating representations	RQ2-RQ3
Outcome and evidence	Cognitive outcome, research design, instrument or data source, and evidence strength	RQ3-RQ4

Screening proceeded through database filtering, title screening, abstract screening, and full text eligibility assessment. Of the 297 initial records, 142 remained after bibliographic filtering; 63 were excluded during title screening and 9 during abstract screening. Full texts were sought for 70 reports. Twenty reports could not be retrieved or did not provide sufficient full text information, leaving 50 reports for eligibility assessment. Eight were excluded at this stage, resulting in 42 studies for thematic synthesis (Figure 1).

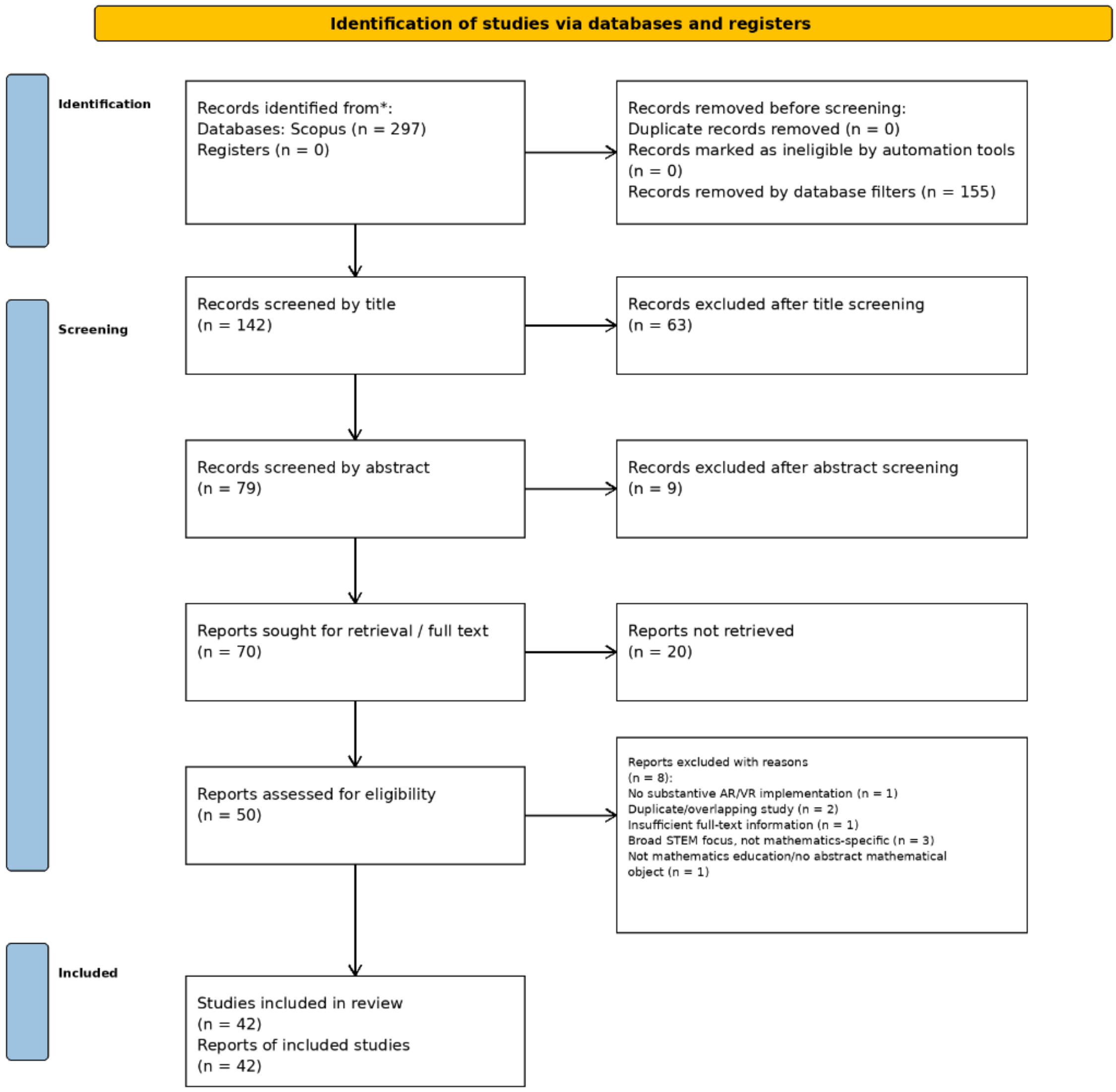

Figure 1. PRISMA 2020-based study selection flow diagram adapted from Page et al. (2021)

Data were coded using the matrix in Table 1. Mathematical objects included solids, surfaces, graphs, functions, vectors, transformations, systems of equations, derivatives, and trigonometric representations. Pedagogical applications and technological affordances included dynamic 3D visualization, interactive manipulation, spatial navigation, multiple representations, embodied interaction, gamification, collaboration, accessibility, and visual scaffolding. Cognitive outcomes included spatial visualization, spatial ability, conceptual understanding, representational fluency, problem solving, memory retention, and cognitive load. Engagement, motivation, usability, and technology acceptance were treated as supporting evidence rather than direct evidence of cognitive improvement.

Table 2. Evidence-strength classification

Criterion	Strong evidence	Moderate evidence	Weak evidence
Research design	Experimental, quasi-experimental, comparison group, or clear pre-test-post-test design	Empirical implementation with limited comparison or design control	Prototype, demonstration, usability study, or exploratory pilot
Outcome measurement	Direct cognitive measurement using validated or clearly described instruments	Cognitive or learning outcomes measured with partial limitations	Mainly perception, acceptance, usability, engagement, or feasibility data
Reporting clarity	Clear participants, tasks, procedures, duration, and context	Some implementation details incomplete	Limited explanation of educational implementation

Evidence was classified as strong, moderate, or weak (Table 2) according to research design, clarity of participants and educational context, clarity of instruments or data sources, and alignment between reported outcomes and claims. Experimental and quasi experimental studies primarily supported claims about cognitive outcomes. Qualitative and semiotic studies were used to interpret meaning making and representational processes, whereas prototype and usability studies informed technological affordances, feasibility, and research gaps. A meta analysis was not conducted because the studies did not provide sufficiently comparable quantitative data, effect sizes, instruments, intervention structures, or outcome measures.

RESULT AND DISCUSSION

The final corpus comprised 42 studies. Of the 50 full texts assessed, eight were excluded: three because broad STEM data could not be separated for mathematics education, two because they duplicated or overlapped with more complete reports, and one each because it lacked substantive AR/VR implementation, provided insufficient full text information for systematic extraction, or did not match the focus on mathematics education and abstract mathematical objects. Reports that could not be retrieved were not counted as eligibility exclusions.

Table 3. Mapping of included studies by mathematical domain

Domain	Included studies
Geometry and solid geometry	(Carlos-Chullo et al., 2020; J. Cerqueira et al., 2018; Dalager & Majgaard, 2022; Gargrish et al., 2021, 2022; Mandala et al., 2025; Mondido et al., 2024; Nadzeri et al., 2024; Rashevska et al., 2020; Sarkar et al., 2018; Sudirman et al., 2025; Zekeik et al., 2025)
Calculus and multivariable representations	(Bikner-Ahsbahs, 2025; Capone & Lepore, 2020; Cheong et al., 2023; Franco Esparraguera et al., 2024; Herrera et al., 2019; Medina Herrera et al., 2019; Paulo et al., 2021; Pinter & Siddiqui, 2024; Quintero et al., 2015; Salinas Martínez & González Mendívil, 2017; Wong et al., 2022)
Functions, graphs, algebra, and trigonometry	(Alzoubi, 2024; J. M. Cerqueira et al., 2020; Del Cerro Velázquez & Méndez, 2021; Espinosa-Pinos et al., 2023; Gaggi et al., 2023; Li et al., 2022; Singh & May, 2024; Yasmin, 2024)
Vectors, linear algebra, and transformations	(Cabanag, 2023; Chang et al., 2025; Giancaspro et al., 2025; Nowak et al., 2025; Schutera et al., 2021; Shaghaghian et al., 2022)
Semiotic, embodied, accessibility, and inclusion	(Arzarello & Bagossi, 2025; Bikner-Ahsbahs, 2025; Gaggi et al., 2023; Keller et al., 2018; Shaghaghian et al., 2022)

Table 3 shows that the studies were unevenly distributed across mathematical domains. Geometry and solid geometry formed the largest cluster, followed by calculus, functions, vectors, and transformations, with additional studies addressing semiotic processes, embodiment, accessibility, and inclusion. Because some studies addressed more than one analytical domain, individual studies could appear in more than one category; therefore, category frequencies are not additive. This distribution indicates a gradual expansion from explicitly spatial objects toward more relational and representational forms of mathematics.

Table 4. Synthesis of results by research question

RQ	Main finding	Interpretation
RQ1	AR/VR was most frequently applied to geometry and solid geometry, followed by calculus, functions, vectors, transformations, algebra, and trigonometry.	The dominant pattern reflects strong alignment between AR/VR affordances and mathematical domains with high visual-spatial demands.
RQ2	Pedagogical applications included dynamic 3D visualization, manipulation, multiple	AR/VR is strongest when embedded in explicit mathematical tasks rather

RQ	Main finding	Interpretation
	representations, embodied interaction, gamification, collaboration, accessibility, and visual scaffolding.	than used as attractive technology alone.
RQ3	Spatial ability, spatial visualization, and conceptual understanding were the most consistently reported cognitive outcomes.	These outcomes are associated with externalization of abstract objects, interactive manipulation, and coordination among representations.
RQ4	The evidence base was positive but heterogeneous, ranging from quasi-experiments to prototypes and usability studies.	Claims about effectiveness should be grounded primarily in stronger designs and valid cognitive instruments.

For RQ1, geometry and solid geometry formed the dominant category. Studies in this group visualized cubes, prisms, cylinders, cones, spheres, polyhedra, nets, surface area, and volume. This concentration is consistent with the spatial character of geometric objects and with AR/VR affordances for rotation, manipulation, and perspective taking (Dalager & Majgaard, 2022; Gargrish et al., 2021; Nadzeri et al., 2024; Zekeik et al., 2025). In calculus and multivariable calculus, represented objects included solids of revolution, surfaces, derivatives, partial derivatives, level curves, and local changes on surfaces (Cheong et al., 2023; Salinas Martínez & González Mendívil, 2017). Other studies addressed functions, graphs, vectors, systems of linear equations, and transformations, indicating that AR/VR applications are extending from spatial objects toward more abstract mathematical relationships (Li et al., 2022; Nowak et al., 2025; Shaghaghian et al., 2022).

This pattern suggests that AR/VR is particularly aligned with mathematical objects whose spatial relationships are difficult to infer from static diagrams. In geometry and calculus, for example, learners must coordinate object shape, orientation, movement, and symbolic notation. AR/VR can support this coordination by allowing learners to observe objects from multiple viewpoints, manipulate parameters, and connect visual and spatial features with mathematical definitions. At the same time, the concentration of studies in explicitly spatial topics indicates a current limitation: domains in which abstraction is primarily relational, logical, or symbolic remain comparatively underexplored.

For RQ2, AR/VR was implemented through dynamic 3D visualization, interactive manipulation, multiple representations, embodied interaction, game based learning, collaborative learning, accessibility, and visual scaffolding. Interactive manipulation allowed learners to rotate objects, change scale, modify parameters, and explore mathematical structures. The use of multiple representations was important because AR/VR could connect symbolic, graphical, visual, spatial, and verbal forms. Embodied interaction further supported visualization through gestures, bodily movement, spatial orientation, and interaction with virtual objects (Arzarello & Bagossi, 2025; Shaghaghian et al., 2022).

The synthesis further indicates that these affordances become pedagogically valuable when embedded in purposeful mathematical activity. Dynamic visualization may help learners notice structures, but interpretation requires more than observation. Interactive manipulation becomes meaningful when learners predict changes, compare cases, explain invariant properties, and connect observed changes with symbolic or graphical representations. Likewise, embodied interaction can support reasoning when bodily movement is explicitly connected with mathematical transformation, direction, orientation, or variation. The pedagogical value of AR/VR therefore depends less on the presence of immersive features than on how those features are organized through tasks, questions, reflection, and feedback.

For RQ3, the reported cognitive outcomes included conceptual understanding, spatial ability, spatial visualization, representational fluency, problem solving, memory retention, and cognitive load. The most consistent evidence concerned spatial ability, spatial visualization, and conceptual understanding, which is consistent with the prevalence of three dimensional objects and tasks requiring spatial coordination. AR geometry learning assistants were associated with understanding of 3D objects and memory retention (Gargrish et al., 2021, 2022), while AR for linear functions supported representational fluency (Li et al., 2022). Evidence for representational fluency, problem solving, memory retention, and cognitive load was less consistent because fewer studies used specific validated instruments for these outcomes.

The prominence of spatial ability, spatial visualization, and conceptual understanding is theoretically coherent because these outcomes correspond closely to the affordances most frequently used in AR/VR environments: three dimensional rendering, object manipulation, observation from multiple perspectives, and coordination among representations. Evidence for higher order outcomes such as problem solving and representational fluency, as well as for cognitive load regulation, was less stable because these constructs require more precise measurement and more sustained instructional sequences. Accordingly, stronger claims about reasoning, problem solving, or transfer require tasks and instruments that directly capture explanation, justification, and coordination across representations.

For RQ4, the evidence base was heterogeneous in both design and evidentiary strength. Quasi experimental studies and designs with pretest and posttest measures provide stronger support for cognitive claims, whereas prototype, usability, and pilot studies are more appropriate for establishing technological affordances and design potential. The main gaps concern theory, methodology, pedagogy, and technology. Future work therefore requires designs informed by theories of representation, embodied cognition, cognitive load, and spatial reasoning, together with validated instruments and systematic mathematical task design.

Across the four research questions, AR/VR can be conceptualized as a representational mediation environment in which abstract mathematical objects are externalized through visual, spatial, dynamic, and interactive representations that learners coordinate with formal mathematical ideas. In this sense, AR/VR functions not only as a display environment but also as a mediator among mathematical objects, learner actions, multiple representations, and cognitive processes (Figure 2).

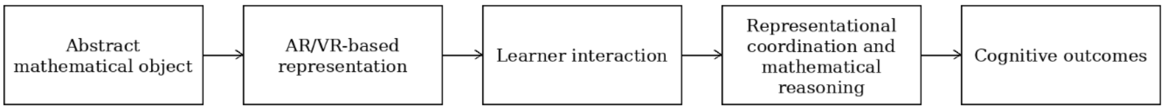

Figure 2. Conceptual Model Of AR/VR As A Representational Mediation Environment In Mathematics Education

This construct rests on three assumptions. First, mathematical understanding requires coordination among representations rather than observation of a single visual form. Second, visual and spatial interaction can support cognitive processing when learners manipulate, compare, explain, and connect representations. Third, immersive interaction becomes educationally meaningful when it is integrated with mathematical tasks, scaffolding, reflection, and teacher orchestration. Cognitive benefits should therefore not be attributed to visual richness or immersion alone, but to the interaction among characteristics of the mathematical object, representational design, learner activity, and pedagogical support. Rotating a solid, for example, can contribute to mathematical reasoning when learners identify invariant properties, relate rotation to volume or cross sections, or connect the visual form with symbolic expressions.

The concentration of studies in geometry and solid geometry shows both the current strength and the boundary of the field. Statistics, probability, proof, abstract algebra, and higher order mathematical reasoning remain underrepresented. Future designs should therefore support a transition from seeing mathematical objects to reasoning about mathematical structures through tasks that require learners to conjecture, justify, compare representations, explain relationships, validate solutions, and generalize mathematical ideas.

The synthesis also distinguishes technological affordance from pedagogical effectiveness. Immersion, interactivity, spatial manipulation, gamification, and embodiment do not automatically produce mathematical learning. Effective implementation requires alignment among the mathematical relationship to be learned, the representation used to make that relationship accessible, the learner action elicited, and the explanation or justification expected from that action. Questions, discussion, formal notation, assessment, and reflection are therefore needed to consolidate mathematical meaning. Without this pedagogical consolidation, AR/VR may remain visually engaging but conceptually shallow.

Accordingly, this review positions AR/VR as a representational mediation environment operating through four mechanisms: externalizing abstract objects, enabling interactive exploration, supporting cognitive reorganization, and requiring pedagogical consolidation. In practice, AR/VR appears particularly useful when learners must construct relations that are difficult to infer from static representations, including spatial orientation, transformation, surface behavior, vector direction, and connections between equations and graphs. Guided exploration and reflection after the activity are essential for transforming interaction into mathematical explanation.

LIMITATIONS AND RECOMMENDATIONS FOR FUTURE RESEARCH

This review has several limitations. First, the search was limited to Scopus, so relevant studies indexed exclusively in other databases may not have been captured. Second, inclusion was restricted to English language studies published from 2015 to 2025. Third, a thematic rather than meta analytic synthesis was necessary because the studies differed substantially in research design, sample characteristics, educational level, mathematical content, AR/VR platform, intervention duration, instruments, and reported outcomes. Consequently, the review does not provide a pooled effect size or statistical estimate of effectiveness.

A further limitation concerns the interpretive classification of evidence strength, which did not constitute a full risk of bias assessment for each study. Future reviews could complement this approach with standardized quality appraisal tools, interrater reliability procedures, and sensitivity analyses. Empirical research should prioritize stronger comparison designs, validated cognitive measures, larger samples, longer intervention periods, and less studied domains such as statistics, probability, proof, abstract algebra, and real analysis. Further work should also examine adaptive scaffolding, learning analytics, teacher orchestration, and explicit representational tasks as mechanisms for strengthening mathematical learning with AR/VR.

CONCLUSION

This systematic review of 42 studies indexed in Scopus and published from 2015 to 2025 shows that AR/VR is most established in mathematics domains with substantial visual and spatial demands, particularly geometry and solid geometry, with growing applications in calculus, functions, vectors, linear algebra, transformations, algebra, and trigonometry. The most consistently reported cognitive outcomes were spatial ability, spatial visualization, and conceptual understanding. Evidence for representational fluency, problem solving, memory retention, and cognitive load was more variable.

The main contribution of this review is to conceptualize AR/VR as a representational mediation environment rather than merely as a visualization tool. Its educational value arises when abstract mathematical objects are externalized, explored interactively, coordinated across representations, and connected with formal mathematics through theoretically grounded tasks, scaffolding, reflection, teacher orchestration, and rigorous empirical evaluation. AR/VR should therefore be designed not simply to make abstract objects visible, but to support reasoning about mathematical structures, relationships, and meanings.

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