Kernel and Properties in Up Translate and Down Translate Intuitionistic Ring Fuzzy
Kernel Dan Sifatnya Pada Translasi Naik Dan Turun Ring Fuzzy Intuitionistik

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ABSTRAK
Kata Kunci : ring fuzzy intuitionistik, kernel, translasi, ideal fuzzy intuitionistik.

ABSTRACT
Intuitionistic fuzzy set is an extension of fuzzy set, where there are membership function and non-membership function when added up is worth less than one. It is used in a vague and ambiguous condition. Also, in the algebraic concept intuitionistic fuzzy set can be collaborated with an existing structure such as ring. Intuitionistic fuzzy ring is a generalized of ring combined with intuitionistic fuzzy sets under conditions to be met. In ring studies, it is known by the ideal terms that inevitably impact intuitionistic fuzzy ideal. In addition, the study of fuzzy sets there are operators that are subject to membership and non membership functions. One of the known operators is translates . Let a ring homomorphism, then kernel of homomorphism must ideal. That generalized process, should be discussed in terms of the kernel properties of ring homomorphism over intuitionistic fuzzy ring whether it still ideal or has another structure. After that the structure of the kernel is also examined if the intuitionistic fuzzy ring is subjected to translates operator.
Keyword(s): intuitionistic fuzzy ring, kernel, translate, intuitionistic fuzzy ideal.

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PRELIMINARY

Set theory has always been an interesting material in every era development. Given the rapid pace of science, starting from basic things to quantum things. As well as a firm set which states that every member included in the set is 1 and the member that is not included in the set is 0. The values of 1 and 0 are called the degree of membership of the element. It turns out that it does not work as it should in everyday life. Many things are called "uncertainty" so that the structure of the set develops into a fuzzy set (Fuzzy Sets). This set raises a membership function which is an extension of the degree of membership. The fuzzy set is characterized by the presence of a membership function in each of its elements where the value of the membership function is in the interval [0,1]. A firm set (Crisp Sets) is a set structure that has a membership function of zero and one (degree of membership). L.A Zadeh generalizes the degree of membership to be a membership function at intervals of zero to one (Zadeh, 1965).

In this fuzzy set concept, it can also be carried out in mathematical algebraic theory such as groups, semigroups, rings, and modules. Some literature on fuzzy studies is combined with algebra (Alam, 2015; Dixit et al., 1992; Kuroki, 1991; Pan, 1988; Rosenfeld, 1971; Utama, 2016). For example group theory, fuzzy group theory is a generalization of the form of groups that exist in firm sets. The axioms in group theory can be refined so that a group is a special example of a fuzzy group. Likewise for other structural theories such as semigroups, rings, and modules. Each of these axioms can be refined into a fuzzy set.

In certain cases, there are some things that cannot be applied through fuzzy sets. The fact that the membership function and the non-membership function of a fuzzy set is always one. In the case of the election of 2 leading candidates, we cannot guarantee that everyone must exercise their right to vote. This resulted in an element of abstention (impartiality), so that the total percentage of the selection of the 2 candidates was not equal to 1. If it is brought into the study of membership functions and non-membership functions, the sum of the two is not always 1 (less than 1). Therefore, the fuzzy concept has been expanded to become intuitionistic fuzzy (T.Atanassov, 1986). Given any non-empty universal set X, we define an intuitionistic fuzzy set $A$ of $X$.

**Definition 1.1 (Atanassov, 1999b)** Let any $X \neq 0$. Given any $X \neq 0$. The set $A$ is an intuitionistic fuzzy set from $X$ written $A$ with the following definition,

$$A = \{(x, \mu_A(x), v_A(x)) : x \in X\}$$

provided that $\mu_A : X \to [0,1]$ and $v_A : X \to [0,1]$ are called membership functions and non-membership functions of $X$ that satisfy $0 < \mu_A(x) + v_A(x) \leq 1$ and denoted $\breve{A} = (\mu_A, v_A)$.

Henceforth, the intuitionistic fuzzy set is written as SFI. This set introduces a non-membership function whose sum with a membership function is less than one.
In the same case, intuitionistic can also be combined with group theory (P.K.Sharma, 2011) (Doda, 2013) (Sharma, 2014) and also some research on the structure of intuitionistic fuzzy (Annamalai, 2014; Atanassov, 1999c; Ejegwa et al., 2014). Some applications of intuitionistic fuzzy in other sets (Atanassov, 1999a; Kim & Jun, 2002; Marashdeh & Salleh, 2011) and in several fields other than mathematics, especially in decision making (Abdullah et al., 2020; Abdullah & Goh, 2019; Ahmad et al., 2020; Das et al., 2013; Maji et al., 2002; Şahin & Karabacak, 2020).

In addition to the field of application, fuzzy sets can also be studied through algebraic structures such as groups and rings. In previous studies, the properties of Image and Pre-Image translate of intuitionistic fuzzy groups (Pratama, 2016) and the intuitionistic ring fuzzy (Pratama, 2020). The consistency of the intuitionistic ring fuzzy continues on the ring structure that we are familiar with on the firm set. Before entering into ring theory, then first group theory. Given a non-empty set G, with the binary operation " + " and written \((G, +)\). The set \(G\) with the + operation is called a group if it fulfills 4 axioms including closed, associative, neutral, inverse and is called an abelian group if it is commutative. This group theory developed rapidly and became the basis for the emergence of other sets and also motivated the formation of other structures, namely rings. Ring is a set structure with two binary operations that satisfy several axioms. Given any non-empty set \(R\), and the binary operations " + " and " \(\times\) " written \((R, +, \times)\) are called rings if only if applicable,

- \((R, +)\) fulfills closed, associative, neutral and inverse axioms
- \((R, \times)\) satisfies the closed and associative axioms
- \((R, +, \times)\) satisfies the distributive axiom (right and left)

The concept of groups and rings is commonly used in classical (firm) sets, and this time it will be applied to fuzzy sets, especially intuitionistic fuzzy sets as in definition 1.1. Previously, we will give an intuitionistic fuzzy group.

**Definition 1.2 (Sharma, 2011)** Let \((G, +)\) group dan SFI A of \(G\). The set \(\tilde{A} = (\mu_{\tilde{A}}, v_{\tilde{A}})\) is called intuitionistic fuzzy subgroup of \(G\) if and only if :

- i) \(\mu_{\tilde{A}}(x - y) \geq \min\{\mu_{\tilde{A}}(x), \mu_{\tilde{A}}(y)\}\)
- ii) \(v_{\tilde{A}}(x - y) \leq \max\{\mu_{\tilde{A}}(x), \mu_{\tilde{A}}(y)\}\)

for every \(x, y \in G\).
If the set used is a ring, then the intuitionistic fuzzy ring set of \( X \) is defined as follows,

**Definition 1.3 (Sharma, 2011)** Let ring \((R,+\times)\) and SFI \( A \) of \( R \). The set \( \tilde{A} = (\mu_{\tilde{A}}, v_{\tilde{A}}) \) is called intuitionistic fuzzy subring of \( R \) if and only if:

i) \( \mu_{\tilde{A}}(x - y) \geq \min\{\mu_{\tilde{A}}(x), \mu_{\tilde{A}}(y)\} \) and \( v_{\tilde{A}}(x - y) \leq \max\{\mu_{\tilde{A}}(x), \mu_{\tilde{A}}(y)\} \)

ii) \( \mu_{\tilde{A}}(xy) \geq \min\{\mu_{\tilde{A}}(x), \mu_{\tilde{A}}(y)\} \) and \( v_{\tilde{A}}(xy) \leq \max\{\mu_{\tilde{A}}(x), \mu_{\tilde{A}}(y)\} \)

for every \( x, y \in R \).

Intuitionistic ring fuzzy is extension of the ring structure which will be weakened in several axioms. This concept is very interesting because it can give rise to other types of fuzzy structures in algebraic studies. This will open up the idea that the algebraic structure we know so far is only a special case of fuzzy algebra. For example, in a ring structure if any ring homomorphism is given, the kernel of the ring homomorphism is a subring and furthermore is ideal. If the set structure we use has an intuitionistic fuzzy ring, it will give rise to an intuitionistic fuzzy ideal. The concept is known as the ideal term (right and left ideal), where any subset \( I \) of a ring \((I \subseteq R)\) is called ideal if it has special properties that is closed to the multiplication of elements outside the subring. If this is brought to an intuitionistic fuzzy set, it becomes an intuitionistic fuzzy ideal.

**Definition 1.4 (Malik & Mordeson, 1991)** Let ring \((R,+\times)\) and SFI \( A \) of \( R \). The set \( \tilde{A} = (\mu_{\tilde{A}}, v_{\tilde{A}}) \) is called intuitionistic fuzzy ideal of \( R \) if and only if:

i) \( \mu_{\tilde{A}}(x - y) \geq \min\{\mu_{\tilde{A}}(x), \mu_{\tilde{A}}(y)\} \) and \( v_{\tilde{A}}(x - y) \leq \max\{\mu_{\tilde{A}}(x), \mu_{\tilde{A}}(y)\} \)

ii) \( \mu_{\tilde{A}}(xy) \geq \min\{\mu_{\tilde{A}}(x), \mu_{\tilde{A}}(y)\} \) and \( v_{\tilde{A}}(xy) \leq \max\{\mu_{\tilde{A}}(x), \mu_{\tilde{A}}(y)\} \)

for every \( x, y \in R \).

Henceforth, the ring \( R(+\times) \) is simply written as \( R \) and the intuitionistic fuzzy subring is written as SRFI. Furthermore, the intuitionistic fuzzy ideal is written as IFI, besides that it is necessary to define the image and pre-image of a mapping in SFI as follows,

**Definition 1.5 (Sharma, 2011)** Let \( X,Y \neq \emptyset \), the set SFI \( \tilde{A} \) and \( \tilde{B} \) of \( X \) and \( Y \) and the mapping \( f:X \rightarrow Y \). Image of \( \tilde{A} \) to the mapping \( f \) is denoted \( f(\tilde{A}) \) defined \( f(\tilde{A})(y) = (\mu_{f(\tilde{A})}(y), v_{f(\tilde{A})}(y)) \)

with condition, \( \mu_{f(\tilde{A})}(y) = \max\{\mu_{\tilde{A}}(x)\}, x \in f^{-1}(y) \) and \( v_{f(\tilde{A})}(y) = \min\{v_{\tilde{A}}(x)\}, x \in f^{-1}(y) \). Pre-image of \( \tilde{B} \) is denoted \( f^{-1}(\tilde{B}) \) defined as \( f^{-1}(\tilde{B})(x) = (\mu_{f^{-1}(\tilde{B})}(x), v_{f^{-1}(\tilde{B})}(x)) \)

provide that \( \mu_{f^{-1}(\tilde{B})}(x) = \mu_{\tilde{B}}(f(x)) \) and \( v_{f^{-1}(\tilde{B})}(x) = v_{\tilde{B}}(f(x)) \).
This Definition 1.5 is the basis for defining Kernel on ring homomorphism in intuitionistic fuzzy ring.

**Definition 1.6 (Dummit & Foote, 2003)** Given $R_1(+, \times_1)$ and $R_2(+, \times_2)$ and the mapping $f: R_1 \to R_2$. The mapping $f$ is called ring homomorphism if and only if $(\forall r_1, s_1 \in R_1)$ satisfies,

1) $f(r_1 + s_1) = f(r_1) + f(s_1)$
2) $f(r_1 \times s_1) = f(r_1) \times f(s_1)$

It is defined that a set whose members are pre-images of a homomorphism to neutral elements is called $\text{Ker}(f)$ with the condition $\text{Ker}(f) = \{ r \in R_1 \mid f(r) = e_{R_2} \}$

In addition, the use of operators in the intuitionistic fuzzy set is a change in the membership and non-membership functions. Each element in the intuitionistic fuzzy set has a single membership and non-membership function, if we make changes to the function with some rules that do not violate the membership and non-membership functions, it becomes a new intuitionistic fuzzy set. The operator used this time is the translation operator, which is adding or subtracting membership and non-membership functions with a constant with certain rules. After being given the definition of structure in SRFI, then the definition of translation will be given in SRFI.

**Definition 1.7 (Souriar, 1993)** Given a non-empty set $X$, $\alpha \in [0,1]$ and $\bar{A} = (\mu_{\bar{A}}, v_{\bar{A}})$ SFI of $X$.

The ascending translate operator (up operator) $\bar{A}$ by $\alpha$ is written $\bar{A}^{\alpha+} = T_{\alpha+}(\bar{A})$ is defined as follow :

$$\bar{A}^{\alpha+} = T_{\alpha+}(\bar{A})(x) = (\mu_{\bar{A}^{\alpha+}}(x), v_{\bar{A}^{\alpha+}}(x))$$

provide that $\mu_{\bar{A}^{\alpha+}}(x) = \min\{\mu_{\bar{A}}(x) + \alpha, 1\}$, and $v_{\bar{A}^{\alpha+}}(x) = \max\{v_{\bar{A}}(x) - \alpha, 0\}$, The descending operator (down operator) $\bar{A}$ by $\alpha$ is written $\bar{A}^{\alpha-} = T_{\alpha-}(\bar{A})$ is defined as follow :

$$\bar{A}^{\alpha-} = T_{\alpha-}(\bar{A})(x) = (\mu_{\bar{A}^{\alpha-}}(x), v_{\bar{A}^{\alpha-}}(x))$$

provide that $\mu_{\bar{A}^{\alpha-}}(x) = \max\{\mu_{\bar{A}}(x) - \alpha, 0\}$, and $v_{\bar{A}^{\alpha-}}(x) = \min\{v_{\bar{A}}(x) + \alpha, 1\}$,

If given any SRFI $A$ and $\alpha \in [0,1]$, it is obtained that $A^{\alpha+}$ is SRFI and vice versa as explained below.

**Theorem 1.8 (Sharma, 2014)** If SRFI $\bar{A}$ of ring R then $\bar{A}^{\alpha+}$ SRFI of ring R for every $\alpha \in [0,1]$.  

**Theorem 1.9 (Pratama, 2020)** Given SFI $\bar{A}$ of ring R. If $\bar{A}^{\alpha+}$ SRFI of R with $\alpha < \min\{1 - p^*, q^*\}$, then $\bar{A}$ is SRFI of R with condition $p^* = \max\{\mu_{\bar{A}}(x) ; x \in R - R_{\bar{A}}\}$ and $q^* = \min\{v_{\bar{A}}(x) ; x \in R - R_{\bar{A}}\}$. and $R_{\bar{A}} = \{x \in R \mid \mu_{\bar{A}}(x) = \mu_{\bar{A}}(0) \text{ and } v_{\bar{A}}(x) = \mu_{\bar{A}}(0)\}$
**Collorary 1.10 (Pratama, 2020)** Given $SFI \tilde{A}$ of ring $R$. If $\tilde{A}^{0\times}$ SRFI of $R$ with $\alpha < \min\{1-p^*, q^*\}$, then $\tilde{A}$ is SRFI of $R$ with condition

$$p^* = \max\{v_\tilde{A}(x) : x \in R - R_\tilde{A}\} \quad \text{and} \quad q^* = \min\{\mu_\tilde{A}(x) : x \in R - R_\tilde{A}\}. \quad \text{and} \quad R_\tilde{A} = \{x \in R \mid \mu_\tilde{A}(x) = \mu_\tilde{A}(0) \text{ and } v_\tilde{A}(x) = v_\tilde{A}(0)\}$$

In accordance with the previous results in Theorem 1.8 and Theorem 1.9 and Effect 1.10, therefore this paper will show the consistency of the intuitionistic fuzzy ideal structure in the intuitionistic fuzzy ring kernel. In addition, the structure of the intuitionistic fuzzy ideal will also be studied if the intuitionistic fuzzy ring set is subject to translate operators.

**METHOD**

The method used is the study of literature from scientific books and journals, especially those related to kernels, intuitionistic fuzzy rings, and fuzzy set translation operators. The process begins with defining an intuitionistic fuzzy ring which is a generalization of a fuzzy ring. Next, define the intuitionistic fuzzy ideal and some applicable axioms. Then it is applied to any homomorphism on the intuitionistic fuzzy ring so as to get the kernel. The next step is to test the structure of the kernel which is an intuitionistic fuzzy subring or an intuitionistic fuzzy ideal. The structure of the set is then changed using the translate operator on the intuitionistic fuzzy ring. The last step is to prove the structure of the kernel on the modified intuitionistic fuzzy ring set using translation operators. Visually it will be illustrated in the following chart:

RESULT AND DISCUSSION

**Kernel’s Properties in Intuitionistic Fuzzy Ring**

As in ring theory, the kernel of a ring homomorphism between rings is a subring, at this stage it will be proven whether the kernel of an intuitionistic fuzzy ring homomorphism is an intuitionistic fuzzy ring. Previously, we will show the shape of the kernel in the intuitionistic fuzzy ring. Given any ring homomorphism $f: X \rightarrow Y$ on intuitionistic fuzzy ring $\tilde{A}$ and $\tilde{B}$ of $X$
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and it’s clear that \( \overline{\text{Ker}}(f) \subseteq f^{-1}(B) \). According to the kernel definition, we get

\[
\overline{\text{Ker}}(f) = \left( \mu_{f^{-1}(B)}(x), v_{f^{-1}(B)}(x) \right)
\]

provide that \( \mu_{f^{-1}(B)}(x) = \mu_B(f(x)) = \mu_B(0_y) \) and \( v_{f^{-1}(B)}(x) = v_B(0_y) \), so \( \overline{\text{Ker}}(f) = \left( \mu_B(0_y), v_B(0_y) \right) \). More complete,

\[
\overline{\text{Ker}}(f)(x) = \left\{ \left( x_i, \mu_B^x(0_y), v_B^x(0_y) \right) \mid f(x_i) = 0_y, i \in I \right\}
\]

**Theorem 3.1.** If given SRFI \( \tilde{A} \) and \( B \) of \( R_1 \) and \( R_2 \) and ring homomorphism \( f; R_1 \to R_2 \) so \( \overline{\text{Ker}}(f) \) is SRFI of \( R_1 \).

**Proof:**

\( \because \) To show \( \overline{\text{Ker}}(f) \) is SRFI, it's enough to prove that \( C_{\delta, \theta}(\overline{\text{Ker}}(f)) \) is subring of \( R_1 \) for every \( \delta, \theta \in [0,1] \) with condition \( \delta + \theta \leq 1 \) or in other words if we take any \( x_1, x_2 \in C_{\delta, \theta}(\overline{\text{Ker}}(f)) \), it will be proved that \( x_1 - x_2 \in C_{\delta, \theta}(\overline{\text{Ker}}(f)) \) and \( x_1, x_2 \in C_{\delta, \theta}(\overline{\text{Ker}}(f)) \).

Let \( x_1, x_2 \in C_{\delta, \theta}(\overline{\text{Ker}}(f)) \), so we get \( \mu_{\overline{\text{Ker}}(f)}(x_1) \geq \delta, v_{\overline{\text{Ker}}(f)}(x_1) \leq \theta \) dan \( \mu_{\overline{\text{Ker}}(f)}(x_2) \geq \delta, v_{\overline{\text{Ker}}(f)}(x_2) \leq \theta \)

\[
\Leftrightarrow \mu_B^{x_1}(0_y) \geq \delta, v_B^{x_1}(0_y) \leq \theta \quad \text{and} \quad \mu_B^{x_2}(0_y) \geq \delta, v_B^{x_2}(0_y) \leq \theta
\]

\[
\Leftrightarrow \min\{\mu_B^{x_1}(0_y), \mu_B^{x_2}(0_y)\} \geq \delta \quad \text{and} \quad \max\{v_B^{x_1}(0_y), v_B^{x_2}(0_y)\} \leq \theta
\]

It is known that \( \tilde{A} \) is SRFI on \( R_1 \) and \( x_1, x_2 \in C_{\delta, \theta}(\overline{\text{Ker}}(f)) \), so we get

\[
\Leftrightarrow \mu_{\overline{\text{Ker}}(f)}(x_1 - x_2) \geq \min\{\mu_{\overline{\text{Ker}}(f)}(x_1), \mu_{\overline{\text{Ker}}(f)}(x_2)\}
\]

\[
\geq \min\{\mu_B^{x_1}(0_y), \mu_B^{x_2}(0_y)\}
\]

\[
\geq \delta,
\]

and

\[
v_{\overline{\text{Ker}}(f)}(x_1 - x_2) \leq \max\{v_{\overline{\text{Ker}}(f)}(x_1), v_{\overline{\text{Ker}}(f)}(x_2)\}
\]

\[
\leq \max\{v_B^{x_1}(0_y), v_B^{x_2}(0_y)\}
\]

\[
\leq \theta
\]

\[
\Leftrightarrow \mu_{\overline{\text{Ker}}(f)}(x_1x_2) \geq \min\{\mu_{\overline{\text{Ker}}(f)}(x_1), \mu_{\overline{\text{Ker}}(f)}(x_2)\}
\]

\[
\geq \min\{\mu_B^{x_1}(0_y), \mu_B^{x_2}(0_y)\}
\]

\[
\geq \delta,
\]

and

\[
v_{\overline{\text{Ker}}(f)}(x_1 - x_2) \leq \max\{v_{\overline{\text{Ker}}(f)}(x_1), v_{\overline{\text{Ker}}(f)}(x_2)\}
\]

\[
\leq \max\{v_B^{x_1}(0_y), v_B^{x_2}(0_y)\}
\]

\[
\leq \theta
\] (1)

and

\[
v_{\overline{\text{Ker}}(f)}(x_1 - x_2) \leq \max\{v_{\overline{\text{Ker}}(f)}(x_1), v_{\overline{\text{Ker}}(f)}(x_2)\}
\]

\[
\leq \max\{v_B^{x_1}(0_y), v_B^{x_2}(0_y)\}
\]

\[
\leq \theta
\] (2)
From the forms (1) and (2) we get \( x_1 - x_2 \in C_{\delta, \theta}(\overline{\text{Ker}}(f)) \) and \( x_1x_2 \in C_{\delta, \theta}(\overline{\text{Ker}}(f)) \). So it is appropriate that \( C_{\delta, \theta}(\overline{\text{Ker}}(f)) \) is subring of \( R_1 \) so that \( \overline{\text{Ker}}(f) \) is SRFI of \( R_1 \).

As in ring theory, that the kernel is an ideal of a ring, it will be proven that this applies also to intuitionistic fuzzy rings.

**Corollary 3.2** If given SRFI \( \overline{A} \) and \( \overline{B} \) of \( R_1 \) and \( R_2 \) and ring homomorphism \( f: R_1 \to R_2 \) then \( \overline{\text{Ker}}(f) \) is IFI of \( R_1 \).

**Proof:**

It has been proven in Theorem 3.1 that \( \text{Ker}(f) \) is SRFI, then it remains only to prove that for the condition,

\[
\mu_{\overline{\text{Ker}}(f)}(x_1x_2) \geq \max\{\mu_B^x(0_y), \mu_B^y(0_y)\} \geq \delta \text{ and }
\nu_{\overline{\text{Ker}}(f)}(x_1x_2) \leq \min\{\nu_B^x(0_y), \nu_B^y(0_y)\} \leq \theta.
\]

Given \( \overline{\text{Ker}}(f) = \{(x_i, \mu_B^x(0_y), \nu_B^x(0_y)) \mid f(x_i) = 0_y, i \in I\} \), so if \( x_1, x_2 \in \overline{\text{Ker}}(f) \), then we get \((x_1, \mu_B^x(0_y), \nu_B^x(0_y)) \) and \((x_2, \mu_B^x(0_y), \nu_B^x(0_y)) \).

The properties of \( \mu_A(e) \geq \mu_A(x) \) and \( \nu_A(e) \leq \nu_A(x) \) for all \( x \in X \), therefore we get,

\[
\mu_B^x(0_y) = \mu_B^y(0_y) = \mu_B^y(0_y) \text{ and } \nu_B^x(0_y) = \nu_B^y(0_y) = \nu_B(0_y).
\]

According to the nature of neutral elements in intuitionistic fuzzy, then the form (2) in the proof of Theorem 3.1 can also be written as \( \mu_{\overline{\text{Ker}}(f)}(x_1x_2) \leq \max\{\mu_B^x(0_y), \mu_B^y(0_y)\} \geq \delta \) \( \nu_{\overline{\text{Ker}}(f)}(x_1x_2) \leq \min\{\nu_B^x(0_y), \nu_B^y(0_y)\} \leq \theta \). So it can be concluded that \( \overline{\text{Ker}}(f) \) includes the IFI of \( R_1 \).

Now we will prove that if an SRFI is assigned a translation operator (either up or down), we will prove that the kernel of the translation of the SRFI remains as an SRFI and furthermore is an IFI.

**Theorem 3.3** If given SRFI \( \overline{B}^\alpha \) of \( R_2 \) for every \( \alpha \in [0,1] \) and ring homomorphism \( f: R_1 \to R_2 \) then \( \overline{\text{Ker}}(f) \) is SRFI of \( R_1 \).

**Proof:**

\( \vdash \) To show that \( \overline{\text{Ker}}(f) \) is SRFI, it is enough prove that \( C_{\delta, \theta}(\overline{\text{Ker}}(f)) \) is subring of \( R_1 \) for every \( \delta, \theta \in [0,1] \) with \( \delta + \theta \leq 1 \).

Given any \( x_1, x_2 \in C_{\delta, \theta}(\overline{\text{Ker}}(f)) \), we will prove that \( x_1 - x_2 \in C_{\delta, \theta}(\overline{\text{Ker}}(f)) \) and \( x_1x_2 \in C_{\delta, \theta}(\overline{\text{Ker}}(f)) \). Given by \( \mu_{\overline{\text{Ker}}(f)}(x_1) \geq \delta \), \( \nu_{\overline{\text{Ker}}(f)}(x_1) \leq \theta \) and \( \mu_{\overline{\text{Ker}}(f)}(x_2) \geq \delta \), \( \nu_{\overline{\text{Ker}}(f)}(x_2) \leq \theta \).
\[
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\]
\[
\mu_{\overline{B}}(x_1(0)) \geq \delta, \quad \mu_{\overline{B}}(x_2(0)) \leq \theta \quad \text{and} \quad \mu_{\overline{A}}(y) \geq \delta, \quad v_{\overline{B}}(y) \leq \theta
\]
\[
\min\{\mu_{\overline{B}}(x_1(0)), \mu_{\overline{A}}(y)\} \geq \delta \quad \text{and} \quad \max\{v_{\overline{B}}(x_1(0)), v_{\overline{A}}(y)\} \leq \theta
\]
\[
\min\{\mu_{\overline{B}}(x_1(0)) + \alpha, 1\}, \min\{\mu_{\overline{B}}(x_2(0)) + \alpha, 1\} \geq \delta \quad \text{and}
\]
\[
\max\{v_{\overline{B}}(x_1(0)) - \alpha, 0\}, \max\{v_{\overline{B}}(x_2(0)) - \alpha, 0\} \leq \theta
\]
\[
\min\{\mu_{\overline{B}}(x_1(0)), \mu_{\overline{B}}(y) \geq \delta \quad \text{and}
\]
\[
\max\{v_{\overline{B}}(x_1(0)) - \alpha, v_{\overline{B}}(y) - \alpha\}, \{\alpha, 0\} \leq \theta
\]
\[
\min\{\mu_{\overline{B}}(x_1(0)), \mu_{\overline{B}}(y) \geq \delta \quad \text{and}
\]
\[
\max\{v_{\overline{B}}(x_1(0)) - \alpha, v_{\overline{B}}(y) - \alpha\}, \{\alpha, 0\} \leq \theta
\]
\[
\text{It is know that } \bar{A} \text{ is SRFI on } R_1 \text{ and } x_1, x_2 \in C_{\delta, \theta}(\overline{Ker}(f)), \text{ so}
\]
\[
\mu_{\overline{Ker}(f)}(x_1 - x_2) \geq \min\{\mu_{\overline{Ker}(f)}(x_1), \mu_{\overline{Ker}(f)}(x_2)\}
\]
\[
\geq \min\{\mu_{\overline{B}}(x_1(0)), \mu_{\overline{A}}(y)\}
\]
\[
\geq \min\{\min\{\mu_{\overline{B}}(x_1(0)), \mu_{\overline{A}}(y)\} + \alpha, 1\}
\]
\[
\geq \delta
\]
\[
\nu_{\overline{Ker}(f)}(x_1 - x_2) \leq \max\{\nu_{\overline{Ker}(f)}(x_1), \nu_{\overline{Ker}(f)}(x_2)\}
\]
\[
\leq \max\{v_{\overline{B}}(x_1(0)), v_{\overline{A}}(y)\}
\]
\[
\leq \max\{\max\{v_{\overline{B}}(x_1(0)), v_{\overline{A}}(y)\} - \alpha, 1\}
\]
\[
\leq \theta
\]

Then we get \( x_1 - x_2 \in C_{\delta, \theta}(\overline{Ker}(f)) \) and \( x_1x_2 \in C_{\delta, \theta}(\overline{Ker}(f)) \). So \( C_{\delta, \theta}(\overline{Ker}(f)) \) is subring of \( R_1 \) and it it can be conclusion that \( \overline{Ker}(f) \) is SRFI of \( R_1 \). \( \blacksquare \)
Teorema 3.4 If given SRFI $\overline{B^\alpha}$- of $R_2$ for every $\alpha \in [0,1]$ and ring homomorphism $f : R_1 \to R_2$ then $Ker(f)$ is SRFI of $R_1$.

Bukti:

.: To show that $\overline{Ker(f)}$ is SRFI, it is enough to show that $C_{\delta, \theta}(\overline{Ker(f)})$ is subring of $R_1$ for every $\delta, \theta \in [0,1]$ with $\delta + \theta \leq 1$.

Given any $x_1, x_2 \in C_{\delta, \theta}(\overline{Ker(f)})$, we will prove that $x_1 - x_2 \in C_{\delta, \theta}(\overline{Ker(f)})$ and $x_1x_2 \in C_{\delta, \theta}(\overline{Ker(f)})$. Given by

\[
\mu_{\overline{Ker(f)}}(x_1) \geq \delta , \nu_{\overline{Ker(f)}}(x_1) \leq \theta \quad \text{and} \quad \mu_{\overline{Ker(f)}}(x_2) \geq \delta , \nu_{\overline{Ker(f)}}(x_2) \leq \theta \\
\iff \mu_{\overline{B^\alpha}}(0_y) \geq \delta , \nu_{\overline{B^\alpha}}(0_y) \leq \theta \quad \text{and} \quad \mu_{\overline{B^\alpha}}(0_y) \geq \delta , \nu_{\overline{B^\alpha}}(0_y) \leq \theta \\
\iff \min\{\mu_{\overline{B^\alpha}}(0_y), \mu_{\overline{B^\alpha}}(0_y)\} \geq \delta \quad \text{and} \quad \max\{\nu_{\overline{B^\alpha}}(0_y), \nu_{\overline{B^\alpha}}(0_y)\} \leq \theta \\
\iff \min\{\max\{\mu_{\overline{B^\alpha}}(0_y) - \alpha, 0\}, \max\{\mu_{\overline{B^\alpha}}(0_y) - \alpha, 0\}\} \geq \delta \quad \text{and} \\
\max\{\min\{\nu_{\overline{B^\alpha}}(0_y) + \alpha, 1\}, \min\{\nu_{\overline{B^\alpha}}(0_y) + \alpha, 1\}\} \leq \theta \\
\iff \min\{\max\{\mu_{\overline{B^\alpha}}(0_y) - \alpha, \mu_{\overline{B^\alpha}}(0_y) - \alpha\}, 0\} \geq \delta \quad \text{and} \\
\max\{\min\{\nu_{\overline{B^\alpha}}(0_y) + \alpha, \nu_{\overline{B^\alpha}}(0_y) + \alpha\}, 1\} \leq \theta \\
\iff \min\{\max\{\mu_{\overline{B^\alpha}}(0_y), \mu_{\overline{B^\alpha}}(0_y)\} - \alpha, 0\} \geq \delta \quad \text{and} \\
\max\{\min\{\nu_{\overline{B^\alpha}}(0_y), \nu_{\overline{B^\alpha}}(0_y)\} + \alpha, 1\} \leq \theta \\
\iff \mu_{\overline{Ker(f)}}(x_1 - x_2) \geq \min\{\mu_{\overline{Ker(f)}}(x_1), \mu_{\overline{Ker(f)}}(x_2)\} \\
\geq \min\{\mu_{\overline{B^\alpha}}(0_y), \mu_{\overline{B^\alpha}}(0_y)\} \\
\geq \min\{\max\{\mu_{\overline{B^\alpha}}(0_y), \mu_{\overline{B^\alpha}}(0_y)\} - \alpha, 0\} \\
\geq \delta \\
\iff \nu_{\overline{Ker(f)}}(x_1 - x_2) \leq \max\{\nu_{\overline{Ker(f)}}(x_1), \nu_{\overline{Ker(f)}}(x_2)\} \\
\leq \max\{\nu_{\overline{B^\alpha}}(0_y), \nu_{\overline{B^\alpha}}(0_y)\} \\
\leq \max\{\min\{\nu_{\overline{B^\alpha}}(0_y), \nu_{\overline{B^\alpha}}(0_y)\} + \alpha, 1\} \\
\leq \theta \\
\iff \mu_{\overline{Ker(f)}}(x_1x_2) \geq \min\{\mu_{\overline{Ker(f)}}(x_1), \mu_{\overline{Ker(f)}}(x_2)\} \\
\geq \min\{\mu_{\overline{B^\alpha}}(0_y), \mu_{\overline{B^\alpha}}(0_y)\} \\
\geq \min\{\max\{\mu_{\overline{B^\alpha}}(0_y), \mu_{\overline{B^\alpha}}(0_y)\} - \alpha, 0\} \\
\geq \delta 
\]
\[ v_{\overline{\text{Ker}}(f)}(x_1x_2) \leq \max\{v_{\overline{\text{Ker}}(f)}(x_1), v_{\overline{\text{Ker}}(f)}(x_2)\} \]
\[ \leq \max\{v_{\overline{\alpha^+}(0_y)}, v_{\overline{\alpha^+}(0_y)}\} \]
\[ \leq \max\{\min\{v_{\overline{\alpha^+}(0_y)}, v_{\overline{\alpha^+}(0_y)}\} + \alpha, 1\} \]
\[ \leq \theta \]

We get \( x_1 - x_2 \in C_{\delta, \theta}(\text{Ker}(f)) \) and \( x_1x_2 \in C_{\delta, \theta}(\overline{\text{Ker}}(f)) \). So \( C_{\delta, \theta}(\overline{\text{Ker}}(f)) \) is subring of \( R_1 \) so that \( \text{Ker}(f) \) is SRFI of \( R_1 \).

Next, it will be proved for the translation of the intuitionistic fuzzy ideal kernel.

**Collorary 3.5** If given SRFI \( \overline{\alpha^+} \) of \( R_2 \) for every \( \alpha \in [0,1] \) and ring homomorphism \( f: R_1 \to R_2 \) then \( \overline{\text{Ker}}(f) \) is IFI of \( R_1 \).

**Bukti:**
It has been proven in Theorem 3.3 that \( \overline{\text{Ker}}(f) \) is SRFI if condition \( \overline{\alpha^+} \) SRFI, then it remains only to prove that for the condition

\[ \mu_{\overline{\text{Ker}}(f)}(x_1x_2) \geq \max\{\mu_{\overline{\alpha^+}(0_y)}, \mu_{\overline{\alpha^+}(0_y)}\} \geq \delta \text{ and } v_{\overline{\text{Ker}}(f)}(x_1x_2) \leq \min\{v_{\overline{\alpha^+}(0_y)}, v_{\overline{\alpha^+}(0_y)}\} \leq \theta. \]

Given that \( \overline{\alpha^+} \) is SRFI of \( R_2 \) then \( \overline{\text{Ker}}(f) = \{(x_i, \mu_{\overline{\alpha^+}(0_y)}, v_{\overline{\alpha^+}(0_y)}) \mid f(x_i) = 0_y, i \in I\}, \) so that \( x_1, x_2 \in \overline{\text{Ker}}(f) \), then we get

\[ (x_1, \mu_{\overline{\alpha^+}(0_y)}, v_{\overline{\alpha^+}(0_y)}) \text{ and } (x_2, \mu_{\overline{\alpha^+}(0_y)}, v_{\overline{\alpha^+}(0_y)}) \]

The properties of \( \mu_A(e) \geq \mu_A(x) \) and \( v_A(e) \leq v_A(x) \) for all \( x \in X \), therefore we get

\[ \mu_{\overline{\alpha^+}(0_y)} = \mu_{\overline{\alpha^+}(0_y)} = \mu_{\overline{\alpha^+}(0_y)} \text{ dan } v_{\overline{\alpha^+}(0_y)} = v_{\overline{\alpha^+}(0_y)} = v_{\overline{\alpha^+}(0_y)}. \]

In accordance with the nature of neutral elements in intuitionistic fuzzy, then the form (2) in the proof of Theorem 3.3 can also be written as \( \mu_{\overline{\text{Ker}}(f)}(x_1x_2) \leq \max\{\mu_{\overline{\alpha^+}(0_y)}, \mu_{\overline{\alpha^+}(0_y)}\} \geq \delta \), and \( v_{\overline{\text{Ker}}(f)}(x_1x_2) \leq \min\{v_{\overline{\alpha^+}(0_y)}, v_{\overline{\alpha^+}(0_y)}\} \leq \theta \). So it can be concluded that \( \overline{\text{Ker}}(f) \) is IFI of \( R_1 \).

**Collorary 3.6** If given SRFI \( \overline{\alpha^-} \) of \( R_2 \) for every \( \alpha \in [0,1] \) and ring homomorphism \( f: R_1 \to R_2 \) then \( \overline{\text{Ker}}(f) \) is IFI of \( R_1 \).

**Proof:**
It has been proven in Theorem 3.3 that \( \overline{\text{Ker}}(f) \) is SRFI if condition \( \overline{\alpha^+} \) SRFI, then it remains only to prove that for the condition
\[\mu_{\overline{\text{Ker}}(f)}(x_1x_2) \geq \max\{\mu_{\overline{B}^a_\alpha}(0_y), \mu_{\overline{B}^a_\alpha}(0_y)\} \geq \delta \text{ and} \]
\[v_{\overline{\text{Ker}}(f)}(x_1x_2) \leq \min\{v_{\overline{B}^a_\alpha}(0_y), v_{\overline{B}^a_\alpha}(0_y)\} \leq \theta.\]

Given that $\overline{B}^a$ is SRFI of $R_2$ then $\overline{\text{Ker}}(f) = \left\{(x_i, \mu_{\overline{B}^a_\alpha}(0_y), v_{\overline{B}^a_\alpha}(0_y)) \mid f(x_i) = 0_y, i \in I\right\}$, so that if $x_1, x_2 \in \overline{\text{Ker}}(f)$, then we get
\[(x_1, \mu_{\overline{B}^a_\alpha}(0_y), v_{\overline{B}^a_\alpha}(0_y)) \text{ and } (x_2, \mu_{\overline{B}^a_\alpha}(0_y), v_{\overline{B}^a_\alpha}(0_y)).\]

The properties of $\mu_\alpha(e) \geq \mu_\alpha(x)$ and $v_\alpha(e) \leq \mu_\alpha(x)$ for all $x \in X$, therefore we get
\[\mu_{\overline{B}^a_\alpha}(0_y) = \mu_{\overline{B}^a_\alpha}(0_y) = \mu_{\overline{B}^a_\alpha}(0_y) \text{ dan } v_{\overline{B}^a_\alpha}(0_y) = v_{\overline{B}^a_\alpha}(0_y) = v_{\overline{B}^a_\alpha}(0_y).\]

According to the nature of neutral elements in intuitionistic fuzzy, then the form (2) in the proof of Theorem 3.4 can also be written as $\mu_{\overline{\text{Ker}}}(f)(x_1x_2) \leq \max\{\mu_{\overline{B}^a_\alpha}(0_y), \mu_{\overline{B}^a_\alpha}(0_y)\} \geq \delta$, and $v_{\overline{\text{Ker}}(f)}(x_1x_2) \leq \min\{v_{\overline{B}^a_\alpha}(0_y), v_{\overline{B}^a_\alpha}(0_y)\} \leq \theta$. So it can be conclude that $\overline{\text{Ker}}(f)$ is IFI of $R_1$.

**CONCLUSION**

Given any intuitionistic fuzzy subring (SRFI) $\overline{A}$ and $\overline{B}$ of $R_1$ and $R_2$, respectively, and the ring homomorphism $f: R_1 \rightarrow R_2$ then the structure of $\overline{\text{Ker}}(f)$ is SRFI of $R_1$ and so is the intuitionistic fuzzy ideal (IFI) of $R_1$. Furthermore, it is defined that ascending operator translate (up translate) at SRFI $\overline{B}$ is $\alpha (B^a_\alpha^+)$ and descending operator translate (down translate) at SRFI $\overline{B}$ is $\alpha (B^a_\alpha^-)$, then the structure of $\overline{\text{Ker}}(f)$ still remains the SRFI of $R_1$ (so is the IFI) of $R_1$.

This process can still be developed by replacing other types of operators from intuitionistic fuzzy sets or combining 2 or more operators. In addition, it can also test other algebraic structures such as groups, modules and vector spaces with several other special sets.

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