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RIEMANN–LIOUVILLE FRACTIONAL DERIVATIVES OF SINE, COSINE, AND TANGENT

Syifaul Janan[1*], Dimas Rizky Ananda[2], Bhaskara Panji Aditya Wardhana[3], Arya Hadi Saputra[4], Muhammad Fathur Firdaus[5], Witha Widyadana[6]
[1,2,3,4,5,6]Department of Mechanical Engineering, Universitas Pembangunan Nasional Veteran Jakarta, Jakarta, 16515, Indonesia

*Correspondence: syifaul.janan@upnvj.ac.id

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ABSTRACT

This study derives explicit fractional power-series representations of the Riemann–Liouville (RL) derivatives of the sine, cosine, and tangent functions for $0 < \alpha < 1$ by applying the left-sided RL operator with lower terminal at zero term by term to the Maclaurin series of each function. Each result is $x^{1-\alpha}$ or $x^{-\alpha}$ times a power series in x^2; for sine and cosine these are two-parameter Mittag–Leffler functions, and for tangent the coefficients involve the absolute values of the Bernoulli numbers. A Cauchy–Hadamard argument shows that the operator preserves the formal radius of convergence, so the series converge absolutely with $R = \infty$ for sine and cosine and $R = \pi/2$ for tangent, while the real domain is $x > 0$ for sine and cosine and $0 < x < \pi/2$ for tangent. Comparison with direct numerical evaluation of the RL definition gave relative differences below 10^{-10}, and plots illustrate the passage from the function ($\alpha = 0$) to the ordinary derivative ($\alpha = 1$).

Keywords: Fractional Derivative, Riemann–Liouville, Maclaurin Series, Convergence Analysis, Trigonometric Functions.

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PRELIMINARY

Fractional calculus extends differentiation and integration to non-integer orders and describes systems with memory and nonlocal effects (Mainardi, 2022). Among the many definitions (Ortigueira, 2024), the Riemann–Liouville (RL) derivative suits analytical work because it acts on power functions through the Gamma function (Zhou, 2023), which makes it compatible with power series that can be truncated for computation (Brociek et al., 2022).

RL-type operators are used to solve fractional differential equations (Mahatekar & Deshpande, 2025), to model viscoelastic materials (Bonfanti et al., 2020), and to design signal filters (Muresan et al., 2021), where their numerical evaluation must be done with care (Cardone & Conte, 2020; Diethelm, Garrappa, & Stynes, 2020). The RL derivative is

also the prototype of general fractional derivatives with Sonine kernels (Luchko, 2021), whereas derivatives with nonsingular kernels have been criticised (Diethelm, Garrappa, Giusti, et al., 2020).

Because the RL operator changes both the exponents and the coefficients of a Maclaurin expansion, it is unclear whether the classical convergence domain is preserved. Trigonometric functions test this well: sine and cosine are entire, whereas tangent has singularities at $x = \pm\pi/2$ that limit its series to $|x| < \pi/2$.

The Maclaurin–RL approach has been applied to exponential and hyperbolic functions (Janan et al., 2025; Janan, 2025; Janan & Janan, 2024), to $\ln x$, where the fractional integral converges on $(0,2]$ but the derivative only on $(0,2)$ (Janan et al., 2026), and to inverse trigonometric functions, whose radius of convergence equals that of the original function (Khoirunisa et al., 2026). For integer order, (Guo et al., 2022) gave Maclaurin expansions of powers of inverse (hyperbolic) sine and tangent, and Bernoulli-number expansions of tangent-type functions are still being studied (Li et al., 2023; Sun et al., 2023; Zhang & Qi, 2024).

The RL derivatives of sine and cosine are known to be expressible through the two-parameter Mittag–Leffler function (Garrappa et al., 2019; Gorenflo et al., 2020; Mainardi, 2020), and, to the authors' knowledge, no explicit series with a convergence analysis has been reported for tangent. The studies above also treat convergence function by function, without separating the formal radius of convergence from the real domain of the left-sided operator.

The series for sine and cosine are therefore not claimed as new. The contribution of this study is (i) a general Cauchy–Hadamard argument that term-by-term RL differentiation preserves the formal radius of any power series, applied to sine, cosine, and tangent; (ii) the separation of this radius from the real domain $x > 0$ of the left-sided operator, including the endpoint orders $\alpha = 0$ and $\alpha = 1$; (iii) the explicit tangent series with absolute Bernoulli numbers; and (iv) validation against direct evaluation of the RL definition.

METHODS

This study uses an analytical derivation followed by numerical verification, as summarised in Figure 1. The derivation starts from the classical Maclaurin series (Spiegel et al., 1968), where B_{2k} are the signed Bernoulli numbers ($B_2 = 1/6$, $B_4 = -1/30$), so that $|B_{2k}| = (-1)^{k-1}B_{2k}$:

$$\sin x = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} x^{2k+1}, \qquad |x| < \infty \tag{1}$$

$$\cos x = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} x^{2k}, \qquad |x| < \infty \tag{2}$$

$$\tan x = \sum_{k=1}^{\infty} \frac{2^{2k}(2^{2k}-1)B_{2k}}{(2k)!} x^{2k-1}, \qquad |x| < \frac{\pi}{2} \tag{3}$$

Throughout, D^{α} denotes the left-sided RL derivative with lower terminal at zero. For non-integer α with $n-1 < \alpha < n$, and $\Gamma(\cdot)$ the gamma function, it is defined by (Oldham & Spanier, 1974)

$$D^{\alpha} f(x) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dx^n} \int_0^x (x-s)^{n-\alpha-1} f(s)\, ds \tag{4}$$

For integer $\alpha = n$ it is the ordinary derivative d^n/dx^n, and $D^0 f = f$ by convention; Eq. (4) is used with $n = 1$ for $0 < \alpha < 1$, and $\alpha = 1$ is the ordinary derivative. The operator is linear (Oldham & Spanier, 1974): for constants $\lambda, u \in R$,

$$D^{\alpha}(\lambda f(x) + ug(x)) = \lambda D^{\alpha} f(x) + \mathrm{u} D^{\alpha} g(x) \tag{5}$$

and for a power function with $\alpha > 0$ it gives (Oldham & Spanier, 1974),

$$D^{\alpha} x^k = \frac{\Gamma(k+1)}{\Gamma(k-\alpha+1)} x^{k-\alpha} \tag{6}$$

Equations (5) and (6) allow the operator to act term by term; this interchange is justified in Proposition 1 (Results). The radius and domain of convergence of each fractional series are then obtained from the Cauchy–Hadamard formula and checked with the ratio test. Finally, the series are validated against direct numerical evaluation of Eq. (4), and plotted for $0 < \alpha < 1$ together with $\alpha = 0$ and $\alpha = 1$.

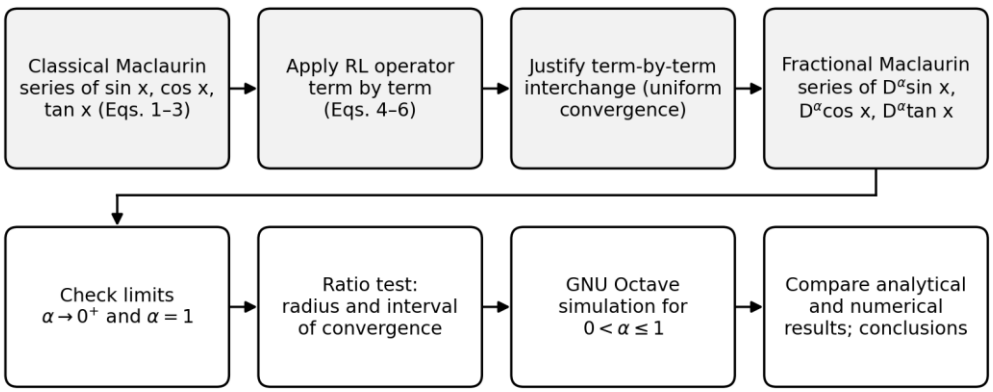


Figure 1. Research Flowchart

RESULTS AND DISCUSSION

1. Term-by-Term RL Differentiation and Radius of Convergence

Proposition 1. Let $f(x) = \sum_{n=0}^{\infty} c_n x^n$ have radius of convergence $R > 0$ and let $0 < \alpha < 1$. Then, for $0 < x < R$,

$$D^{\alpha} f(x) = x^{-\alpha} \sum_{n=0}^{\infty} c_n \cdot \frac{\Gamma(n+1)}{\Gamma(n+1-\alpha)} \cdot x^n \tag{7}$$

and the power series in (7) has the same radius of convergence R.

Proof. Put $w_n = \frac{\Gamma(n+1)}{\Gamma(n+1-\alpha)}$. Since $\frac{w_{n+1}}{w_n} = \frac{n+1}{n+1-\alpha} \to 1$, $w_n^{1/n} \to 1$, and by the Cauchy–Hadamard formula $\limsup_{n\to\infty} |c_n w_n|^{1/n} = \limsup_{n\to\infty} |c_n|^{1/n} = \frac{1}{R}$; the same holds for the weights $\frac{\Gamma(n+1)}{\Gamma(n+2-\alpha)}$. Now fix $0 < b < R$. On $[0, b]$ the series of f converges uniformly and $(x-s)^{-\alpha}$ is integrable, so the sum and the integral in $I^{1-\alpha} f(x) = \frac{1}{\Gamma(1-\alpha)} \int_0^x (x-s)^{-\alpha} f(s) ds$ may be interchanged. With $\int_0^x (x-s)^{-\alpha} s^n ds = \frac{\Gamma(1-\alpha)\Gamma(n+1)}{\Gamma(n+2-\alpha)} x^{n+1-\alpha}$ this gives

$$I^{1-\alpha} f(x) = \sum_{n=0}^{\infty} c_n \frac{\Gamma(n+1)}{\Gamma(n+2-\alpha)} x^{n+1-\alpha} \tag{8}$$

The termwise derivative of (8) is the series in (7), which converges uniformly on compact subintervals of $(0, R)$ by the radius argument. Hence (8) may be differentiated term by term, and $D^{\alpha} f = \frac{d}{dx} I^{1-\alpha} f$ gives (7). ■

Remark. Because $x^{-\alpha}$ is real only for $x > 0$, the real domain of the left-sided operator is $0 < x < R$, whereas R is a formal property of the coefficients. For the endpoint orders, $D^0 f = f$ by convention (and (7) tends to f as $\alpha \to 0+$), while for $\alpha = 1$ Eq. (7) gives f' because $\frac{1}{\Gamma(0)} = 0$ removes the term $c_0 \frac{x^{-\alpha}}{\Gamma(1-\alpha)}$.

2. Fractional Series of Sine, Cosine, and Tangent

Theorem 1. For $0 < \alpha < 1$, the RL derivatives of sine, cosine, and tangent are

$$D^{\alpha} \sin x = \sum_{k=0}^{\infty} \frac{(-1)^k}{\Gamma(2k+2-\alpha)} x^{2k+1-\alpha}$$

$$D^{\alpha} \cos x = \sum_{k=0}^{\infty} \frac{(-1)^k}{\Gamma(2k+1-\alpha)} x^{2k-\alpha}$$

$$D^{\alpha} \tan x = \sum_{k=1}^{\infty} \frac{2^{2k-1}(2^{2k}-1)|B_{2k}|}{k\Gamma(2k-\alpha)} x^{2k-\alpha-1}$$

Each series converges absolutely on $x > 0$ (sine, cosine) and on $0 < x < \pi/2$ (tangent),

with the radius of the classical series: $R = \infty$ and $R = \pi/2$, respectively.

Proof. Apply Eq. (7) to Eqs. (1)–(3), i.e. with $n = 2k+1$, $n = 2k$, and $n = 2k-1$. Since $\Gamma(2k+2) = (2k+1)!$, $\Gamma(2k+1) = (2k)!$, and $\Gamma(2k) = (2k-1)!$, the factorials cancel; for tangent, $\frac{2^{2k}\Gamma(2k)}{(2k)!} = \frac{2^{2k-1}}{k}$. The radii are those of (1)–(3) by Proposition 1 and are confirmed by the ratio test in Section 3. ■

Remark. With $E_{2,\beta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(2k+\beta)}$, Eqs. (9) and (10) equal $x^{1-\alpha}E_{2,2-\alpha}(-x^2)$ and $x^{-\alpha}E_{2,1-\alpha}(-x^2)$ (Garrappa et al., 2019; Gorenflo et al., 2020; Mainardi, 2020), whereas the tangent series, whose coefficients contain Bernoulli numbers, is not of Mittag-Leffler type. As $\alpha \to 0+$ the series give $\sin x$, $\cos x$, and $\tan x$, and as $\alpha \to 1$ they give $\cos x$, $-\sin x$, and $(\sec x)^2$ (for cosine the term $k = 0$ vanishes because $\frac{1}{\Gamma(1-\alpha)} \to 0$), consistent with the classical operators.

3. Convergence, Real Domain, and Endpoint Behaviour

Let a_k be the k-th term of each series and $x > 0$. By $\Gamma(z+2) = (z+1)z\Gamma(z)$, the ratios for sine and cosine are

$$\left|\frac{a_{k+1}}{a_k}\right| = \frac{x^2}{(2k+3-\alpha)(2k+2-\alpha)}, \quad \left|\frac{a_{k+1}}{a_k}\right| = \frac{x^2}{(2k+2-\alpha)(2k+1-\alpha)}$$

respectively. Both tend to 0 as $k \to \infty$, so the series converge absolutely for every $x > 0$, in agreement with $R = \infty$. For tangent,

$$\left|\frac{a_{k+1}}{a_k}\right| = x^2 \frac{k}{k+1} \frac{\Gamma(2k-\alpha)}{\Gamma(2k+2-\alpha)} \frac{2^{2k+1}(2^{2k+2}-1)|B_{2k+2}|}{2^{2k-1}(2^{2k}-1)|B_{2k}|}$$

and the asymptotics $|B_{2k}| \sim \frac{2(2k)!}{(2\pi)^{2k}}$ give

$$\lim_{k\to\infty} \left|\frac{a_{k+1}}{a_k}\right| = \frac{4x^2}{\pi^2}$$

which is smaller than 1 if and only if $x < \pi/2$. Hence $R = \pi/2$, as predicted by Proposition 1; the series diverges for $|x| > \pi/2$ in the formal sense, and the endpoint $x = \pi/2$ was not examined.

Because of the factor $x^{-\alpha}$, the series are real only for $x > 0$: the real domains are $(0, \infty)$ for sine and cosine and $(0, \pi/2)$ for tangent, and $-\pi/2 < x < 0$ is not covered by the left-sided operator. The passage from $\alpha = 0$ to $\alpha = 1$ is not uniform near $x = 0$: $D^{\alpha}\sin x \sim \frac{x^{1-\alpha}}{\Gamma(2-\alpha)} \to 0$ whereas $\cos 0 = 1$, and $D^{\alpha}\cos x \sim \frac{x^{-\alpha}}{\Gamma(1-\alpha)}$ is unbounded whereas $-\sin 0 = 0$. The latter reflects that the RL derivative of a constant c is $c\frac{x^{-\alpha}}{\Gamma(1-\alpha)}$, not zero as for the Caputo

derivative.

4. Numerical Validation and Graphs

The series were checked against two evaluations that do not use them, following the practice of independent verification advised for fractional-order computations (Diethelm, Garrappa, & Stynes, 2020) . (a) The RL definition, Eq. (4) with $n = 1$, was evaluated as

$$D^{\alpha}f(x) = g'(x), \qquad g(x) = \frac{1}{\Gamma(1-\alpha)}\int_0^x (x-s)^{-\alpha} f(s)ds$$

by tanh-sinh quadrature and central differences (step 10^{-6}). (b) The equivalent form

$$D^{\alpha}f(x) = \frac{f(0)x^{-\alpha} + \int_0^x (x-s)^{-\alpha} f'(s)ds}{\Gamma(1-\alpha)}$$

valid for $f \in C^1$, was evaluated by quadrature. The series used 80 terms (sine, cosine) and 400 terms (tangent), with Bernoulli numbers computed in 50-digit arithmetic (Python, mpmath). Over all 27 points ($\alpha = 0.25, 0.5, 0.75$; $x = 0.5, 2, 6$ for sine and cosine, $x = 0.5, 1, 1.4$ for tangent) the largest relative difference was 2.7×10^{-11} against (a), where finite-difference error dominates, and 1.9×10^{-13} against (b). Table 1 lists representative values; the script is provided as supplementary material.

Table 1. Series versus direct evaluation of the RL derivative, methods (a) and (b), at representative points; all 27 points are in the supplementary file.

Function	α	x	Series	Direct (a)	Rel. diff. vs (a)	Rel. diff. vs (b)
sin x	0.25	2.0	0.61729159	0.61729159	1.9e-13	4.6e-42
sin x	0.5	2.0	0.28045646	0.28045646	2.3e-13	1.7e-27
sin x	0.75	2.0	-0.07523737	-0.07523737	3.0e-14	1.9e-13
cos x	0.25	2.0	-0.70405367	-0.70405367	1.8e-13	8.9e-42
cos x	0.5	2.0	-0.90100806	-0.90100806	1.8e-13	1.2e-27
cos x	0.75	2.0	-0.97552794	-0.97552794	1.8e-13	3.3e-14
tan x	0.25	1.0	1.89865116	1.89865116	1.4e-12	7.4e-42
tan x	0.5	1.0	2.30050957	2.30050957	1.7e-12	1.2e-27
tan x	0.75	1.0	2.79122897	2.79122897	2.2e-12	3.6e-14

Figures 2–4 show $D^{\alpha}f$ for $\alpha = 0.1, \ldots, 0.9$ together with f ($\alpha = 0$) and f' ($\alpha = 1$), generated from the series on grids of 300 points ($0.02 \le x \le 12$ for sine, $0.15 \le x \le 12$ for cosine, $0.02 \le x \le 1.45$ for tangent). The curves move from the function towards its ordinary derivative as α increases, with the non-uniform behaviour near $x = 0$ noted above. For tangent they increase without bound as $x \to \frac{\pi}{2}^-$, consistent with the singularity; as a check, the curve for $\alpha = 1$ equals $(\sec 1.25)^2 \approx 10.06$ at $x = 1.25$.

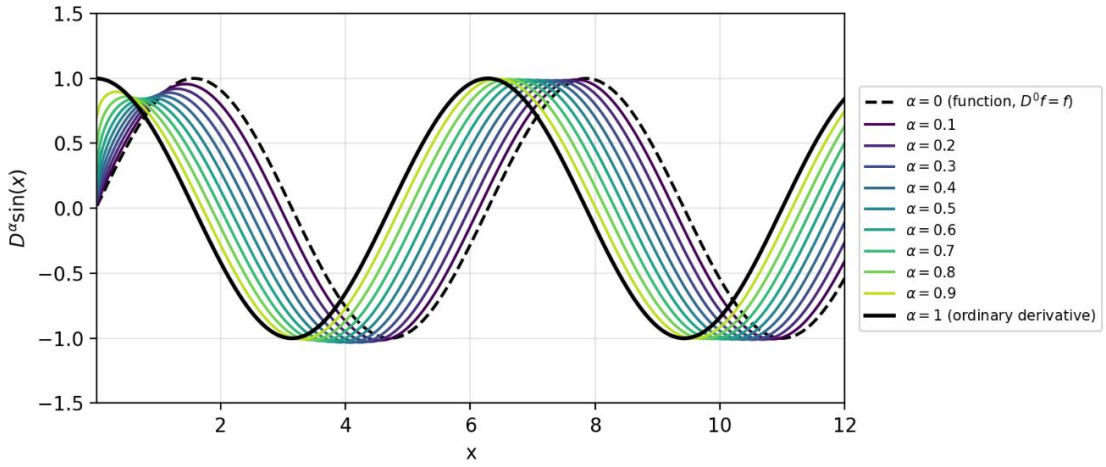


Figure 2. Graph of $D^\alpha \sin x$

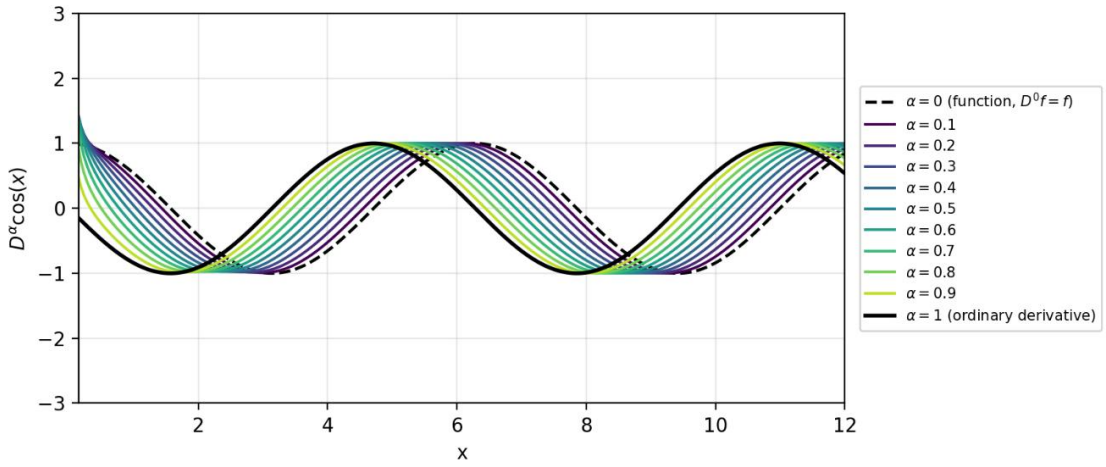


Figure 3. Graph of $D^\alpha \cos x$

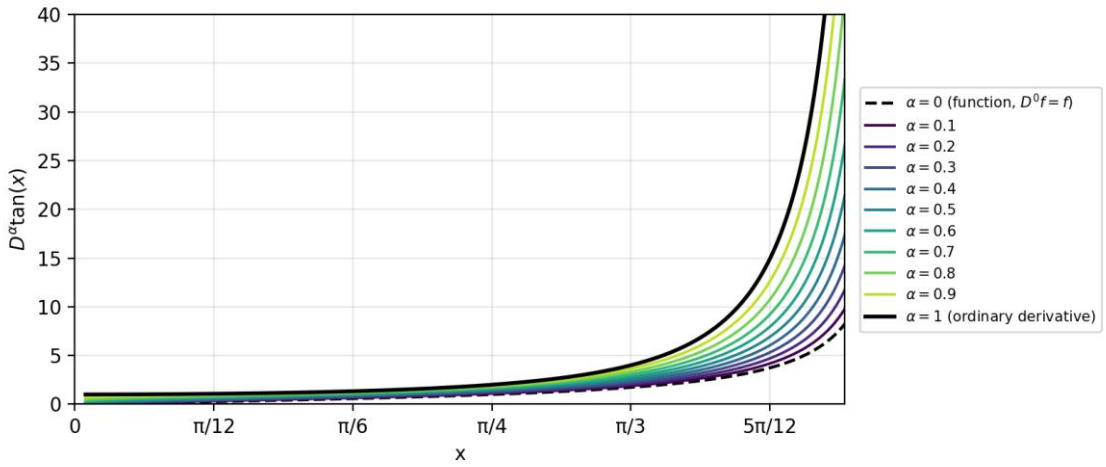


Figure 4. Graph of $D^\alpha \tan x$

CONCLUSION

Explicit RL fractional series of sine, cosine, and tangent for $0 < \alpha < 1$ were obtained, with the classical radii $(\infty, \infty, \pi/2)$, real domains $x > 0$, $x > 0$, $0 < x < \pi/2$, and agreement with direct numerical evaluation of the RL definition. The study is limited to the left-sided RL derivative with lower terminal zero, $0 < \alpha < 1$, and real $x > 0$, with convergence shown by the radius and ratio test and no truncation-error bounds. Future work may derive truncation-error bounds and treat the Caputo derivative and the cotangent, secant, and cosecant functions.

DECLARATIONS

Author Contribution	:	Syifaul Janan: Conceptualization, Methodology, Writing – Original Draft, Supervision. Dimas Rizky Ananda: Formal Analysis, Investigation, Visualization. Bhaskara Panji Aditya Wardhana: Formal Analysis, Investigation, Visualization. Arya Hadi Saputra: Formal Analysis, Investigation, Visualization. Muhammad Fathur Firdaus: Writing – Review & Editing. Witha Widyadana: Writing – Review & Editing.
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