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SIQS Epidemic Model With Bilinear Incidence

SIQS Model Epidemi Dengan Insiden Bilinear

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ABSTRAK

Penyakit adalah sebuah kondisi tidak normal pada tubuh atau pikiran yang menyebabkan ketidaknyamanan. Terdapat dua jenis penyakit berdasarkan cara penularannya, yaitu penyakit menular dan penyakit tidak menular. Penyakit menular dapat mewabah pada sebuah populasi. Wabah penyakit pada kurun waktu yang singkat disebut epidemi. Jika wabah penyakit berada pada sebuah populasi dalam kurun waktu yang lama, maka disebut endemik. Model epidemik, merupakan sebuah model yang dapat menggambarkan fenomena penyebaran penyakit menular. Dalam perkembangannya, model penyebaran penyakit tidak hanya model SI saja, akan tetapi juga berkembang menjadi model SIQS dan model – model lainnya. Dalam suatu model epidemik, diperlukan sebuah populasi yang tercampur dengan baik, sehingga setiap individu yang terinfeksi mempunyai peluang yang sama dalam menularkan penyakitnya pada individu rentan. Kondisi seperti itu dinamakan dengan kejadian bilinier. Penelitian ini bertujuan untuk mengetahui faktor / variabel yang mempengaruhi model epidemik dengan tipe SIQS dengan kejadian bilinier. Hasil penelitian menunjukkan bahwa Model Epidemik SIQS dengan Kejadian Bilinier memiliki bilangan rasio reproduksi dasar $R_0 = \frac{\Lambda\beta}{(\alpha+\sigma+\mu)\mu}$. Berdasarkan hasil simulasi diperoleh bahwa beberapa cara yang dapat digunakan agar penyakit hilang dari populasi adalah dengan mengurangi kontak dengan individu terinfeksi dan/atau menekan laju kelahiran.
Kata Kunci : Penyakit; Epidemik, Model, Bilinier

ABSTRACT

Illness is an abnormal condition in the body or mind that causes discomfort. There are two types of diseases based on the way they are transmitted, namely infectious diseases and non-communicable diseases. Infectious diseases can occur in a population. An outbreak of disease over a short period of time is called an epidemic. If a disease outbreak persists in a population for a long period of time, it is endemic. Epidemic model, is a model that can describe the phenomenon of the spread of infectious diseases. During its development, the disease spread model is not only the SI model, but also develops into the SIQS model and other models. In an epidemic model, a well-mixed population is needed, so that each infected individual has an equal chance of transmitting the disease to susceptible individuals. Such conditions are called bilinear incidence. In the initial step, the formulation of the model was formed, then determine the disease-free and endemic equilibrium point of the formed model. Next determine the basic reproduction ratio number. Then an analysis of the stability of the disease-free equilibrium point and the endemic equilibrium point is carried out, which is then carried out on the simulation of the model. This study aims to determine the factors / variables that affect the epidemic model with the SIQS type with bilinear incidence. The results showed that the SIQS epidemic model with bilinear incidence has a basic reproduction ratio number $R_0 = \frac{\Lambda\beta}{(\alpha+\sigma+\mu)\mu}$. Based on the simulation results, it is found that several ways that can be used to eliminate disease from the population are by reducing contact with infected individuals and or reducing the birth rate.

Keywords : Illness, Epidemic, Model, Bilinear Incidence

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PRELIMINARY

Disease is an abnormal condition in the body or mind that causes discomfort. The disease is divided into two types, namely infectious diseases and non-communicable diseases (Bustan, 1997). Infectious disease is a disease that is spread caused by germs, such as amoebas, bacteria, fungi, and viruses that attack the human body. Some examples of infectious diseases, among others, influenza, malaria, measles, avian flu, etc. Communicable diseases are diseases that spread is not caused by germs, but due to the constraints of physiological or metabolic processes that occur in the tissues of the human body, including dizziness, mouth sores, abdominal pain, etc. Infectious disease may be endemic in a population. Outbreak of the disease in a short period of time called an epidemic. If the outbreak of the disease in a population in a long time, then it is called endemic.

Epidemic Model, is a model that can describe the phenomenon of the spread of infectious diseases. Epidemic Model the simplest model is the SI. In this model, the population observed is divided into two parts, namely the group of susceptible individuals, S (susceptible), and groups of individuals infected, I (infectives). Kermack-Mckendrick expand the model THE model SIR by adding a group of individuals who recover R (recovery) in 1927. In 1932, the Kermack-Mckendrick also formulate the SIS model, where in this model, individuals who are infected will be vulnerable again after recovering from the disease (Padilah, T., 2017; Chen S. et al, 2018). If on the model SIS, do the quarantine on the infected individual, then the obtained model SIQS (Ma and Li, 2009). In the model SIQS, the letters Q (quarantine) represent a group of individuals who are quarantined after being infected. In developments, some of the diseases that requires the quarantine process as one way to heal and prevent widespread transmission of the disease. However, it is possible that any individual who has been cured of the disease can be re-infected by the same disease, in other words, each individual is cured back into the group of susceptible individuals S (susceptible). One example is the case of illness that requires quarantine to prevent the spread of infection and accelerate the healing process is Covid-

19. The characteristics of covid-19 that resembles influenza leads to an opportunity for any individual who has been cured of the disease is re-infected by the same disease (Parra-Lucares et. Al, 2022; Zayet, S., et al, 2020). In the case of the spread of the disease Covid-19, any individual who has been infected have the same opportunities in the spread of the disease to every individual on vulnerable groups. In an epidemic model, condition as it is called with the incidence of bilinear (bilinear incidence) (A. A. Ayoade et al., 2019; Baba, I. A. and Hincal, E., 2017; Ghosh J.K., et al, 2019). Therefore, the writer is interested to conduct a research about the model of the SIQS Epidemic Model with Bilinear Incidence. This study aims to determine the factors / variables that affect the epidemic model with type SIQS with the incidence of bilinear.

METHODS

On the first step carried out the study of literature supporting that consists of several journals, lecture notes, books and articles come from the internet. After review, the journal and other supporting materials, the authors conducted a formulation of the model in accordance with the assumptions used in the SIQS Epidemic Model with Bilinear Incidence. After the formulation of the model, next determine the equilibrium point of disease-free and endemic of the model. Next determine the number ratio of the basic reproduction. After the equilibrium point is determined, the next step is to analysis the stability of the equilibrium point of disease-free and the equilibrium point is endemic. Then the simulation model is done using Matlab software.

RESULTS AND DISCUSSION

The assumptions used in the formation of the SIQS Epidemic Model with Bilinear Incidence is as follows :

1. There are birth and natural death in each population in a row of Λ and μ,
2. Each individual entered into a new sub-population vulnerable,
3. The rate of spread of the disease is random means that each infected individual has an equal opportunity to transmit the disease in each individual vulnerable to the pace of the spread of the disease by βSI,
4. Fatal disease (cause of death) with a mortality rate of σ,
5. There is no emigration in each sub-population,
6. Each infected individuals have to go through the quarantine process with the rate of

displacement of a population of α,

7. S Any individual who healed after quarantined back to the sub-population of S, with a cure rate of γ.

The following is a diagram of the compartment of the SIQS Epidemic Model with Bilinear Incidence based on the assumptions above :

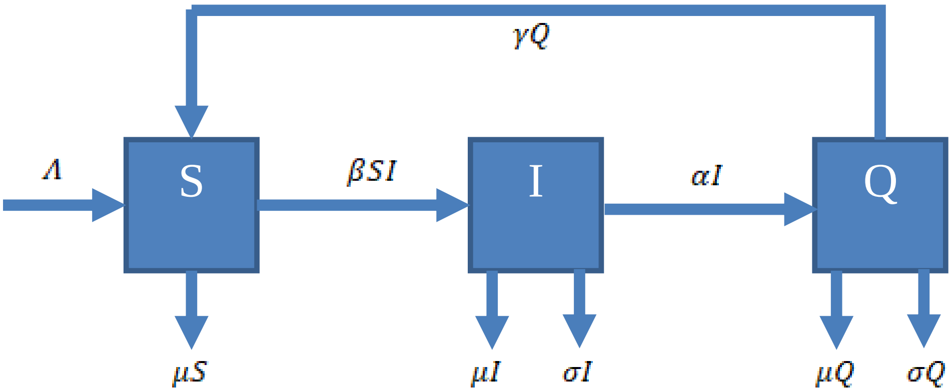

Figure 1. Diagrams Compartment of the SIQS Epidemic Model with Bilinear Incidence

$$\frac{dS}{dt} = \Lambda - \beta SI - \mu S + \gamma Q$$
$$\frac{dI}{dt} = \beta SI - (\sigma + \mu + \alpha)I \qquad (1)$$
$$\frac{dQ}{dt} = \alpha I - (\sigma + \mu + \gamma)Q$$

Theorem 1. *Given the domain of the System (1), namely* $\Omega = \left\{(S,I,Q)|0 \le S \le \frac{\Lambda}{\mu}, 0 \le I \le \frac{\Lambda}{\mu}, 0 \le Q \le \frac{\Lambda}{\mu}, S + I + Q \le \frac{\Lambda}{\mu}\right\}$. *Set* Ω *is the set of finite and a is the set of invariant positive.*

Proof. Taken any initial value $(S(t_0), I(t_0), Q(t_0)) \in \Omega$. Note that $N = S + I + Q$, then by summing the third equation in System (1) is obtained

$$\frac{dN}{dt} = \Lambda - \mu N - \sigma(I + Q) \le \Lambda - \mu N \qquad (2)$$

So, for the value of the $t \to \infty$ obtained $N \to \frac{\Lambda}{\mu}$. In the other word

$$\lim_{t\to\infty}(S + I + Q) = \lim_{t\to\infty} N \le \frac{\Lambda}{\mu}.$$

Note Inequalities (2), if $N > \frac{\Lambda}{\mu}$, then $\frac{dN}{dt} < 0$. Conversely, if the $N < \frac{\Lambda}{\mu}$, then $\frac{dN}{dt} > 0$. Next note that $t > 0$, obtained $S + I + Q = N \le \frac{\Lambda}{\mu}$. So, it is proved that the Set of Ω is finite and is the set of invariant positive.

Theorem 2. *The disease-free equilibrium point from the System (1) is* $E_0 = (S_0, I_0, Q_0) = \left(\frac{\Lambda}{\mu}, 0, 0\right)$.

Theorem 3. *If* $R_0 = \frac{\Lambda\beta}{(\alpha+\sigma+\mu)\mu} > 1$, *then the endemic equilibrium point of the System (1) is*

$$E^* = (S^*, I^*, Q^*) = \left(\frac{\alpha+\sigma+\mu}{\beta}, \frac{(\Lambda\beta-(\alpha+\sigma+\mu)\mu)(\sigma+\gamma+\mu)}{\beta(\alpha\sigma+\alpha\mu+\sigma^2+\sigma\gamma+2\sigma\mu+\gamma\mu+\mu^2)}, \frac{\alpha(\Lambda\beta-(\alpha+\sigma+\mu)\mu)}{\beta(\alpha\sigma+\alpha\mu+\sigma^2+\sigma\gamma+2\sigma\mu+\gamma\mu+\mu^2)}\right)$$

Theorem 4. *If* $R_0 = \frac{\Lambda\beta}{(\alpha+\sigma+\mu)\mu} < 1$, *then the disease-free equilibrium point* E_0 *is locally asymtotic stable.*

Proof. By performing the process of the linearization of System (1) at the point E_0, obtained by the Jacobian Matrix as follows,

$$J_0 = \begin{pmatrix} -\mu & -\beta S_0 & \gamma \\ 0 & \beta S_0 - (\sigma+\mu+\alpha) & 0 \\ 0 & \alpha & -(\sigma+\mu+\gamma) \end{pmatrix}$$

So, obtained the eigen value of

$$\lambda_1 = -\mu,\ \lambda_2 = \beta S_0 - (\sigma+\mu+\alpha) = \frac{\Lambda\beta}{\mu} - (\sigma+\mu+\alpha) = (\sigma+\mu+\alpha)(R_0-1), \text{ and}$$

$$\lambda_3 = -(\sigma+\mu+\gamma).$$

If $R_0 < 1$, then $\lambda_2 = (\sigma+\mu+\alpha)(R_0-1) < 0$. So, the disease-free equilibrium point E_0 is locally asymptotic stable.

Theorem 5. *If* $R_0 = \frac{\Lambda\beta}{(\alpha+\sigma+\mu)\mu} > 1$, *then the endemic equilibrium point* E^* *is locally asymptotic stable.*

Proof. By performing the process of the linearization of System (1) at the point E^*, obtained by the Jacobian Matrix as follows,

$$J^* = \begin{pmatrix} -(\beta I^*+\mu) & -\beta S^* & \gamma \\ \beta I^* & \beta S^* - (\sigma+\mu+\alpha) & 0 \\ 0 & \alpha & -(\sigma+\mu+\gamma) \end{pmatrix}$$

So that the obtained characteristic equation of the Jacobian Matrix J^* as follows,

$$\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3.$$

with

$$a_1 = \alpha + 2\sigma + \gamma + 3\mu + \beta I^* - \beta S^* > 0$$

$$a_2 = \alpha\sigma + \alpha\gamma + \sigma\gamma + 2\alpha\mu + 4\sigma\gamma + 2\gamma\mu + \sigma^2 + 3\mu^2 + \alpha\beta I^* + 2\sigma\beta I^* + \gamma\beta I^* + 2\mu\beta I^* - \sigma\beta S^* - \gamma\beta S^* - 2\mu\beta S^* > 0$$

$$a_3 = \alpha\mu^2 + 2\sigma\mu^2 + \sigma^2\mu + \gamma\mu^2 + \mu^3 - \mu^2\beta S^* + \alpha\sigma\mu + \alpha\gamma\mu + \sigma\gamma\mu + \sigma^2\beta I^* + \mu^2\beta I^* - \sigma\mu\beta S^* - \gamma\mu\beta S^* + \alpha\sigma\beta I^* + \gamma\sigma\beta I^* + \alpha\mu\beta I^* + 2\sigma\mu\beta I^* + \gamma\mu\beta I^* > 0$$

because $a_1 > 0, a_2 > 0, a_3 > 0$ and $a_1 a_2 - a_3 > 0$, then with Routh-Hurwitz Criterion, it is obtained that all the eigenvalues of the System (1) has a part number of negative real. So the endemic equilibrium point E^* *is locally asymptotic stable*.

SIMULATION MODEL

The simulation model is given with the aim of complementing the analytical results obtained in the previous discussion. The simulation model is numerically carried out with the help of Matlab software R2020b. The simulation model is done by changing the parameters Λ and β, because these parameters are parameters that have a considerable influence on changes in the value of R_0

Table 1. The Parameter Values

No	Notation	Value	Source
1	Λ	138 person per day	BPS Banyumas (2020)
2	β	$1{,}58 \text{ x } 10^{-13}$ / day	Satuan Tugas Penanganan Covid-19 (2020)
3	μ	$\frac{1}{70 \times 365}$ / day	it is assumed the average human lifespan of 70 years
4	σ	$1{,}31 \text{ x } 10^{-4}$ / day	Satuan Tugas Penanganan Covid-19 (2020)
5	α	0,00053 / day	Pemerintah Kabupaten Banyumas (2020)
6	γ	0,00001 / day	Vitale J., et al (2021)

If used parameters as in Table 1., then obtained the values of $R_0 = 0{,}0007984 < 1$. So, according to the Theorem 4. The disease-free equilibrium point E_0 is locally asymptotic stable.

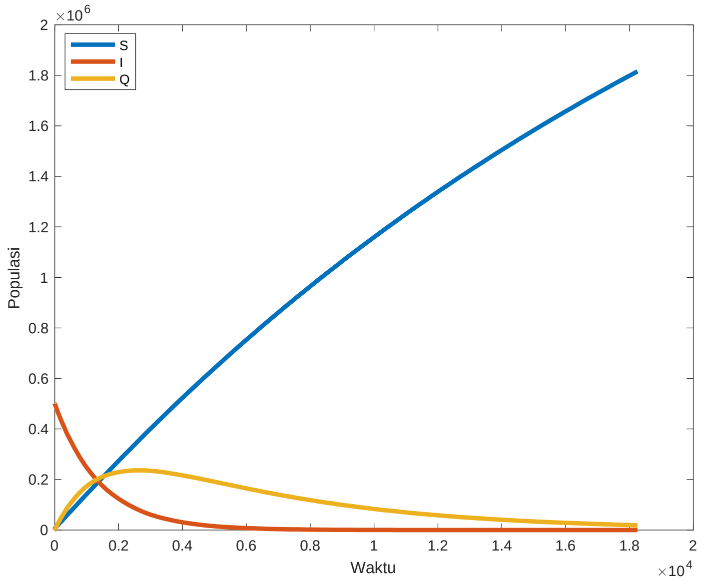

Figure 2. Chart of the System (1) when $R_0 = 0{,}0007984 < 1$

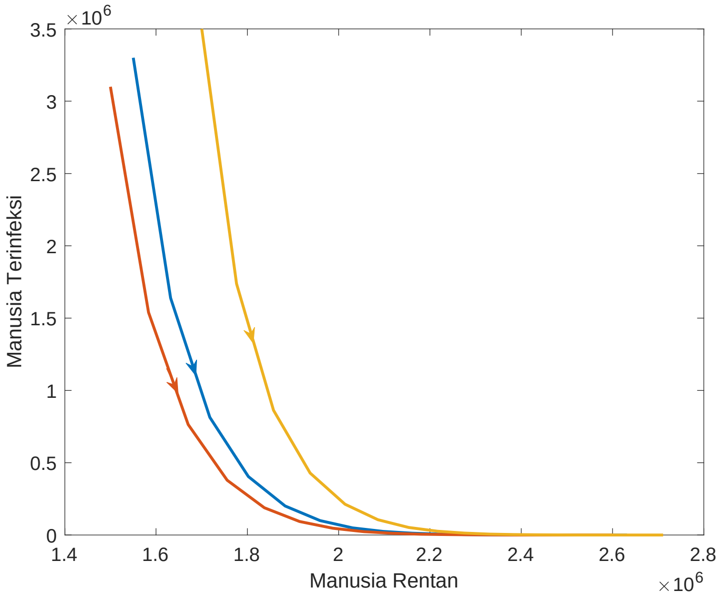

Figure 3. Phase Portrait pf the System (1) when $R_0 = 0{,}0007984 < 1$

Figure 2 and Figure 3 is a chart and a phase portrait of the System (1) with the initial value of $S_0 = 6000, I_0 = 500000$ and $Q_0 = 72$. According to the both of the Figure above obtained that the disease-free equilibrium point $E_0 = (3525900{,}0{,}0)$ is locally asymptotic stable. So it can be said that the disease will disappear from the population.

The simulation is then performed again with the parameter values used in the Table 2.

Table 2. The Parameter Values

No	Notation	Value	Source
1	Λ	20000 person per day	assumed
2	β	$5 \text{ x } 10^{-11}$ / day	assumed
3	μ	$\frac{1}{70 \, x \, 365}$ / day	it is assumed the average human lifespan of 70 years
4	σ	$1{,}31 \text{ x } 10^{-4}$ / day	Satuan Tugas Penanganan Covid-19 (2020)
5	α	0,00053 / day	Pemerintah Kabupaten Banyumas (2020)
6	γ	0,00001 / day	Vitale J., et al (2021)

If used parameters as in Table 2., then obtained the values of $R_0 = 36{,}4781 > 1$. So, according to the Theorem 5. The endemic equilibrium point E^* is locally asymptotic stable.

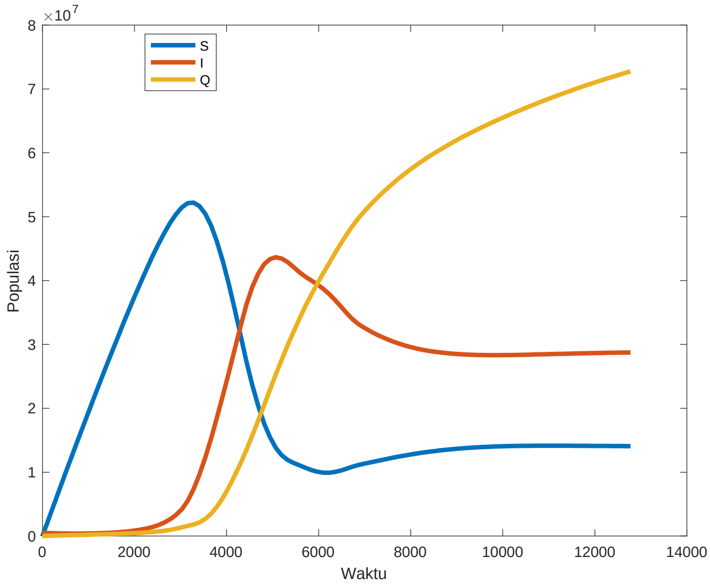

Figure 4. Chart of the System (1) when $R_0 = 36{,}4781 > 1$.

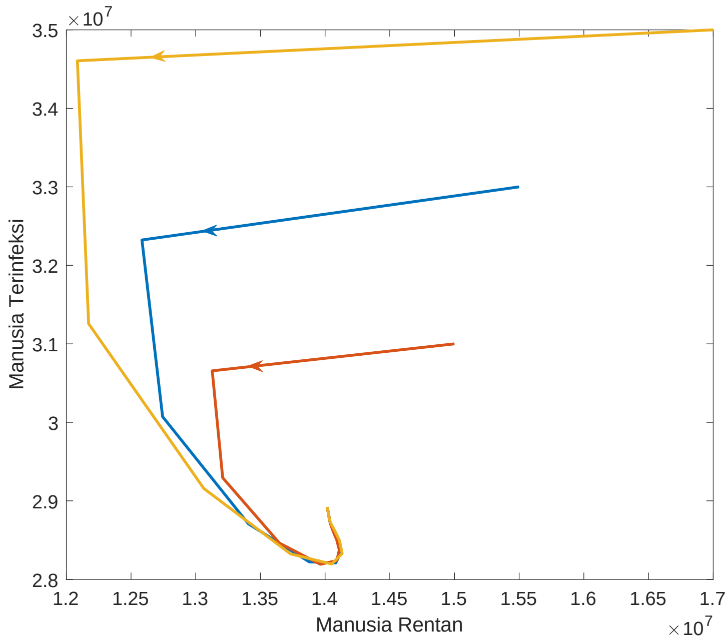

Figure 5. Phase Portrait pf the System (1) when $R_0 = 36{,}4781 > 1$

Figure 4 and Figure 5 is a chart and a phase portrait of the System (1) with the initial value of $S_0 = 6000, I_0 = 5000000$ and $Q_0 = 72000$. According to the both of the Figure above obtained that the endemic equilibrium point $E^* = (14008000{,}28987000{,}85153000)$ is locally asymptotic stable. So, it can be concluded that occur endemic, or in other words, the disease persists in the population.

CONCLUSION

The conclusions obtained from this study are as follows:

1. The SIQS Epidemic Model with bilinear incidence stated in the System (1), with the number ratio of the basic reproduction (R_0) is $\frac{\Lambda\beta}{(\alpha+\sigma+\mu)\mu}$.

   a. If $R_0 < 1$, the disease-free equilibrium point $E_0 = \left(\frac{\Lambda}{\mu}, 0{,}0\right)$ is locally asymptotic stable.

   b. If $R_0 > 1$, then the endemic equilibrium point

$$E^* = \left(\frac{\alpha+\sigma+\mu}{\beta}, \frac{(\Lambda\beta-(\alpha+\sigma+\mu)\mu)(\sigma+\gamma+\mu)}{\beta(\alpha\sigma+\alpha\mu+\sigma^2+\sigma\gamma+2\sigma\mu+\gamma\mu+\mu^2)}, \frac{\alpha(\Lambda\beta-(\alpha+\sigma+\mu)\mu)}{\beta(\alpha\sigma+\alpha\mu+\sigma^2+\sigma\gamma+2\sigma\mu+\gamma\mu+\mu^2)}\right)$$

   is locally asymptotic stable.

2. Based on the simulation results obtained that the smaller the rate of population growth (Λ) and the smaller the level of effectiveness of the contact between vulnerable

individuals with infected individuals (β) will cause the smaller the value of R_0. On the contrary the greater the rate of population growth (Λ) and the greater the level of effectiveness of the contact between vulnerable individuals with infected individuals (β) will lead to greater value of R_0. Thus, some of the ways that can be used so that the disease disappear from the population is to reduce contact with the infected individual and/or reduce the rate of birth.

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