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A MATHEMATICAL PROFIT-SHARING MODEL FOR MUDARABA SUKUK FINANCING

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ABSTRACT

Sukuk is one of the popular financial instruments today. However, existing return distribution mechanisms do not explicitly reflect the dynamics of investment profitability. This study develops a mathematical profit-sharing model for a mudaraba sukuk financing scheme that implements periodic return distributions linked to the profitability of the underlying investment. Geometric Brownian Motion is employed to generate investment income and represent its dynamics, while investment returns are determined using a time value of money approach adapted to the sukuk financing structure, and sensitivity analysis is conducted under varying profitability conditions. The simulation results demonstrate that the proposed model can implement periodic profit-sharing distributions while ensuring principal repayment at maturity. This model offers a mathematical perspective on sukuk research and contributes to the existing literature on mathematical finance.

Keywords: Profit-Sharing Mathematical Model, Mudaraba, Geometric Brownian Motion, Sensitivity Analysis.

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PRELIMINARY

The implementation of Islamic principles in financial services is increasing in tandem with the increased recognition and desire for Islamic financial instruments (Almaskati, 2022; Trianto & Masrizal, 2021). Among Islamic financial instruments, sukuk has become one of the most widely adopted financing instruments (Aman et al., 2021; Yıldırım et al., 2020). The increasing acceptance of sukuk has contributed to the rapid expansion of the global sukuk market (Baita et al., 2025; Helmi et al., 2025). The global sukuk market continued to grow and demonstrate resilience, as evidenced by the active market and issuances in both existing Sukuk issuing countries and new participants (Refinitiv, 2021). Sukuk continues to be a significant catalyst for growth, representing 24.43% or USD 0.794 trillion of the total industry value of USD 3.25 trillion (IIFM, 2023). A similar trend has also been observed in

Indonesia: the number of outstanding corporate sukuk increased by 16.93% compared with the previous year (OJK, 2023).

The growing popularity of sukuk has also stimulated extensive research on this instrument. Latest literature on *Sukuk* emphasises empirical or legal aspects, shariah regulation, and supervisory frameworks can be found in (Alam et al., 2023; Mseddi, 2023; Murniati et al., 2024; Paltrinieri et al., 2023; Razak et al., 2019), while quantitative approaches to sukuk financing mechanisms remain limited. However, a mathematical approach to sukuk financing mechanisms becomes important amidst economic uncertainty. Sumarti et al. (2014) developed a mathematical model that analyses investments under a musharaka contract. Furthermore, Ehsan Baaquie et al. (2023) argued that existing sukuk financing structures do not fully incorporate the principles of risk and profit-sharing. Therefore, this study develops a novel mathematical profit-sharing model for a sukuk investment scheme that links periodic return distributions to investment profitability while ensuring principal repayment at maturity.

Islamic economic activities are based on various Sharia-compliant contracts that incorporate the profit-sharing principle (Handarbeni et al., 2023). Profit-sharing refers to a contractual arrangement between two or more parties in which profits are distributed according to an agreed ratio and contractual terms (Kamilah et al., 2021; Lessy et al., 2021). In Indonesia, sukuk issuance is governed by Fatwa DSN–MUI No. 137/DSN-MUI/IX/2020, which specifies the permissible sharia contracts for issuance (DSN-MUI , 2020). Mudaraba is one of the most widely adopted financing schemes (Otoritas Jasa Keuangan, 2019). Under a mudaraba contract, the capital provider (*rabb al-mal*) provides funds to the investment manager (*mudharib*), with profits shared according to an agreed ratio and losses borne by the capital provider unless caused by the manager's negligence or misconduct. (Dubai International Financial Centre, 2009; Ramadhanty, 2022). Therefore, this study adopts mudaraba as the underlying contract for developing the proposed mathematical model.

The proposed model is evaluated through numerical simulations based on the characteristics of Indonesian corporate mudaraba sukuk, with investment income generated using Geometric Brownian Motion (GBM) and sensitivity analysis conducted under varying profitability conditions. The study provides a mathematical framework for sukuk financing and highlights the need for explicit modeling of profit-sharing mechanisms under uncertain economic conditions. The numerical analysis is intended solely to illustrate the application of the proposed mathematical model and does not constitute a valuation, investment recommendation, or assessment of the performance of the sukuk instrument or its issuer.

METHODS

This study applies an analytical approach to examine the notion and legislation of sukuk, with a specific emphasis on the mudaraba contract. Based on the characteristics of the sukuk instrument and a literature review, a mathematical model of the profit-sharing mechanism is developed. Sumarti et al. (2014) provides the mathematical foundation for the proposed model, which extends the variable profit-sharing mechanism and repayment obligations under uncertain investment outcomes to the mudaraba sukuk structure, linking periodic returns to underlying investment profitability while preserving principal repayment at maturity.

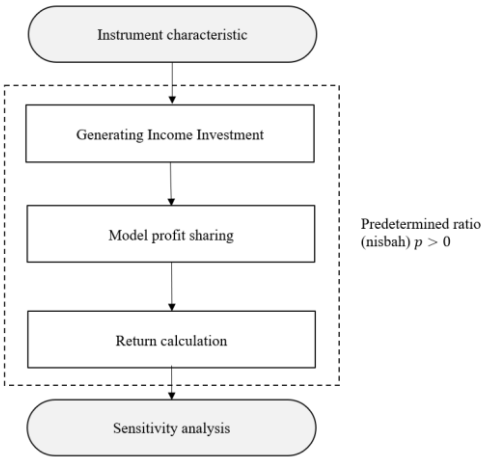

Figure 1. Research Methodology Flowchart

The investment income in this study is generated using the GBM model (Irwin, 2007; Miranda & Brandão, 2013; Sumarti et al., 2015), given in Equation (1) to represent the dynamic behavior of investment income.

$$w_t = w_0 e^{(\mu)t+\sigma\sqrt{t}\,Z}, Z \sim N(0,1), \quad (1)$$

where μ represents the historical investment margin of the investment manager in the year of sukuk issuance, used as the drift parameter to characterize the initial profitability and growth of the underlying investment. The margin is defined as the ratio of profit to revenue, given by

$$M_t = \frac{Profit_t}{Revenue_t} \quad (2)$$

While σ represents the volatility parameter capturing the uncertainty in investment income and is varied for the sensitivity analysis. Accordingly, w_0 is defined as the total basic financing obligations for the mudaraba sukuk contract, comprising the repayment of the principal amount and associated costs. The proposed model distinguishes between the repayment of the sukuk principal as investment capital and the distribution of investment

profits, with periodic return distributions determined based on the realized investment income. This formulation is consistent with the mudaraba principle described in Dubai International Financial Centre (2009), under which investment profits depend on the outcome of the underlying investment and are not guaranteed. Furthermore, sensitivity analysis is conducted to evaluate the model's ability to fulfill financing obligations under varying investment profitability conditions. The simulation is performed using MATLAB R2023b. The general methodological framework of the study is presented in Figure 1.

RESULT AND DISCUSSION

Profit-Sharing Mathematical Model for Mudaraba Financing of Sukuk

Sukuk issuance is carried out under a contract that is in accordance with Islamic sharia. One of the contracts that is allowed to be the basis for sukuk issuance is mudaraba (Dubai International Financial Centre, 2009). Mudaraba is a partnership agreement in which one party provides the capital while the other manages the investment, with profits shared according to a predetermined profit-sharing ratio (nisbah) and capital losses borne by the capital provider (Dubai International Financial Centre, 2009; Kusuma & Silva, 2014; Murniati et al., 2024; Razak et al., 2019). As a Sharia-compliant investment instrument, sukuk emphasizes the sharing of profits and risks among the parties involved. In practice, however, sukuk return distributions are commonly structured with predetermined amounts. Ehsan Baaquie et al. (2023) argue that predetermined periodic return distributions do not fully reflect the concept of profit and risk sharing. In this context, periodic return distributions would more appropriately reflect the underlying investment mechanism if they were linked to the profitability of the underlying investment. Motivated by this limitation, this study develops a mathematical model in which periodic return distributions are determined by the profitability of the underlying investment mudaraba sukuk.

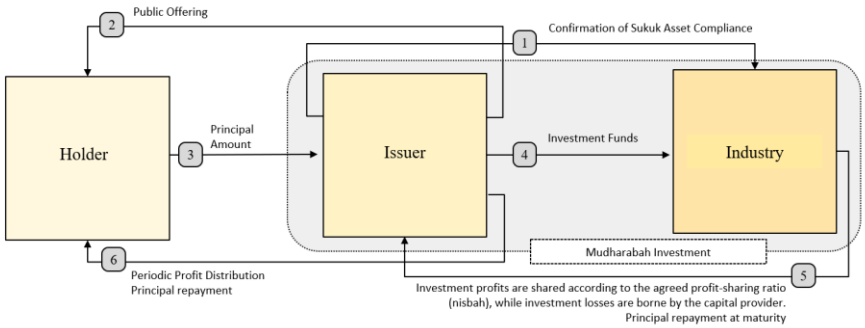

Figure 2. General Scheme of Sukuk Mudaraba Financing

The development of the proposed mathematical profit-sharing model for mudaraba sukuk financing begins with establishing the cash flow scheme of the underlying financing structure. Figure 2 illustrates the general cash flow scheme of mudaraba sukuk financing adapted from Fatwa DSN-MUI Nomor: 07/DSN-MUI/IV/2000 Tentang Pembiayaan Mudharabah (Qiradh) (2000), Fatwa DSN-MUI Nomor: 33/DSN-MUI/IX/2002 Tentang Obligasi Syariah Mudharaba (2002), Fatwa DSN-MUI Nomor: 115/DSN-MUI/LX/2017 Tentang Akad Mudharabah (2017), Dubai International Financial Centre (2009) dan Himpunan Skema Sukuk Otoritas Jasa Keuangan (2019). In this study, a mudaraba contract is established between the issuer and the industry, which acts as the investment manager (*mudharib*). The investment funds originate from the principal contributed by the sukuk holders and are channeled by the issuer to the industry. The issuer primarily serves an administrative and payment intermediary role by channeling the investment funds to the industry and subsequently distributing the holder's share of investment returns to the sukuk holders. Throughout the investment period, the industry generates investment income, which is allocated according to the predetermined profit-sharing ratio (nisbah). The portion of investment income allocated under the profit-sharing arrangement is used to fulfill the relevant financing obligations, while any remaining income is distributed to the sukuk holders as periodic profit. At maturity, the sukuk holders receive the principal redemption from the issuer, which is funded by the principal installments paid by the industry. More details of the cash flow structure are provided in Table 1.

Table 1. Cash flow structure of mudaraba among all parties over the sukuk investment period

Period	Holder	Issuer	Industry
$t = 0$	F	$-(F-G)$	$-G$
$t = 1,2,\ldots$	$-\sum R_t$	$-\sum I_t - \sum PD_t$	$\sum I_t + \sum PD_t + \sum R_t$
$t = T$	$-\sum I_t - \sum PD_t - D_T$	$\sum I_t + \sum PD_t + D_T - D_T$	D_T

Note: (+) denotes cash outflow and (-) denotes cash inflow.

Table 1 presents the cash flow structure of the mudaraba sukuk financing scheme among all parties over the investment period. At the beginning of the investment, $t = 0$, the sukuk holder provides the principal amount F to the issuer, subject to the specified public offering cost $O = F - G$. The issuer subsequently channels the available financing amount G to the industry, which undertakes the underlying investment. During each investment period $t = 1,2,\ldots$, the industry has to fulfill the relevant financing liabilities using the profit-sharing of investment income, consisting of the redemption value installment I_t, any payable

debt PD_t, nd the remaining profit-sharing income after these obligations are fulfilled is distributed to the holder as periodicreturn R_t. Furthermore, at maturity, $t = T$, the principal amount is scheduled for redemption by the issuer to the sukuk holder. Under the mudaraba principle, an outstanding amount may reflect a capital loss arising from the underlying investment that is not attributable to negligence, misconduct, or breach of contract by the industry, in which case the loss is borne by the investor. However, in the sukuk structure considered in this study, the principal amount is contractually scheduled to be fully redeemed to the sukuk holders at maturity. Therefore, any portion of the scheduled principal redemption that remains unpaid due to insufficient funds is represented by D_T in the cash flow structure and is considered for settlement at maturity.

Suppose that the sukuk holders provide the principal amount F to the issuer, who allocates the funds G to the underlying investment after deducting the public offering cost O. During the sukuk investment term, the industry makes periodic payments to the issuer for $t = 1,2,\dots,T$. Accordingly, the total installment TI_t at period t is defined as follows;

$$TI_t = O_t + I_t + PD_t, \tag{3}$$

where O_t denotes the issuance-related cost with the basic $O_b = \frac{O}{T}$, I_t denotes the principal redemption installment, and PD_t denotes the amount of outstanding debt that can be repaid in the period t. These liabilities are settled using the profit-sharing of investment income, $ws_t = p \times w_t$, where w_t denotes the investment income and p is the predetermined profit-sharing ratio (nisbah).

Meanwhile, the installment payment for the redemption value that the holder must receive at the end of the period with the basic installment is $I_b = \frac{F}{T}$, satisfies Equation (3).

$$I_t = \begin{cases} I_b & , ws_t - O_t \geq I_b, \\ ws_t - O_t & , 0 < ws_t - O_t < I_b, \\ 0 & , ws_t - O_t \leq 0, \end{cases} \tag{4}$$

where the unpaid portion of the principal redemption is deferred to the subsequent periods. Under this mechanism, debt D_t arises when the investment is unable to fully satisfy its financing obligations. The outstanding debt is carried forward to the subsequent period without any penalty. In each period, the payable debt PD_t is determined as the amount of outstanding debt that can be repaid from the available investment income (Sumarti et al., 2014b).

In this mudaraba model, the return distributed to the sukuk holders is determined from the remaining investment income after the settlement of the financing obligations. It

should be noted that the return distribution dates specified in the sukuk prospectus do not necessarily coincide with the periodic settlement of the financing obligations. Suppose that return distributions are made at periods $n = 1,2,\dots,N$, with R_n denoting the return distributed at the n-th distribution date. The holder's implied rate of return r_s satisfies the following,

$$F = \frac{R_1}{1+r_s} + \cdots + \frac{R_N}{(1+r_s)^N} + \frac{\sum_{t=1}^{T} I_t}{(1+r_s)^T} + \frac{\sum_{t=1}^{T} PD_t}{(1+r_s)^T} + \frac{D_T}{(1+r_s)^T}. \quad (5)$$

R_n represents the accumulated remaining investment income over each distribution interval and is obtained by summing the corresponding periodic amounts R_t. Equation (5) equates the principal fund F with the present value of all cash flows received by the sukuk holder over the investment period. The rate r_s therefore represents the implied return of the holder that equates the present value of the modeled cash flows to the principal fund of the sukuk. Meanwhile, the industry's return is determined according to Equation (6).

$$p_s = \frac{\sum_{t=1}^{T}(w_t - ws_t) - D_T}{\sum_{t=1}^{T} w_t}. \quad (6)$$

Based on Equation (6), the maximum return received by the industry is determined by the agreed profit-sharing ratio. The returns of both the sukuk holders and the industry depend on the realized investment income, whereas the issuer receives a predetermined public offering fee.

Model Simulation Result

The numerical simulations were conducted using the characteristics of the Indonesian mudaraba sukuk SMSMFP01CN3 issued by PT Sarana Multigriya Finansial (Persero). The sukuk was issued on February 10, 2021, with a total principal value of IDR 100,010,000,000. It offers a profit-sharing return of 4.75% per annum, based on a predetermined profit-sharing ratio (nisbah) of 66.43%. In accordance with Articles 1131 and 1132 of the Indonesian Civil Code, the issuer guarantees its present and future movable and immovable assets as collateral for the sukuk obligations (PT Sarana Multigriya Finansial (Persero), 2021b). The detailed characteristics of the sukuk are summarized in Table 2.

Table 2. Instrument Characteristics of Mudarabah Sukuk

	Characteristic
Code	SMSMFP01CN3
Issue date	February 10, 2021
Due date	February 20, 2022
Total fund	IDR 100,010,000,000.00
Profit-sharing	4.75%
Nisbah	66.43%

	Characteristic
Payment coupon date	Quarterly pay
First coupon payment	May 10, 2021
Last coupon payment	May 20, 2022
Public offering cost	IDR 181,644,663.00

Source: Sukuk Prospectus and Corporate Annual Report

Based on the prospectus, the SMSMFP01CN3 mudaraba sukuk designates the issuer as both the sukuk issuer and the investment manager, while a trustee acts on behalf of the sukuk holders throughout the financing period to safeguard their interests. Therefore, the proposed financing scheme is modified, as illustrated in Figure 2, to incorporate the role of the trustee in the sukuk issuance process and the issuer's dual function as both issuer and investment manager, industry. These structural modifications affect the allocation and calculation of returns and costs among the parties but do not alter the underlying mechanism of the proposed mathematical model. In the empirical application, the holder's implied return r_s is determined using Equation (4), based on the cash flows generated by the proposed model. The issuer, which also acts as the investment manager (industry), receives the profit corresponding to its share of the investment income under the predetermined profit-sharing ratio. Meanwhile, the trustee, which assumes the corresponding administrative role in the empirical structure, receives the public offering cost.

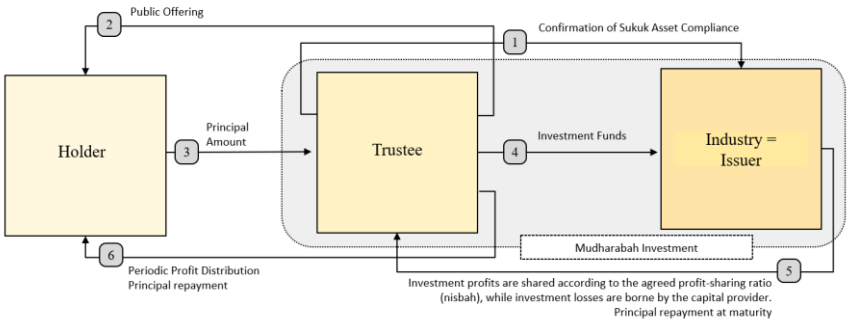

Figure 3. Sukuk mudaraba scheme with the involvement of trustee

The generation of investment income is one of the subsequent steps in the model simulation. For the numerical implementation, the GBM is parameterized using the characteristics of the mudaraba sukuk summarized in Table 2. The basic principal redemption installment is I_b = IDR 8,334,166,666.67 per month with a financing period of 12 months. While the monthly basic public offering cost is $O_b = IDR$ 15,137,055.25 (PT Sarana Multigriya Finansial (Persero), 2021a). Accordingly, the minimum investment

income for the GBM simulation is set to $w_0 = O_b + I_b = \text{IDR } 8{,}349{,}303{,}721.92$, which corresponds to the total monthly financing obligations that must be fulfilled by the issuer.

The drift parameter is set at $\mu = 25.53\%$, the historical margin of PT Sarana Multigriya Finansial (Persero). Based on Equation (2), the margin is defined as the ratio of profit to revenue measured over the same period t. Given the underlying business characteristics of the investment manager, the monthly margins are expected to exhibit characteristics broadly comparable to the annual margin calculated from the corresponding annual aggregates. Therefore, the historical annual margin is used as a representation of the profitability characteristic in the monthly simulation. PT Sarana Multigriya Finansial (Persero) acts as both the issuer and investment manager of the mudaraba sukuk at the time of issuance. The value μ represents the investment profitability characteristic at the beginning of the investment period and serves as the baseline for generating the subsequent dynamics of investment income through the GBM.

Meanwhile, the volatility parameter $\sigma = 30\%$ represents the uncertainty and variability of investment income. Both parameters are applied in the GBM consistent with the monthly installment mechanism. The stochastic component is represented by independent standard normal random variables $Z \sim N(0,1)$, with the number of random variables corresponding to the number of months in the investment period. These random numbers are combined with the specified drift parameter, volatility parameter, and minimum investment income, and are then used in the GBM formulation described in Equation (1) to generate the investment income over the investment period.

Table 3: Illustrative Simulation Model Result (in IDR millions)

t	w_t	ws_t	O_t	I_t	D_t	PD_t	TI_t	R_n
1	15,704.04	10,432.20	15.14	8,334.17	-	-	8,349.30	-
2	7,860.25	5,221.56	15.14	5,206.43	3,127.74	-	5,221.56	-
3	6,074.07	4,035.00	15.14	4,019.86	7,442.04	-	4,035.00	2,082.89
4	19,082.43	12,676.46	15.14	8,334.17	3,114.89	4,327.16	12,676.46	-
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
11	21,289.38	14,142.53	15.14	8,334.17	-	5,559.54	13,908.85	-
12	12,397.05	8,235.36	15.14	8,220.22	113.95	-	8,235.36	233.69

Table 3 illustrates the application of the proposed mathematical model to the mudaraba sukuk. The profit-sharing investment income, ws_t is obtained by multiplying the simulated investment income generated by the GBM model by the predetermined profit-sharing ratio (nisbah) of 66.43% specified in the sukuk prospectus. This amount is then used to settle the financing obligations during the investment period. The simulation demonstrates that the financing obligations vary depending on the availability of investment income.

When the available income is insufficient to fully satisfy the financing obligations, outstanding debt D_t is created and carried forward to the subsequent period. Conversely, when sufficient income is available in later periods, part or all of the outstanding debt is repaid through PD_t. For example, in period $t = 2$, the profit-sharing investment income is sufficient to cover the public offering cost and most of the principal redemption installment, resulting in an outstanding debt of IDR 3,127.74 million. The outstanding debt is subsequently settled in period $t = 4$, using the investment income generated during that period through $PD_{t=4}$. The same mechanism is applied throughout the investment period until maturity.

Based on the simulation results, the returns are subsequently calculated according to the modified financing scheme and the characteristics of the sukuk prospectus using a Monte Carlo simulation with 1,000 replications. The holder's implied rate of return r_s is determined using Equation (5), which represents the cash flows generated by the proposed model over the investment financing period. In this study, the value of r_s is obtained using the Internal Rate of Return method, which determines the rate of return that equates the present value of the simulated cash flows to the initial investment. Furthermore, return distributions to the sukuk holders R_n are made only on the scheduled distribution dates specified in the sukuk structure and are calculated from the remaining investment income after all financing obligations have been settled. Since profit distributions under the mudaraba sukuk are scheduled every three months, the first return distribution R_1, representing the accumulated distributable profit over the first three months after all financing obligations have been settled. Under this mechanism, the resulting implied annual return to the sukuk holders is 2.353% per annum. Meanwhile, the issuer, which also acts as the industry in the proposed financing scheme, is allocated a 33.49% share of the investment income, determined using Equation (6). The trustee receives the specified public offering fee, which corresponds to 0.182% of the initial principal fund. These results illustrate how the proposed financing model allocates investment income and associated fees among all parties according to their respective contractual arrangements, while ensuring that all financing obligations are settled by maturity.

For comparison, the implied rate of return obtained from the proposed model in the simulated scenario is lower than the fixed coupon rate specified in the prospectus, which is 4.75% per annum. This difference illustrates that different profit distribution mechanisms can result in different return characteristics. In the prospectus, the coupon rate is predetermined, whereas the proposed model links the profit distribution to the profitability

of the underlying investment. Nevertheless, this result remains illustrative, as it is obtained from a particular simulated realization. Therefore, further research is required, particularly on optimizing the profit-sharing ratio, as the proportion of profit-sharing affects the cash flows generated by the proposed model and, consequently, the returns received by the respective parties. Thus, the proposed model provides an alternative return mechanism that is more responsive to the performance of the underlying investment, while the fixed-coupon mechanism provides greater certainty regarding the predetermined periodic return.

Theoretically, under a mudaraba contract, the issuer, as the industry, is not obligated to guarantee the return of the principal amount, and financial losses are borne by the capital provider unless such losses arise from the issuer's negligence or breach of contract. However, the sukuk prospectus examined in this study stipulates full repayment of the principal at maturity. Consequently, the profit-sharing returns received by all parties are determined based on the proposed profit-sharing mechanism, while the principal is repaid in full at maturity in accordance with the terms set out in the prospectus. From the perspective of the simulation results, if the generated investment income is insufficient to fulfill the financing obligations by maturity, an outstanding amount may remain. Compared with a pure mudaraba arrangement, in which an unrecovered principal may constitute a capital loss borne by the capital provider, this finding highlights the importance of appropriate risk mitigation mechanisms to address potential shortfalls in principal repayment while preserving the profit-sharing nature of the sukuk structure.

Sensitivity Analysis Result

In this study, sensitivity analysis is conducted to evaluate the ability of the proposed model to fulfill the investment financing obligations under different investment conditions. The simulations are performed under multiple scenarios by varying the drift and volatility parameters used to generate investment income GBM. Both parameters are defined over the normalized interval [0,1], representing a broad range of profitability and uncertainty conditions from an economic perspective, from relatively low to high levels. The parameter space is discretized into 1,000 values, with each parameter setting evaluated through 1,000 independent Monte Carlo replications to estimate the corresponding probability of fulfilling the financing obligations.

Figure 4 presents the sensitivity of the probability of fully settling the financing obligations at maturity to the drift parameter μ. The results show a pronounced increasing pattern: the probability remains close to zero at relatively low values of μ, then increases progressively and eventually approaches a high level at larger values of μ. At approximately

$\mu = 30\%$, the probability begins to increase substantially. This pattern indicates that higher investment profitability increases the likelihood that the generated investment income can fulfill the financing obligations and eliminate outstanding debt by maturity.

Consistent with the sensitivity analysis of the drift parameter, Figure 5 shows that the probability of fully settling the financing obligations is zero at the initial volatility level, corresponding to a stable investment-income condition. This indicates that, under the baseline profitability represented by the observed margin, the generated investment income is insufficient to fulfill all financing obligations even when investment income is stable. Therefore, improving the profitability level, as indicated by the sensitivity analysis of μ, is more relevant to increasing the probability of full settlement than relying on changes in volatility.

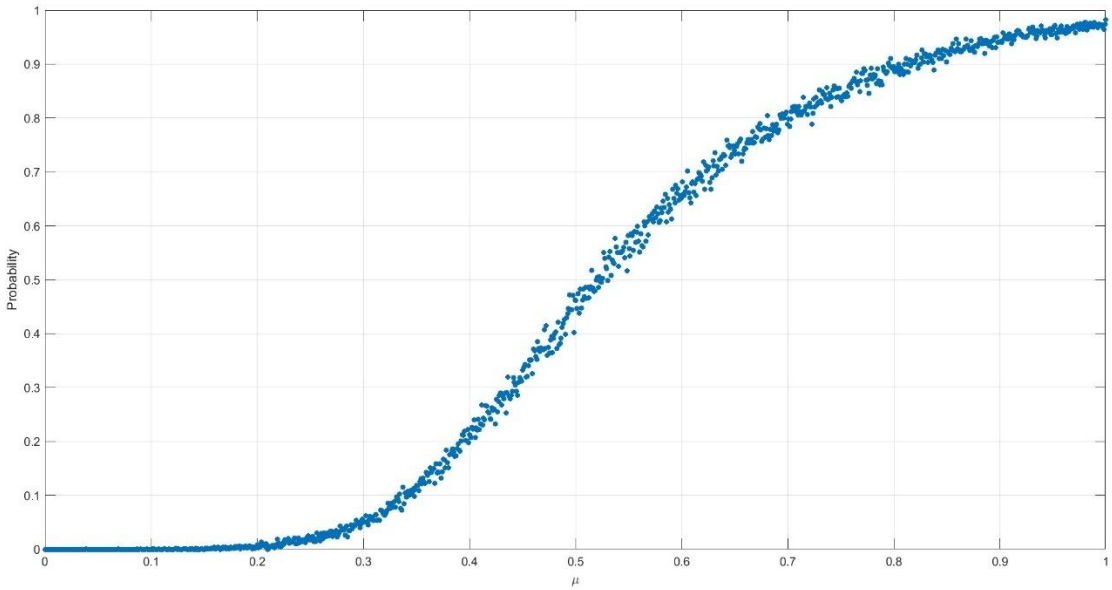

Figure 4. Probability of Full Liability Fulfillment under Variations of Drift

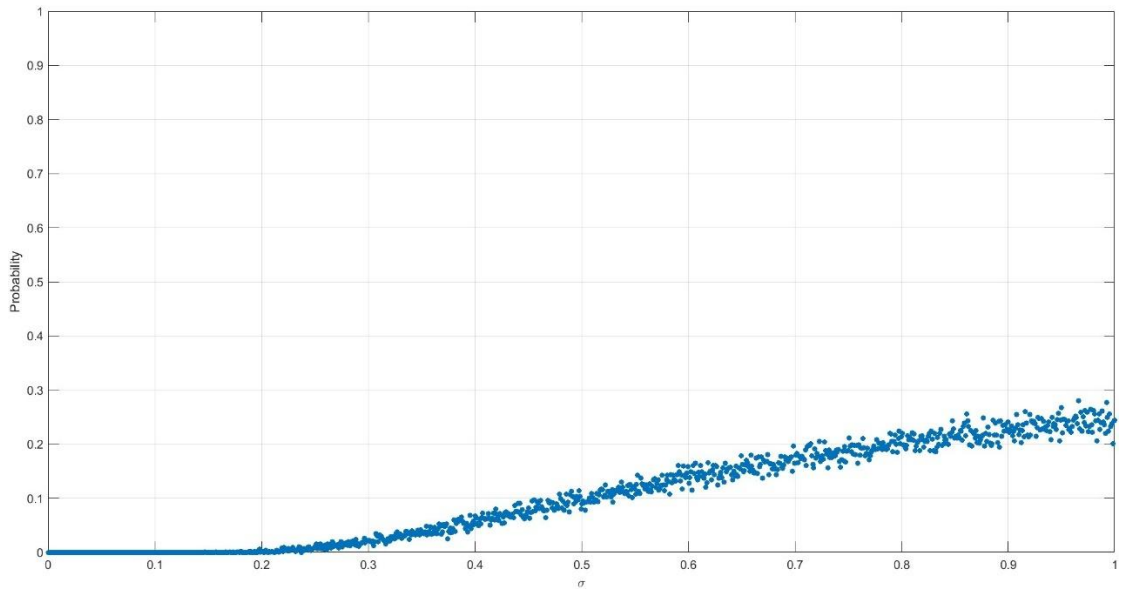

Figure 5. Probability of Full Liability Fulfillment under Variations of Volatility

The above condition provides a deterministic benchmark for the profitability level required to fulfill the financing obligations under the specified profit-sharing ratio. Based on the GBM formulation presented in Equation (1), and with a predetermined profit-sharing ratio (nisbah) of $p = 66.43\%$ on the stable condition ($\sigma = 0$), the break-even drift is approximately $\mu^* = 40.9\%$. Therefore, when $\mu < 40.9\%$, the investment income allocated through the profit-sharing mechanism is insufficient to cover the financing obligations under the stable condition. Since the empirical drift used in the simulation is $\mu = 25.53\%$, the baseline profitability is below this break-even level.

This result is consistent with the sensitivity analysis. When volatility is introduced, investment income becomes more dispersed, allowing some simulated realizations to generate sufficiently high income to settle the outstanding obligations. This results in a slight increase in the probability of full settlement as σ increases. However, the probability remains low because the underlying profitability is still below the break-even level. These results indicate that, under the considered profit-sharing ratio, an improvement in the underlying profitability is more important for increasing the probability of fulfilling the financing obligations than an increase in volatility.

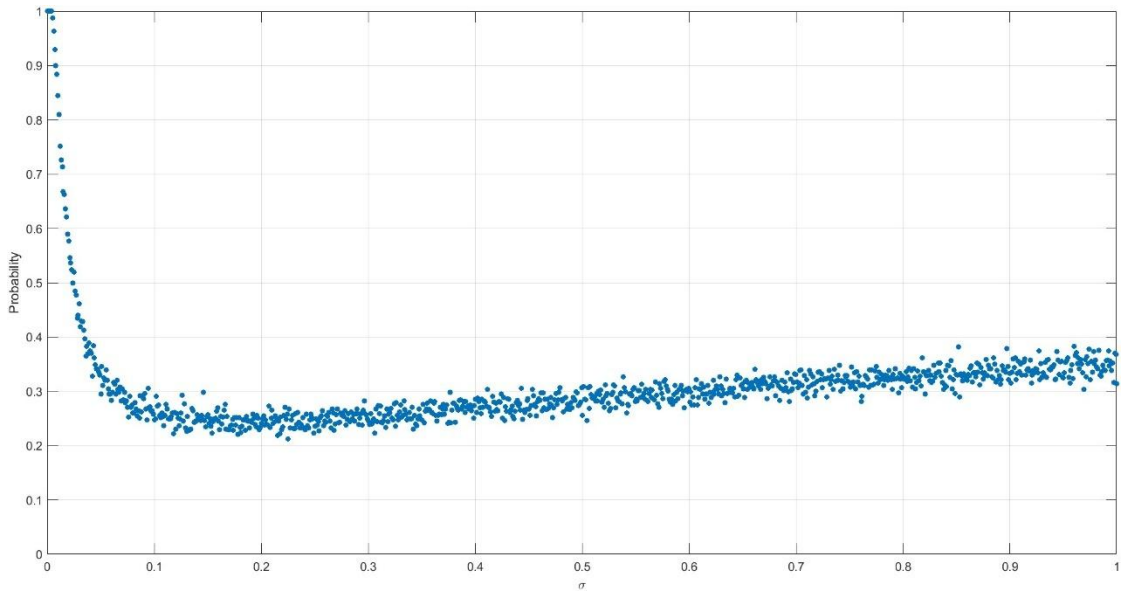

Figure 6. Probability of Full Liability Fulfillment under Variations of Volatility for $\mu = 42\%$

An additional simulation is conducted with the drift parameter set slightly above the deterministic break-even level, as presented in Figure 6. With the drift parameter is set to $\mu = 42\%$, the probability of fully settling the financing obligations is one under the stable condition ($\sigma = 0$). As volatility increases, the probability decreases sharply, particularly at low levels of σ, before gradually flattening at higher volatility levels. This pattern indicates that the financing structure is initially sufficient under stable investment-income conditions but becomes increasingly exposed to income fluctuations as volatility rises. The pronounced decline at low volatility levels is attributable to the relatively small margin between the specified profitability level and the break-even threshold.

Overall, the sensitivity analysis demonstrates that the feasibility of the proposed sukuk financing scheme is primarily influenced by the relationship between investment profitability and investment risk. A higher investment margin of the investment manager improves the ability to fulfill the financing obligations, while greater volatility in investment income increases exposure to outstanding debt, particularly when the profitability level is close to the break-even threshold. Therefore, both parameters should be jointly considered when evaluating the feasibility of a mudaraba sukuk financing scheme.

CONCLUSION

The proposed mathematical model provides a framework for implementing mudaraba sukuk financing with periodic profit-sharing distributions linked to investment profitability, while incorporating principal repayment at maturity in accordance with the

sukuk prospectus. The sensitivity analysis highlights the importance of considering both profitability and investment risk in assessing the fulfillment of investment liabilities. Given the limited literature on mathematical approaches to sukuk, this study contributes a mathematical perspective by providing an illustrative representation of sukuk financing under uncertain investment profitability. Future research may extend the model to other sukuk contracts, incorporate optimization techniques to determine optimal profit-sharing ratios, and develop risk mitigation mechanisms for potential shortfalls in financing obligations. Such extensions may contribute to a broader mathematical framework for analyzing profit-sharing mechanisms across various Sharia-compliant financing structures.

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