Comparison Of Euler Method And Runge Kutta Method In Estimation Of The Number Of Population In Aceh Province

Perbandingan Metode Euler Dan Metode Runge Kutta Dalam Estimasi Jumlah Penduduk Di Provinsi Aceh

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ABSTRACT
Residents are people who are in an area and are controlled by applicable rules and interact with each other continuously or continuously. If population growth increases, this will cause many problems, including: poverty levels, unemployment, school dropout rates and increasing crime rates in each region. In anticipating the problem of increasing the population of Aceh Province, information about the population is needed. Therefore, this study will discuss several applications of mathematical methods in estimating the population of Aceh Province in the coming year using the Euler method and the Runge Kutta method of order 4. The results of the estimation of the population of Aceh Province using the Runge Kutta method are 5,030,503 inhabitants in 2021, 5,076,307 in 2022, and 5,122,486 in 2023. The estimation results using the Euler method are 5,026,162 in 2021, 5,071,745 in 2022, and 5,117,700 in 2022. The conclusion in this study is that estimation using the Runge Kutta method is a method that is better applied than the method Euler on the problem of estimating the population of Aceh Province. This is because the error value of the Runge Kutta method is smaller than the Euler method with an error value of about 0.04%.

Key words: Runge Kutta, Euler, Population.
PRELIMINARY

Residents are people who are in an area and are controlled by applicable rules and interact with each other continuously or continuously. (Pandu, 2020). According to sociology, population is a collection of people who occupy a certain geographic area and space. According to BPS data in 2020, Indonesia is the fifth most populous country in the world. Based on BPS data rankings for 2020, Indonesia is in fourth place with a population of 267,026,366 after China, India and the United States. Meanwhile, in Southeast Asia, Indonesia has the first largest population, and the third largest in the Asian continent (Wulandari, 2020).

The topic of population has become one of the most discussed topics, both through the mass media and in conversation (Basuki, 2021). This is an indication of the growing awareness of the world community that the demographic crisis has become a situation that is getting worse so that it becomes one of the challenges and threats to life as a whole on our earth. In the current era of globalization, the times are progressing rapidly (Wirasasmita, 2020). In addition, population growth is also increasing rapidly. The monetary crisis and the moral crisis that occurred also contributed to the decline of the Indonesian nation. This condition causes a lot of unemployment and crime. The lack of investment, high unemployment makes the Indonesian people poorer. In addition to poverty, the problem faced by the Indonesian people today is population density (Helvira, 2020).

If population growth increases this will cause many problems, including: poverty rates, unemployment, dropout rates and increasing crime rates in each region. Population growth can be affected by reproductive changes in birth and death rates. Therefore, it is necessary to have a handler from the government in an even distribution of the population, in order to reduce the negative impact on every resident (Janiar, 2017). The occurrence of population growth affects the development process and the development of activities in an area and increases the need for space/land (Firdaus, 2018). The increasing population of an area requires the provision of living necessities, both physical needs such as housing,
facilities and infrastructure, as well as non-physical needs such as education, economy, and recreation areas (Fejriani, 2020).

Aceh province is a province that is ranked 13th with the largest population of 34 provinces in Indonesia. In 2020, in 2020 the population of Aceh province reached 5,459,891 people, while in 2019 the population of Aceh province reached 5,371,532 people. 88,369 Souls (Bps, 2021). In the province of Aceh, due to several factors, namely the rise of early marriages carried out by teenagers, families who do not follow family planning procedures and at the same time migration also plays a role in increasing and reducing the population.

The population problem can be categorized as a major national problem which requires immediate problem solving. The population of an area must be balanced with the number of sources of income, so that an increase in national income can be obtained. This shows that the rate of population growth also affects planning in the economic sector (Sunandar, 2020). In anticipating the problem of increasing the population of Aceh Province, information about the population is needed. Therefore, this study will discuss several applications of mathematical methods in estimating the population of Aceh Province in the coming year using the Euler method and the Runge Kutta Order method 4.

**METHODS**

The type of research used in this research is quantitative research. The data used is from the Website of the Central Statistics Agency for Aceh Province. The method used is the Euler method and the Runge Kutta method.

**Euler Method**

At \( x = 0 \), rated \( y = y_0 \). So \( x = 0 \) sebagai \( x_0 \) slope on \( x = x_0 \) is,

\[
\text{Slope} = \frac{y_1 - y_0}{x_1 - x_0} = f(x_0, y_0)
\]

Wherein, \( y_1 = y_0 + f(x_0, y_0)(x_1 - x_0) \)

\( x_1 - x_0 \) step size \( h \), get

\[
y_1 = y_0 + f(x_0, y_0)h
\]

Used value \( y_1 \) (on the approximate \( y \) and \( x = x_1 \)) at \( y_2 \), will be the predictive value of \( x_2 \).

Wherein,

\[
y_2 = y_1 + f(x_1, y_1)h
\]
\[ x_{2} = x_{1} + h \]

Based on the above equation, it is known that the value of \( y = y_{i} \) at \( x_{i} \),

\[ y_{i+1} = y_{i} + f(x_{i}, y_{i})h \]

Runge-Kutta Method

According to writing (Setiawan dan Mungkasi, 2021) that the runge-kutta method is one of the most important sets of methods for solving differential equations with initial conditions

\[ y' = f(x, y) \]

In general, the "step length" depends on the value of \( r \), but for convenience it is constant, it is \( h \). The general form of the order-n runge-kutta method is:

\[ y_{r+1} = y_{r} + \alpha_{1}k_{1} + \alpha_{2}k_{2} + \ldots + \alpha_{n}k_{n} \]

With \( \alpha_{1}, \alpha_{2}, \ldots, \alpha_{n} \) is a constant and

\[ k_{1} = hf(x_{r}, y_{r}) \]
\[ k_{2} = hf(x_{r} + p_{1}h, y_{r} + q_{11}k_{1}) \]
\[ \ldots \]
\[ k_{n} = hf(x_{r} + p_{n-1}h, y_{r} + q_{n-1,1}k_{1} + q_{n-1,2}k_{2} + \ldots + q_{n-1,n-1}k_{n-1}) \]

Score \( \alpha_{i}, p_{i}, q_{ij} \) chosen in such a way as to minimize the error per step, and equation (4) will be equivalent to the Taylor series method of the highest order possible.

With, Error per step runge-kutta method of order \( -n : O(h^{n+1}) \)

Runge-kutta method error of order \( -n : O(h^{n}) \)

Order method = \( n \)

The One Order Runge-Kutta Method

The level one runge-kutta method takes the form of.

\[ k_{1} = hf(x_{r}, y_{r}) \]
\[ y_{r+1} = y_{r} + (\alpha_{1}k_{1}) \]

The Two Order Runge-Kutta Method

The level two runge-kutta method takes the form of.

\[ k_{2} = hf(x_{r} + p_{1}h, y_{r} + q_{11}k_{1}) \]
\[ y_{r+1} = y_{r} + (\alpha_{1}k_{1} + \alpha_{2}k_{2}) \]

The Third Order Runge-Kutta Method

The third-order Runge-Kutta method takes the form of

\[ k_{3} = hf(x_{r} + h, y_{r} - k_{1} + 2k_{2}) \]
The Fourth Order Runge-Kutta Method

The fourth-order runge-kutta method takes the form of

\[ k_4 = hf(x_r + h, y_r + k_3) \]
\[ y_{r+1} = y_r + \left( \frac{k_1 + 4k_2 + 2k_3 + 3k_4}{6} \right) \]

With,

- \( y_r \) = Initial solution on iteration \( r \)
- \( x_r \) = years \( r \)

Verhulst Models

The Verhulst model is a modification of the Malthusian model. The inspiration for the emergence of the Verhulst model is because the solution in the Malthus model is not realistic, that is, it increases or decreases exponentially (Zeng et al., 2020). If the maximum population size that can be maintained is \( \frac{a}{b} \), that \( \frac{a}{b} - N \) will give an indication of how many additional individuals the environment can accommodate, and \( \left( \frac{\frac{a}{b} - N}{a} = \frac{a - bN}{a} \right) \) give an indication of how many fractions \( \frac{a}{b} \) available for population growth.

The modified equation using the new terms is (Mohamed dkk, 2018):

\[
\frac{dN}{dt} = aN \left( \frac{a - bN}{a} \right) = \frac{a^2 N - abN^2}{a} = aN - bN^2
\]

This model is a nonlinear differential equation which has a solution:

\[
\frac{1}{a} \left( \ln N - \ln (a - bN) \right) = t + c
\]

It is known that the current population \( t = 0 = t_0 \) is \( N_0 \), so:

\[
c = \frac{1}{a} \left( \ln N_0 \ln (a - bN_0) \right)
\]

By substituting the value of \( c \), equation (5) becomes:

\[
\ln \frac{N(a - bN_0)}{N_0 (a - bN)} = at
\]

By exponentiating on both sides, we get:

\[
N(t) = \frac{\frac{a}{b}}{1 + \left( \frac{a}{b} \right) - \frac{1}{ae^{at}}}
\]

Suppose \( \frac{a}{b} = k \), that
\[ N(t) = \frac{K}{1 + \left(\frac{K}{N_0} - 1\right)e^{-at}} \]

If \( t \) towards infinity, then obtained:

\[ \ln \frac{N}{N_0} = k \]

This means that the population will grow asymptotically to \( K \) and \( t \) towards infinity. We will find the equilibrium point of the Verhulst model, which is:

\[ \frac{dN(t)}{dt} = 0 \]

So as,

\[ \frac{dN}{dt} = kN \left( 1 - \frac{N}{K} \right) \]

With,

- \( N = \) Total population at time \( t \)
- \( K = \) carrying capacity of an area for the population
- \( k = \) Growth rate

Equation (6) is known as the Verhulst population growth/population growth model (Hasan et al., 2020).

**Mean Absolute Percentage Error (MAPE)**

Mean Absolute Percentage Error (MAPE) is calculated using the absolute error in each period divided by the actual observed value for that period. MAPE is an error measurement that calculates the size of the percentage deviation between the actual data and the forecast data. The MAPE value can be calculated by the following equation.

\[ \text{MAPE} = \frac{1}{n} \sum_{i=1}^{n} \left( \frac{|\text{Actual} - \text{Approximation}|}{|\text{Approximation}|} \right) \times 100\% \]

The value generated through MAPE calculations, shows the approximation criteria shown in the following table (Sinaga dan Irawati, 2018).

<table>
<thead>
<tr>
<th>MAPE Value</th>
<th>Approximation Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{MAPE} \leq 10% )</td>
<td>Very good</td>
</tr>
<tr>
<td>( 10% &lt; \text{MAPE} \leq 20% )</td>
<td>Well</td>
</tr>
<tr>
<td>( 20% &lt; \text{MAPE} \leq 50% )</td>
<td>Pretty good</td>
</tr>
<tr>
<td>( \text{MAPE} \leq 50% )</td>
<td>Enough</td>
</tr>
</tbody>
</table>
RESULTS AND DISCUSSION

To determine the approximation of the amount of coffee production in Aceh Province, the growth rate and carrying capacity will be determined in equation:

1. Growth rate

\[ k = \frac{1}{r} \ln \left( \frac{N(t)}{N_0} \right) \]

\[ k = \frac{1}{1} \ln \left( \frac{410100}{4074900} \right) = 0.01 \]

2. Carrying capacity

Because the amount of coffee production in Aceh Province from 1998-2021 is still below 54000000 tons, it is assumed that \( K = 54000000 \).

On the interval \([0,25]\) with many iterations \( n = 25 \) then,

\[ h = \frac{b-a}{n} = \frac{25-0}{25} = 1 \]

Determine the Verhulst model as a nonlinear differential equation.

\[ \frac{dN}{dt} = k \left( 1 - \frac{N}{K} \right) N \]

with initial value \( N(t_0) = N_0 \)

\[ = 0.01 \left( 1 - \frac{N}{54000000} \right) N \]

The results of both methods are operated using Matlab software. The following is the Matlab program code in determining the results using the Runge Kutta method.

```matlab
% dN/dt=0.01*N*(1-N/54000000) ; 0<=t<=25 ; y(0)=4074900;
f = @(t,N) (0.01*N*(1-N/54000000));
a = input( 'Enter left end point, a: ' );
b = input( 'Enter right end point, b: ' );
n = input( 'Enter no. of subintervals, n: ' ); % n=(b-a)/h
alpha = input( 'Enter the initial condition, alpha: ' );
h = (b-a)/n;
t = a;
N = alpha;
fprintf( ' t y\n' );
fprintf( '%5.3f %11.7f\n', t, N);
for i = 1:n
    k1 = h*f(t,N);
k2 = h*f(t+h/2.0, N+k1/2.0);
k3 = h*f(t+h/2.0, N+k2/2.0);
k4 = h*f(t+h,N+k3);
    N = N+(k1+2.0*(k2+k3)+k4)/6.0;
    t = a+i*h;
    fprintf( '%5.1f %11.9f\n', t, N);
end
```

The following is the Matlab program code in determining the results using the Euler method.

```matlab
% dN/dt=0.01*N*(1-N/54000000) ; 0<=t<=25 ; y(0)=4074900;
f = @(t,N) (0.01*N*(1-N/54000000));
```
Comparison Of Euler Method And Runge Kutta Method In Estimation Of The Number Of Population In Aceh Province

```matlab
a = input('Enter left end point, a: '); 

b = input('Enter right end point, b: '); 

n = input('Enter no. of subintervals, n: '); 

alpha = input('Enter the initial condition, alpha: '); 

h = (b-a)/n; t = a; w = alpha; 

fprintf('%5.4f %11.8f
', t, w); 

for i = 1:n
    w = w+h*f(t, w); 
    t = a+i*h; 
    fprintf('%5.4f %11.8f
', t, w); 
    plot(t,w,"r*"); grid on; 
    xlabel('t values'); 
    ylabel('w values'); 
    hold on; 
end
```

The results of the completion of the Runge Kutta method and the Euler method can be seen in the following table:

**Table 2. The results of the completion of the Runge Kutta method and the Euler method**

<table>
<thead>
<tr>
<th>t</th>
<th>Years</th>
<th>Total population</th>
<th>Runge Kutta Method</th>
<th>Euler Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1998</td>
<td>4074900.0000</td>
<td>4074900.0000</td>
<td>4074900.0000</td>
</tr>
<tr>
<td>1</td>
<td>1999</td>
<td>4110100.0000</td>
<td>4112734.341388110</td>
<td>4112574.035183330</td>
</tr>
<tr>
<td>2</td>
<td>2000</td>
<td>3929234.0000</td>
<td>4150890.757993340</td>
<td>4150567.689387970</td>
</tr>
<tr>
<td>3</td>
<td>2001</td>
<td>4114000.0000</td>
<td>4189371.447750960</td>
<td>418883.141810700</td>
</tr>
<tr>
<td>4</td>
<td>2002</td>
<td>4166040.0000</td>
<td>4228178.613410450</td>
<td>422752.576566630</td>
</tr>
<tr>
<td>5</td>
<td>2003</td>
<td>4218486.0000</td>
<td>4267314.462332090</td>
<td>426648.182492420</td>
</tr>
<tr>
<td>6</td>
<td>2004</td>
<td>4075599.0000</td>
<td>4306781.206278880</td>
<td>430578.152944870</td>
</tr>
<tr>
<td>7</td>
<td>2005</td>
<td>4031589.0000</td>
<td>4346581.061203670</td>
<td>434540.685594940</td>
</tr>
<tr>
<td>8</td>
<td>2006</td>
<td>4153573.0000</td>
<td>4386716.247031450</td>
<td>438536.982216960</td>
</tr>
<tr>
<td>9</td>
<td>2007</td>
<td>4223833.0000</td>
<td>4427188.987436760</td>
<td>442565.624873110</td>
</tr>
<tr>
<td>10</td>
<td>2008</td>
<td>4293915.0000</td>
<td>4468001.509616190</td>
<td>446628.693693090</td>
</tr>
<tr>
<td>11</td>
<td>2009</td>
<td>4363477.0000</td>
<td>4509156.044055880</td>
<td>450725.530648970</td>
</tr>
<tr>
<td>12</td>
<td>2010</td>
<td>4494410.0000</td>
<td>4550654.824294050</td>
<td>454856.975325080</td>
</tr>
<tr>
<td>13</td>
<td>2011</td>
<td>4597308.0000</td>
<td>4627188.987436760</td>
<td>462752.576566630</td>
</tr>
<tr>
<td>14</td>
<td>2012</td>
<td>4693934.0000</td>
<td>4692500.086678410</td>
<td>469021.246683000</td>
</tr>
<tr>
<td>15</td>
<td>2013</td>
<td>4791924.0000</td>
<td>4763390.768951000</td>
<td>476031.068786440</td>
</tr>
<tr>
<td>16</td>
<td>2014</td>
<td>4906835.0000</td>
<td>4807002.067988580</td>
<td>480725.430648970</td>
</tr>
<tr>
<td>17</td>
<td>2015</td>
<td>5001953.0000</td>
<td>484789.120367000</td>
<td>484789.120367000</td>
</tr>
<tr>
<td>18</td>
<td>2016</td>
<td>5096248.0000</td>
<td>4807002.067988580</td>
<td>480725.430648970</td>
</tr>
<tr>
<td>19</td>
<td>2017</td>
<td>5189466.0000</td>
<td>4850973.312026130</td>
<td>484789.810586100</td>
</tr>
<tr>
<td>20</td>
<td>2018</td>
<td>5281314.0000</td>
<td>4895306.749851920</td>
<td>489161.198050530</td>
</tr>
<tr>
<td>21</td>
<td>2019</td>
<td>5371532.0000</td>
<td>4940004.630935900</td>
<td>493609.241201530</td>
</tr>
<tr>
<td>22</td>
<td>2020</td>
<td>5459891.0000</td>
<td>4985069.205139640</td>
<td>498097.174382660</td>
</tr>
<tr>
<td>23</td>
<td>2021</td>
<td>5030502.722420910</td>
<td>5026162.232283150</td>
<td>502616.232283150</td>
</tr>
<tr>
<td>24</td>
<td>2022</td>
<td>5076307.432532670</td>
<td>5071745.649645760</td>
<td>507174.649645760</td>
</tr>
<tr>
<td>25</td>
<td>2023</td>
<td>5122485.584716570</td>
<td>5117699.660969120</td>
<td>511769.966096912</td>
</tr>
</tbody>
</table>
```
The results can be viewed using the graph in figure 1.

![Population Data with Runge Kutta Method and Euler Method](image)

**Figure 1. Population Data with Runge Kutta Method and Euler Method**

The accuracy of both methods is measured by MAPE obtained from both methods, where the MAPE value obtained from the Runge Kutta method is 0.039702316% and the Euler method is 0.039839570%. Therefore, the Runge Kutta method is more accurate than the Euler method. The results found for the estimated population of Aceh Province using the Runge Kutta method are 5030502.722420910 souls in 2021, 5076307.432532670 in 2022 and 5122485.584716570 in 2023. The estimation results using the Euler method are 5026162.232283150 in 2021, 5071745.649645760 in 2022 and 5117699.660969120 in 2022. And the results that can be applied to the estimated population of Aceh Province are the estimation results from the Runge Kutta method.

**CONCLUSION**

According to the formulation of the problem and the results obtained from this study, it can be concluded that the estimation results of the population of Aceh Province using the Runge Kutta method are 5,030,503 people in 2021, 5,076,307 in 2022, and 5,122,486 in 2023. The estimation results using the Euler method are 5,026,162 in 2021, 5,071,745 in 2022 and 5,117,700 in 2022. The conclusion in this study is that estimation using the Runge Kutta method is a method that is better applied than the Euler method on the problem estimated population of Aceh Province. This is because the error value of the Runge Kutta method is smaller than the Euler method with an error value of about 0.04%.
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