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MATHEMATICAL MODELING OF GRADE 10 STUDENTS IN SOLVING DERIVATIVE-APPLICATION PROBLEMS IN CAMBRIDGE INTERNATIONAL AS & A LEVEL

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ABSTRACT

This study examined the mathematical modeling of 32 Grade 10 students, working in 16 pairs, in two Cambridge International AS & A Level Mathematics classes. Each pair solved derivative-application problems using an Observe–Model–Differentiate–Verify–Conclude (OMDVC) worksheet and later presented its solution. Written and oral evidence were analyzed separately through directed content analysis across six aspects: understanding the context, formulating the model, applying differentiation, verifying, interpreting, and communicating. Differentiation was the most consistently complete aspect, but a correct calculation did not always rest on a complete model or adequate verification. Written and oral evidence closely matched in only five pairs (31.3%). Among the other 11 pairs, five supplied details during the presentation before any guiding question, four had correct written solutions but could explain only part of their reasoning orally, and two clarified or revised their reasoning only after a guiding question. Looking at both sources revealed reasoning that had not been written down, as well as written evidence that students could not fully explain during the presentation. A correct differentiation procedure, by itself, was not sufficient evidence of a complete modeling process.

Keywords: Mathematical Modeling, Derivative Application, Cambridge AS & A Level, Worksheet, Pair Presentation.

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PRELIMINARY

At the secondary school level, differentiation learning should extend beyond procedural accuracy. More generally, understanding the derivative requires connections among its meanings and representations rather than only the application of differentiation rules (Jones & Watson, 2018). Recent design research at the secondary-school level also showed that contextual activities can support students in understanding the derivative as an instantaneous rate of change while engaging them in mathematical modeling (Kurniadi et al., 2026). Cambridge International AS & A Level Mathematics 9709 also places calculus as a means of explaining changes in dynamic situations. The scope of differentiation includes gradients, tangents and normal lines, increasing and decreasing functions, rates of change, connected rates of change, stationary points, and maximum and minimum problems (Cambridge Assessment International Education, 2025). The syllabus also emphasizes problem solving, communication, and mathematical modeling.

In problems involving the application of differentiation, mathematical functions are not always available in a form that is ready to be differentiated. Students must understand the situation, select relevant information, define variables, formulate relationships between variables, define the domain, and distinguish between objective functions and constraint functions. Once the model is formed, students use differentiation to derive solutions, check their accuracy, and interpret the results based on the initial situation. Thus, solving derivative-application problems involves mathematical modeling activities as well as procedural calculus skills.

Mathematical modeling connects real situations with mathematical representations through the process of understanding the situation, performing mathematization, solving problems in the model, interpreting the results, and checking their conformity to the initial context (Cevikbas et al., 2022). The process does not always take place linearly because students can revisit information, correct relationships between variables, or change models when the results obtained do not match the context (Borromeo Ferri, 2006). Therefore, modeling analysis needs to pay attention to the relationship between contextual situations, constructed models, solution procedures, and solution interpretation.

Correct differentiation alone gives limited evidence of students' conceptual understanding. Sahin et al. (2015), for example, found that students could carry out differentiation successfully but were not always able to explain the relationships involved. Feudel and Biehler (2022) reported a related problem when students had to interpret derivatives in context. In optimization tasks, Machaba et al. (2025) documented confusion between a function and its derivative, along with difficulty interpreting rates of change. Across these settings, a correct calculation is only one part of the evidence needed to judge whether students understand the role of the derivative and the meaning of the result in the modeled situation.

Difficulties can also arise when students translate verbal information into mathematical form. An incomplete understanding of contextual situations may lead to an incomplete or inaccurate mathematical model, even when the necessary mathematical procedures have been mastered (Khusna & Ulfah, 2021). In derivative-application problems, an incomplete model may be indicated when variables are not defined, units and domains are not stated, relationships between quantities are not described, or objective and constraint functions are not distinguished. Therefore, the analysis of derivative-application problems needs to include understanding the context, formulating the model, applying differentiation, verifying the solution, and interpreting the result in context.

To organize these activities, the worksheets followed an Observe–Model–Differentiate–Verify–Conclude (OMDVC) sequence, providing separate spaces for students to identify relevant information, formulate a mathematical model, apply differentiation, verify the solution, and interpret the result in context. Similar step-by-step guidance has been used to support mathematical modeling through solution plans and guiding questions (Schukajlow et al., 2015, 2023). OMDVC is not proposed as a new instructional model or evaluated as an intervention; it was used only to structure the worksheet activities.

Students' worksheets record the models they construct and the reasoning they make explicit on the page. Sahin et al. (2015) examined worksheets together with field notes and student interactions to identify aspects of derivative understanding that calculation accuracy alone could not capture. Aguilar (2021) used written products from pairs to trace the models and strategies developed during problem solving. For Teledahl et al. (2025), the quality of mathematical writing rests on clear definitions, coherent reasoning, and an explicit connection to context. A worksheet, however, is limited to what students choose to write down. A correct solution on the page can still leave the reason for a particular step unclear.

Presentations and responses to questions can complement written evidence by showing the reasons for choosing the model, the meaning of the solutions, and students' ability to justify, clarify, or revise their solutions. Brady and Jung (2022) showed that presentations and question-and-answer sessions could reveal students' modeling actions and how they assessed the completeness of their solutions. Therefore, worksheet responses and presentations are treated as two distinct but complementary forms of evidence.

Previous studies have examined derivative understanding through modeling (Sahin et al., 2015), contextual interpretation of derivatives (Feudel & Biehler, 2022), and errors in calculus optimization problems (Machaba et al., 2025). Related studies have considered mathematical modeling of contextual problems (Khusna & Ulfah, 2021), modeling-oriented learning designs for secondary students (Saputri et al., 2022), instructional support for modeling (Durandt et al., 2022; Schukajlow et al., 2015, 2023), and students' presentation of modeling solutions (Brady & Jung, 2022). Yet little attention has been given to how evidence from structured worksheets compares with what Grade 10 students explain during pair presentations when solving derivative-application problems in Cambridge International

AS & A Level Mathematics. Examining both sources makes it possible to see which elements are already evident in students' written work, which are explained only during presentation, and which appear after a guiding question.

Based on this description, this study aims to examine Grade 10 students' mathematical modeling in solving derivative-application problems within Cambridge International AS & A Level Mathematics through an integrated analysis of worksheet responses and pair presentations. Specifically, this study addresses the following two questions: (1) What characteristics of Grade 10 students' mathematical modeling in solving derivative-application problems are evident across six aspects of modeling? (2) How does evidence from pair presentations, including responses to guiding questions, correspond to, supplement, or differ from the modeling evident in students' worksheets? By comparing students' worksheets with their classroom presentations, this study distinguishes modeling already evident in worksheet responses from reasoning articulated orally and from clarifications or revisions that emerged after guiding questions.

METHODS

This study used a qualitative descriptive design to examine how students modeled derivative-application problems. Directed content analysis organized the evidence under six aspects: understanding the context, formulating the model, applying differentiation, verifying the solution, interpreting the result, and communicating the reasoning. The OMDVC worksheet served as one source of evidence for the analysis; the study did not examine its instructional effect.

The six aspects and their indicators were derived from established accounts of the mathematical modeling process (Borromeo Ferri, 2006; Cevikbas et al., 2022). They formed the initial coding scheme but were not treated as a closed set of categories. A response that was not adequately represented by an existing indicator was examined again before a coding decision was made. This procedure follows directed content analysis, in which prior theory supplies the initial codes and unmatched data are examined further (Hsieh & Shannon, 2005).

Data were collected during regular lessons on applications of differentiation at a private senior high school in Semarang, where Cambridge International AS & A Level Mathematics is taught. The lessons covered optimization, stationary points, and rate-of-change problems as part of the regular semester program. Data collection took place within the existing timetable and required no additional lessons. One of the researchers taught mathematics in both participating classes.

The participants were all 32 Grade 10 students enrolled in two existing classes, with 16 students in each class. Both the school and the classes were selected purposively: the school implemented Cambridge International AS & A Level Mathematics, and the two classes were the only Grade 10 classes studying applications of differentiation during the data-collection period. All students had previously studied basic differentiation, and both classes received the same instruction on its applications.

The students worked in 16 pairs, eight pairs from each class. For analysis, a pair's completed worksheet and classroom presentation were considered together as one case. Pairs from the first class were labeled A1–A8 and those from the second B1–B8. These labels identified cases only; the two classes were not treated as comparison groups. All 16 cases were analyzed together, with attention to differences in how pairs constructed and explained their models.

The pairs solved contextual optimization and rate-of-change problems on worksheets organized by the Observe–Model–Differentiate–Verify–Conclude (OMDVC) sequence. In Observe, they identified relevant information and stated what the problem asked them to find. In Model, they defined variables and units, specified relationships or constraints, and expressed the situation as a function. They then differentiated the resulting function.

During Verify, students checked their calculations and considered whether the result made sense in the original problem. The appropriate check depended on the task and could involve the domain and units, the nature of a stationary point or direction of change, or the reasonableness of the value obtained. In conclusion, students stated the meaning of the result in context.

Each pair had 90 minutes to complete the worksheet. At the next class meeting, they presented their solution using a poster or summary sheet. The presentation was analyzed as oral evidence alongside the worksheet. If an explanation was unclear or incomplete, the teacher asked a guiding question without supplying the mathematical model or final answer. Explanations given before any question were recorded as spontaneous presentation evidence, whereas clarification or revision that appeared only after a question was coded separately.

The primary data consisted of completed worksheets, presentation notes, and students' responses to guiding questions. Posters or summary sheets and observation notes served as complementary data. Worksheets provided evidence of the information students selected, the models they constructed, the differentiation procedures they used, the checks they performed, and the conclusions they recorded. Similar use of student products to examine models and modeling strategies can be found in Aguilar (2021). Presentation evidence was used to identify explanations of model choices, relationships among quantities, and interpretations that were not always recorded in the worksheets; Brady and Jung (2022) similarly used presentations and question-and-answer sessions as evidence in studying mathematical modeling. Rubric scores for the worksheets and presentations were recorded as part of routine classroom assessment; they were not used to assign the coding categories reported below.

The analysis was carried out based on six aspects of modeling and its indicators as presented in Table 1.

Table 1. Aspects and Indicators of Mathematical Modeling Analysis

Aspects	Analyzed indicators
Understanding the context	Selection of relevant information, identification of quantities, and statement of the aim of the problem
Formulating the model	Definition of variables, units, relationships between quantities, objective function, constraints, and domain
Applying differentiation	Correct use of differentiation rules, identification of stationary points, substitution, and calculation
Verifying the solution	Checking accuracy of calculations, nature of stationary points or direction of change, suitability of values within the domain, consistency of units, and reasonableness of the solution
Interpreting in context	Explanation of the meaning of a value, sign, unit, or mathematical result in sentences that relate back to the situation and aim of the problem
Communicating	Clarity of reasoning, use of mathematical representations, and response to guiding questions

The five OMDVC stages correspond to five of the six analytical aspects. Observe corresponds to understanding the context, Model to formulating the model, Differentiate to applying differentiation, Verify to verifying the solution, and Conclude to interpreting the result. Communication was treated as a cross-stage aspect because evidence of reasoning, representation, and responses to questions could occur throughout the modeling process.

Each modeling aspect was classified into one of four categories. Complete and accurate was assigned when the relevant indicators were present and mathematically appropriate to the problem context. Accurate but incomplete was used when the main reasoning was correct but one or more relevant indicators were missing or insufficiently explained. Major error was assigned when an error substantially affected the model, differentiation procedure, verification, or interpretation. Not observed was used when neither the worksheet nor the oral evidence provided sufficient information to evaluate the aspect.

The unit of coding was the smallest meaningful segment that provided evidence for a modeling indicator. A segment could be a written statement, variable definition, equation, diagram, calculation step, oral explanation, or response to a guiding question. Each segment was coded under the indicator and modeling aspect that best matched its content. If a segment expressed more than one analytically distinct idea, the ideas were separated and coded individually. Table 2 illustrates how excerpts were assigned to indicators, modeling aspects, and sources of evidence.

Table 2. Examples of Directed Content Analysis Coding

Data excerpt	Source	Code or indicator	Modeling aspect	Origin of evidence
Given surface area $= 500$ cm²; triangle sides = $5x, 5x$, and $6x$; the task is to express y in terms of x and determine the volume.	A4 written worksheet	Identifies relevant quantities and the target of the problem	Understanding the context	Written evidence
$y = \frac{500 - 24x^2}{10x}$ was formed from the surface area relation, but the domain was not stated.	A4 written worksheet	Establishes a relationship among quantities; domain not specified	Formulating the model	Written evidence
The negative result means that the water height is decreasing.	B6 presentation	Interprets the sign of the rate in relation to the context	Interpreting in context	Oral evidence before a guiding question
After being asked what the maximum value represented, the pair explained its meaning in relation to the original problem situation.	A8 response to a guiding question	Relates a mathematical result to the problem context	Interpreting in context	Evidence elicited after a guiding question

Each pair was treated as one case. Worksheets, posters, observation notes, and presentation notes were first read together to establish an overview of the pair's work. Worksheet and oral evidence were then coded separately using the six modeling aspects and indicators in Table 1. For each indicator, the coding record identified whether the evidence appeared in the worksheet, was expressed spontaneously during the presentation, or emerged only after a guiding question. The two sources were then compared within each pair, and the final category for each modeling aspect was determined after the available evidence had been considered together. After the within-pair analysis, the 16 cases were compared to identify recurring patterns and differences. This procedure is consistent with the use of worksheets and classroom interactions as sources of modeling evidence in Sahin et al. (2015) and with Aguilar's (2021) analysis of student products to examine models and modeling strategies.

Worksheets and presentation notes provided the main evidence for each pair; posters and observation notes were consulted when they helped clarify the classroom context. When the sources did not agree, the relevant materials were reviewed again using the same indicators. This served as data-source triangulation. An audit trail documented coding decisions and the treatment of ambiguous passages so that interpretations could be traced to the supporting data (Nowell et al., 2017). Because the full dataset was coded by one researcher, intercoder agreement was not assessed. The school and students were anonymized, and excerpts are identified only by pair code when they are needed to support an interpretation.

RESULT AND DISCUSSION

1. Student Mathematical Modeling Overview

Table 3. Distribution of Mathematical Modeling Performance Across 16 Pairs

Modeling aspects	Complete and accurate	Accurate but incomplete	Major errors	Not observed
Understanding the context	9 (56.3%)	5 (31.3%)	2 (12.5%)	0 (0.0%)
Formulating the model	6 (37.5%)	6 (37.5%)	4 (25.0%)	0 (0.0%)
Applying differentiation	10 (62.5%)	4 (25.0%)	2 (12.5%)	0 (0.0%)
Verifying	5 (31.3%)	7 (43.8%)	4 (25.0%)	0 (0.0%)
Interpreting in context	6 (37.5%)	7 (43.8%)	3 (18.8%)	0 (0.0%)
Communicating	7 (43.8%)	6 (37.5%)	3 (18.8%)	0 (0.0%)

Table 3 summarizes the 16 pairs' performance across the six modeling aspects using evidence from both worksheets and presentations. Posters and summary sheets were treated as part of the presentation evidence, while observation notes were used only to clarify classroom context. For each case, we also recorded whether an element appeared in the worksheet, was explained during the presentation before a guiding question, or was added only after the teacher asked one.

The clearest strength was the calculus step: ten pairs completed the differentiation aspect accurately. This is understandable because the work in this part resembled familiar Cambridge Mathematics routines - differentiating a function, finding a stationary point, checking a sign, or substituting a value. Verification was less secure: five pairs were complete, seven were partly correct, and four made major errors. The contrast suggests that many pairs knew how to run the calculus procedure, but were less consistent when they had to decide whether the result still fitted the domain, unit, constraint, and context of the problem.

In other words, success in using differentiation did not automatically carry over to formulating the model or verifying the solution. Several pairs worked fluently once a function was available, yet the modelling ground underneath the calculation was not always complete. For that reason, the later case analyses read the derivative work together with the objective function, constraints, domain, and presentation evidence, rather than treating the final answer as a complete picture.

2. Understanding the Context and Formulating the Mathematical Model

For understanding the context, nine of the 16 pairs were classified as complete and accurate. Five pairs identified the relevant information but did not fully explain how it related to what the problem asked them to determine. The other two pairs misinterpreted part of the given conditions. In most incomplete responses, students listed the known quantities but did not clearly connect them to the unknown quantity or to the constraints of the problem.

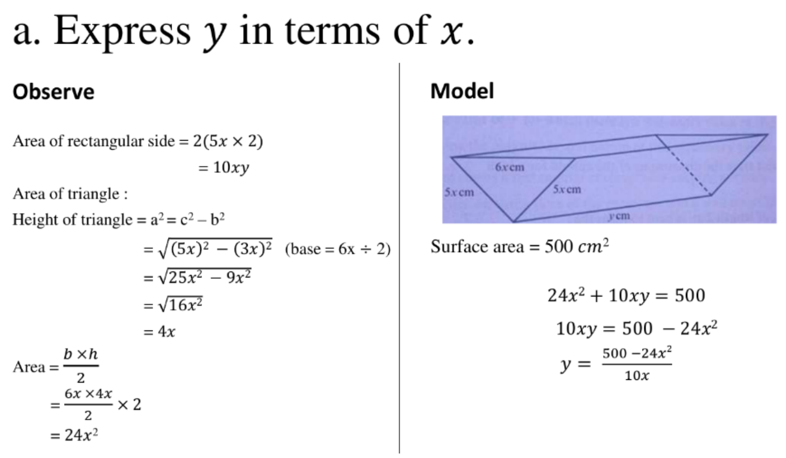

Figure 1. Pair A4 Relates Geometric Information to The Surface-Area Constraint During the Observe and Model Stages

Figure 1 shows why Pair A4 was classified as complete and accurate in understanding the context. Using the dimensions of the triangular cross-section, the pair obtained a height of $4x$ and an area of $12x^2$ for one triangular end. It then combined the area of both triangular ends, $24x^2$ with the area of the two rectangular faces, $10xy$, to form the surface-area equation $24x^2 + 10xy = 500$. Rearranging this equation gave $y = \frac{500-24x^2}{10x}$. The worksheet thus shows how the geometric information was converted into a constraint that could be used to express the volume as a function of x before differentiation.

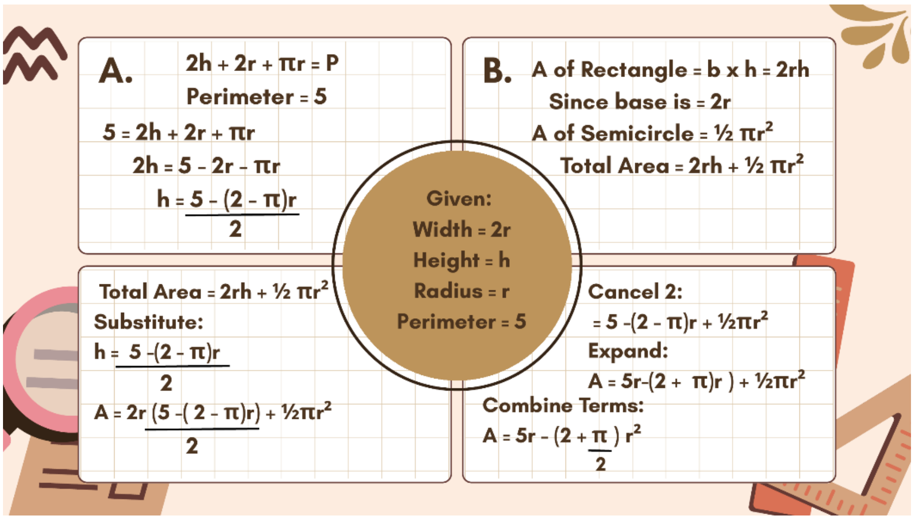

Figure 2. Pair A8 Records the Relevant Quantities but Makes Errors When Rearranging The Perimeter Constraint and Simplifying The Area Function.

As shown in Figure 2, Pair A8 illustrates why recording the relevant quantities was not sufficient for a complete model. In the window problem, the pair correctly wrote the perimeter constraint as $2h + 2r + \pi r = 5$ and the area as $A = 2rh + \frac{1}{2}\pi r^2$. However, it then expressed h as $\frac{5-(2-\pi)r}{2}$, whereas the correct expression is $\frac{5-(2+\pi)r}{2}$. Further errors in the algebraic simplification led to an incorrect objective function; the correct function is $A = 5r - \left(2 + \frac{\pi}{2}\right)r^2$. During the presentation, the pair stated that the aim was to maximize the area but required a guiding question before it could explain how the area function had been constructed.

In the Model stage, six of the 16 pairs produced complete and accurate models. Another six obtained largely correct models but left out variable definitions, units, or the domain. Errors that affected the model occurred in the other four pairs, usually when they applied a constraint, simplified an expression, or selected the function to differentiate. The domain was omitted most often. Several pairs proceeded directly to an objective function without specifying the allowable range of its variable.

Pair A4 fell into the accurate but incomplete category. Its worksheet connected the surface-area constraint and the dimensions of the triangular prism with its volume, and the presentation explained how those relationships led to a one-variable volume function. The oral explanation made several implicit links more explicit, but the domain was still not stated. Pair A6 also reached the correct objective function, although several algebraic steps were unclear in the written solution. For that reason, its model was also coded as accurate but incomplete.

The cases suggest that incompleteness had more than one source. In some work, the difficulty began at the Observe stage because a quantity was copied from the question without its role being clarified. In other work, the difficulty appeared in the Model stage: students obtained a useful expression but left the domain or condition of validity implicit. This distinction matters in Cambridge-style optimisation because the objective function is only meaningful within the constraint that produced it. Comparable difficulties in translating contextual information into mathematical form and interpreting contextual problems have been reported by Khusna and Ulfah (2021) and Anin and Dirgantoro (2023). Using the distinction described by Cevikbas et al. (2022), missing or misread quantities point to weaknesses in situational understanding, whereas an incorrect or insufficiently specified function points to mathematization.

3. Solving the Model Using Differentiation

Ten of the 16 pairs applied differentiation completely and accurately. Four used an appropriate procedure but left parts of the notation or intermediate work unclear, and two made procedural errors that affected the result. This was noticeably stronger than model formulation, which was complete and accurate for only six pairs.

Pair A6 illustrates this contrast. Its model formulation was accurate but incomplete, yet its differentiation procedure was complete. The pair differentiated the objective function correctly, found the stationary point, and used that value to obtain the required dimensions. During the presentation, the students described the stationary point as a candidate minimum and connected it to the dimensions required by the problem. This explanation made the optimization reasoning clearer than it was in the worksheet, although some notation on the poster remained ambiguous.

Pair A8 shows how an earlier modeling error carried into the differentiation stage. Although the pair used a generally appropriate differentiation procedure, it applied the procedure to an area function that contained algebraic errors. Consequently, the calculation could not produce a valid solution to the original problem. The main difficulty therefore originated in formulating the function rather than in applying differentiation. A related difficulty was reported by Jupri et al. (2021) in maximum–minimum problems, particularly in mathematizing the problem through visual representation and the construction of an appropriate mathematical model.

Across the cases, the calculus routine was steadier than the modelling judgement that supported it. This does not mean that differentiation was easy in every case; rather, students seemed more used to symbolic routines than to deciding which function should be differentiated. Pair A8 makes this point visible: the derivative step moved in the expected direction, but the area function itself was not reliable. Hence, in Application of Differentiation, a solution has to be read from the model forward, not from the derivative line alone. This reading is in line with Sahin et al. (2015), who showed that correct differentiation does not necessarily indicate relational understanding of the derivative, and with Feudel and Biehler (2022), who reported difficulties in connecting derivatives with contextual interpretations. The explanation offered here remains cautious because the study was not designed to test causal factors.

4. Verifying and Interpreting Solutions

Five pairs verified their solutions completely and accurately. Complete verification combined a mathematical check, such as a second-derivative test or examination of the derivative's sign, with attention to the domain, units, or reasonableness of the result. Seven pairs completed only part of this work. A common pattern was to use the second derivative without checking whether the resulting value lay in the required domain or carried the correct unit. The other four pairs made errors that substantially affected verification.

Pair A6 substituted the stationary-point value into its model and calculated the required dimensions, but the worksheet did not state the domain and the pair did not fully establish that the point gave the required minimum. Pair A4 connected its result more directly to the maximum-volume condition, although parts of its calculation still needed correction. In both cases, the numerical check received more attention than the conditions needed to make the result valid in the original problem.

Interpretation was similarly uneven. Six pairs wrote conclusions that clearly returned to the problem context, while seven reached an appropriate conclusion but explained it only briefly. The remaining three did not interpret their results in context and were coded as major errors for this aspect. Some missing interpretations appeared during the presentations; in a few cases, they emerged only after a guiding question.

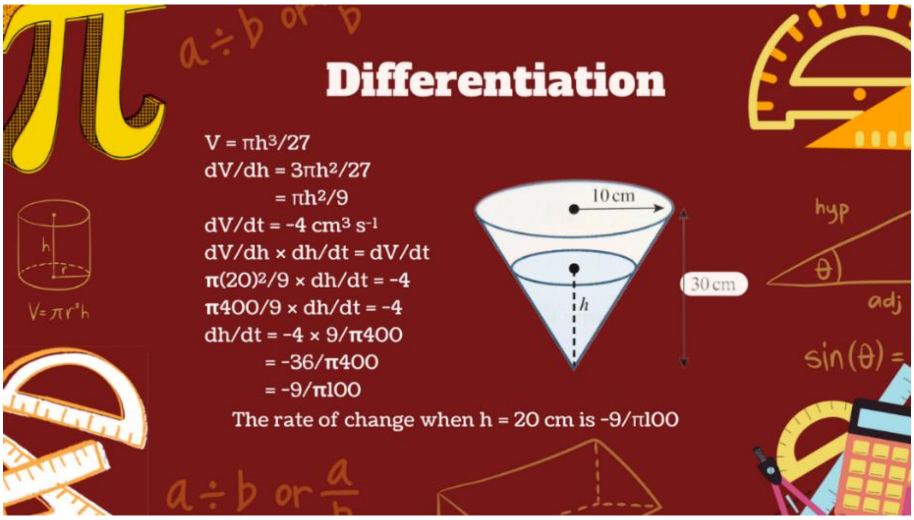

Figure 3. Pair B6 Interprets A Negative Rate as A Decrease in Water Level, but The Final Notation and Unit Are Incomplete

Figure 3 shows Pair B6's solution to a related-rates problem involving the water level in a cone. Using the relationship between the radius and height, the pair obtained $V = \frac{\pi h^3}{27}$ and applied the chain rule to relate $\frac{dV}{dt}$ to $\frac{dh}{dt}$. For $h = 20$ cm and $\frac{dV}{dt} = -4\ \text{cm}^3\text{s}^{-1}$, the calculation gave $\frac{dh}{dt} = -\frac{9}{100\pi}\,\text{cm s}^{-1}$. During the presentation, the students explained that the negative sign indicated that the water level was decreasing as water flowed out of the cone. On the poster, however, the final result appeared as $-9/\pi 100$, without parentheses to indicate that 100π formed the denominator and without the unit cm s^{-1}.

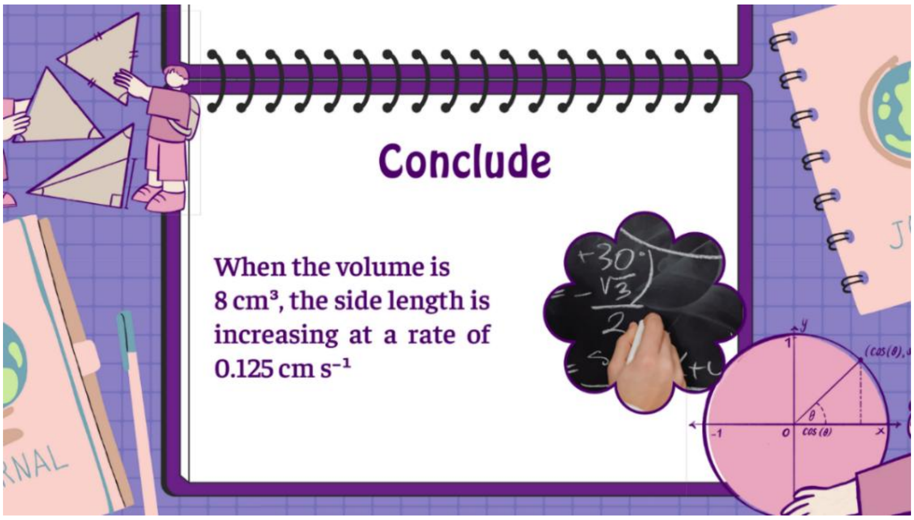

Figure 4. Pair B7 Interprets The Rate of Change in The Problem Context During The Conclude Stage

Pair B7 provided an explicit contextual interpretation in its written conclusion. As shown in Figure 4, the pair stated that when the cube's volume was 8 cm^3 its side length was increasing at 0.125 cm s^{-1}. The conclusion therefore linked the numerical result to the changing quantity, its direction, and the appropriate unit.

Across the 16 pairs, verification was the point where the Cambridge demand of the task became most visible. A second-derivative test or a sign check only answers part of the question; the result still has to be checked against possible values, units, and the wording of the context. Some pairs could explain these points orally, but often after the teacher prompted them. This explains why verification appeared weaker in the qualitative coding: a step might be present on the page, yet the validation of the result was not complete. Cevikbas et al. (2022) treat validation and interpretation as integral parts of mathematical modeling, and the present data show how easily this part can be left unfinished in derivative-application work.

5. Comparison of Worksheets and Pair Presentations

The worksheet and presentation did not always reveal the same parts of a pair's modeling. Worksheets recorded the models, calculations, and conclusions that students chose to write down. During presentation, a pair might add a missing assumption or justification, correct part of a solution, or show that an accurate written answer could not be fully explained orally. Seven pairs were complete and accurate in the communication category in Table 3; the other nine gave incomplete explanations or made major errors. Written accuracy, in other words, did not automatically translate into a complete oral explanation. Table 4 summarizes the main relationship between the two sources for each pair.

Five pairs showed close agreement between worksheet and presentation evidence. Of the other 11, four had written solutions that were more complete than their oral explanations. The remaining seven added information during presentation that was absent from the worksheet: five did so before any guiding question, whereas two clarified or revised their reasoning only afterward. These latter responses were kept separate so that prompted clarification was not counted as reasoning offered independently.

Pair A4 represented the consistent pattern. The worksheet showed how the surface-area constraint was used to express y in terms of x, and the presentation explained why the resulting volume function needed to be differentiated. Pairs A2 and A3, by contrast, supplied information orally that was missing from their worksheets, such as the meaning of a variable or the reason for choosing a particular function. The coding record also noted whether such additions appeared before or after a guiding question.

Table 4. Worksheet–Presentation Evidence Patterns Across 16 Pairs

Relationship patterns	Number of pairs	Features
Consistent worksheet and presentation evidence	5 (31.3%)	The worksheet records the main modeling steps, and the pair explains the same reasoning during the presentation.
Accurate written solution, limited oral explanation	4 (25.0%)	The written model or result is correct, but the pair explains only part of the reasoning used to construct or justify it.
Oral addition before a guiding question	5 (31.3%)	Before any guiding question, the pair explains a modeling element that is absent from the worksheet, such as a domain, unit, assumption, or interpretation.
Clarification or revision following a guiding question	2 (12.5%)	The pair clarifies or revises a modeling element only after the teacher asks a guiding question.

The mismatch between worksheet and presentation evidence changes how the student work should be read. A worksheet is a trace of written mathematics, not a full record of thinking. Some students may know why they used a constraint but leave that reason unwritten. Others may write a correct-looking solution because they follow a routine, but they cannot explain why the function or derivative is appropriate when asked. Related research with Grade 10 students has also reported inconsistencies and incompleteness in written mathematical representations and narratives (Maghfiroh et al., 2026).

Earlier studies have also combined written and oral evidence to examine mathematical reasoning and modelling (Brady & Jung, 2022; Pugalee, 2004). Written tests and interviews have likewise been compared as complementary sources for examining students' mathematical communication (Yunita & Siswanto, 2023). In this study, using both sources changed what could be seen in the communication aspect. Several pairs reached correct written results but could not fully explain the reasoning behind them or connect the result back to the problem context during presentation. The distinction between spontaneous explanation and explanation after a guiding question also helped show how much support each pair still needed.

6. Implications for the OMDVC Worksheet Structure

The OMDVC worksheet separated the work into spaces for Observe, Model, Differentiate, Verify, and Conclude. For students, the worksheet provided separate spaces for working through the problem from contextual information to the derivative calculation. Still, a box or heading did not guarantee complete reasoning. Variable definitions, units, domains, constraints, and contextual checks were still omitted in several worksheets, especially in Model and Verify. Some pairs wrote a function without showing how it followed from the quantities and constraints; in Verify, a recurring pattern was to check the calculation but not whether the result was feasible or meaningful in context.

For the analysis, OMDVC worked in a different way. It helped locate where evidence might appear, but it did not decide the category by itself. The six modelling aspects and their indicators remained the analytical framework, so a completed OMDVC section was not automatically coded as complete. Worksheet evidence was then compared with the presentation record, and oral additions were separated according to whether they appeared before a guiding question or only after one. This separation keeps the instructional role of OMDVC apart from its analytical use.

Because the study did not test the effect of OMDVC on students' performance, the claim has to stay limited. The findings suggest that future worksheets should strengthen the Model and Verify prompts, not that OMDVC has been proven effective. In Model, students could be asked more directly to define variables and units, state the domain or constraints, and show how the quantities lead to the objective function or rate relationship. In Verify, separate prompts could distinguish Cambridge-style mathematical checks, such as the second derivative test or a sign change in the first derivative, from contextual checks involving units, feasibility, and the meaning of the result.

CONCLUSION

This study shows that students' work on Cambridge AS & A Level Application of Differentiation should not be judged only from the final calculation. Many pairs could differentiate, find stationary points, or work with rates of change, but the surrounding modelling was less secure. The weaker parts were usually the decisions before and after the derivative step: forming the objective function or rate relationship, stating the domain and constraints, checking feasibility, and explaining what the answer meant in the original situation.

The main contribution of the study is methodological. Reading worksheets together with pair presentations showed evidence that would be missed from one source alone. Some modelling elements were written clearly, some were explained only during presentation, and some appeared only after a guiding question. This distinction matters because a correct written answer and an independent modelling explanation are not the same thing.

OMDVC is best understood as a worksheet route and an aid for tracing evidence, not as a proven intervention. For students, it organised the stages from Observe to Conclude. For the analysis, it helped locate written and oral evidence across the modelling process. The study is limited to 16 pairs from one school, with presentation notes rather than full transcripts and coding conducted by one researcher. Future studies could use recorded presentations, involve more than one coder, and design rubrics that distinguish between simply showing a modelling step and completing that step well.

REFERENCES

Aguilar, J. J. (2021). Modeling through model-eliciting activities: An analysis of models, elements, and strategies in high school. The cases of students with different level of achievement. *Mathematics Teaching-Research Journal*, *13*(1), 52–70. https://scholarworks.utrgv.edu/tl_fac/50/

Anin, F. R., & Dirgantoro, K. P. S. (2023). Application of Contextual Problems to Help Students Mathematic Understanding Ability in Science and Social Class Grades XI. *Mathline : Jurnal Matematika Dan Pendidikan Matematika*, *8*(2), 329–344. https://doi.org/10.31943/mathline.v8i2.376

Borromeo Ferri, R. (2006). Theoretical and empirical differentiations of phases in the modelling process. *ZDM - Mathematics Education*, *38*(2), 86–95. https://doi.org/10.1007/BF02655883

Brady, C., & Jung, H. (2022). Modeling presentations: Toward an assessment of emerging classroom cultures of modeling. *Educational Studies in Mathematics*, *109*(2), 237–261. https://doi.org/10.1007/s10649-021-10056-x

Cambridge Assessment International Education. (2025). *Cambridge International AS & A Level Mathematics 9709 syllabus for 2026 and 2027* (Version 4). Cambridge University Press & Assessment. https://www.cambridgeinternational.org/Images/697427-2026-2027-syllabus.pdf

Cevikbas, M., Kaiser, G., & Schukajlow, S. (2022). A systematic literature review of the current discussion on mathematical modelling competencies: State-of-the-art developments in conceptualizing, measuring, and fostering. *Educational Studies in Mathematics*, *109*(2), 205–236. https://doi.org/10.1007/s10649-021-10104-6

Durandt, R., Blum, W., & Lindl, A. (2022). Fostering mathematical modelling competency of South African engineering students: Which influence does the teaching design have? *Educational Studies in Mathematics*, *109*(2), 361–381. https://doi.org/10.1007/s10649-021-10068-7

Feudel, F., & Biehler, R. (2022). Students' understanding of the economic interpretation of the derivative in the context of marginal cost. *International Journal of Research in Undergraduate Mathematics Education*, *8*(3), 437–468. https://doi.org/10.1007/s40753-021-00144-x

Hsieh, H.-F., & Shannon, S. E. (2005). Three approaches to qualitative content analysis. *Qualitative Health Research*, *15*(9), 1277–1288. https://doi.org/10.1177/1049732305276687

Jones, S. R., & Watson, K. L. (2018). Recommendations for a "target understanding" of the derivative concept for first-semester calculus teaching and learning. *International Journal of Research in Undergraduate Mathematics Education*, *4*(2), 199–227. https://doi.org/10.1007/s40753-017-0057-2

Jupri, A., Usdiyana, D., & Sispiyati, R. (2021). Teaching and learning process for mathematization activities: The case of solving maximum and minimum problems. *JRAMathEdu (Journal of Research and Advances in Mathematics Education)*, *6*(2), 100–110. https://doi.org/10.23917/JRAMATHEDU.V6I2.13263

Khusna, H., & Ulfah, S. (2021). Kemampuan pemodelan matematis dalam menyelesaikan soal matematika kontekstual. *Mosharafa: Jurnal Pendidikan Matematika*, *10*(1), 153–164. https://doi.org/10.31980/mosharafa.v10i1.649

Kurniadi, E., Zulkardi, Putri, R. I. I., & Darmawijoyo. (2026). Local Instruction Theory: Supporting Derivative Concept Learning Through Climate Change Context. *Mathematics Education Journal*, *20*(1), 27–50. https://doi.org/10.22342/MEJ.V20I1.PP27-50

Machaba, M. F., Jameson, G., & Fasinu, V. G. (2025). Grade 12 students' misconceptions when modelling their calculus ideas into their learning of an optimisation: A real-life problem. *Journal of Research in Science, Mathematics and Technology Education*, *8*(2), 61–89. https://doi.org/10.31756/jrsmte.824

Maghfiroh, L., Mardiyana, M., & Indriati, D. (2026). Analysis of Mathematical Communication Skills in Trigonometry Topic Based on Habits of Mind. *Mathline : Jurnal Matematika Dan Pendidikan Matematika*, *11*(1), 207–224. https://doi.org/10.31943/MATHLINE.V11I1.1107

Yunita, M., & Siswanto, R. D. (2023). Analysis of Mathematical Communication Ability in Solving Story Problems Based on Mathematical Ability and Gender. *Mathline : Jurnal Matematika Dan Pendidikan Matematika*, *8*(1), 181–193. https://doi.org/10.31943/mathline.v8i1.327

Nowell, L. S., Norris, J. M., White, D. E., & Moules, N. J. (2017). Thematic analysis: Striving to meet the trustworthiness criteria. *International Journal of Qualitative Methods*, *16*(1), 1–13. https://doi.org/10.1177/1609406917733847

Pugalee, D. K. (2004). A comparison of verbal and written descriptions of students' problem solving processes. *Educational Studies in Mathematics*, *55*(1–3), 27–47. https://doi.org/10.1023/B:EDUC.0000017666.11367.c7

Sahin, Z., Yenmez, A. A., & Erbas, A. K. (2015). Relational understanding of the derivative concept through mathematical modeling: A case study. *Eurasia Journal of Mathematics, Science and Technology Education*, *11*(1), 177–188. https://doi.org/10.12973/eurasia.2015.1149a

Saputri, N. W., Zulkardi, & Darmawijoyo. (2022). Mathematical Modeling Learning Design with PISA Framework on Grade X Function. *Mathematics Education Journal*, *16*(3), 289–302. https://doi.org/10.22342/JPM.16.3.17228.289-302

Schukajlow, S., Kolter, J., & Blum, W. (2015). Scaffolding mathematical modelling with a solution plan. *ZDM - Mathematics Education*, *47*(7), 1241–1254. https://doi.org/10.1007/s11858-015-0707-2

Schukajlow, S., Krawitz, J., Kanefke, J., Blum, W., & Rakoczy, K. (2023). Open modelling problems: Cognitive barriers and instructional prompts. *Educational Studies in Mathematics*, *114*(3), 417–438. https://doi.org/10.1007/s10649-023-10265-6

Teledahl, A., Kilhamn, C., Ahl, L. M., & Helenius, O. (2025). Defining and measuring quality in students' mathematical writing: A systematic literature review. *Mathematics Education Research Journal*, *37*(3), 557–576. https://doi.org/10.1007/s13394-024-00501-4