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Comparison Of Euler Method And Runge Kutta Method In Estimation Of The Number Of Population In Aceh Province

Perbandingan Metode Euler Dan Metode Runge Kutta Dalam Estimasi Jumlah Penduduk Di Provinsi Aceh

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ABSTRAK

Penduduk merupakan orang-orang yang berada dalam suatu wilayah dan terikat oleh aturan-aturan yang berlaku dan saling berinteraksi satu sama lain secara terus menerus atau kontinu. Apabila pertumbuhan penduduk semakin meningkat hal ini menimbulkan banyak permasalahan antara lain: tingkat kemiskinan, pengangguran, angka putus sekolah dan meningkatnya angka kriminalitas pada setiap wilayah. Dalam mengantisipasi permasalah meningkatnya jumlah penduduk Provinsi Aceh, maka dibutuhkan informasi tentang jumlah penduduk. Oleh sebab itu maka penelitian ini akan membahas beberapa penerapan metode matematis dalam mengestimasi jumlah penduduk Provinsi Aceh pada tahun medatang dengan menggunakan metode euler dan metode runge kutta orde 4. Hasil estimasi jumlah penduduk Provinsi Aceh dengan Menggunakan metode Runge Kutta ialah 5.030.503 Jiwa pada tahun 2021, 5.076.307 jiwa pada tahun 2022, dan 5.122.486 pada tahun 2023. Hasil estimasi dengan menggunakan metode Euler ialah 5.026.162 pada tahun 2021, 5.071.745 pada tahun 2022, dan 5.117.700 pada tahun 2022. Kesimpulan dalam penelitian ini adalah estimasi dengan menggunakan metode Runge Kutta merupakan metode yang lebih baik diterapkan dibandingkan metode Euler pada masalah estimasi jumlah penduduk Provinsi Aceh. Hal ini dikarenakan nilai error metode Runge Kutta lebih lebih kecil dibandingkan metode Euler dengan nilai error sekitar 0,04%.

Kata kunci : Runge Kutta, Euler, Penduduk.

ABSTRACT

Residents are people who are in an area and are controlled by applicable rules and interact with each other Residents are people who are in an area and are bound by applicable rules and interact with each other continuously or continuously. If population growth increases, this will cause many problems, including: poverty levels, unemployment, school dropout rates and increasing crime rates in each region. In anticipating the problem of increasing the population of Aceh Province, information about the population is needed. Therefore, this study will discuss several applications of mathematical methods in estimating the population of Aceh Province in the coming year using the Euler method and the Runge Kutta method of order 4. The results of the estimation of the population of Aceh Province using the Runge Kutta method are 5.030.503 inhabitants in 2021, 5.076.307 in 2022 and 5.122.486 in 2023 The estimation results using the Euler method are 5.026.162 in 2021, 5.071.745 in 2022 and 5.117.700 in 2022. The conclusion in this study is that estimation using the Runge Kutta method is a method that is better applied than the method Euler on the problem of estimating the population of Aceh Province. This is because the error value of the Runge Kutta method is smaller than the Euler method with an error value of about 0.04%.

Key words : Runge Kutta, Euler, Population.

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PRELIMINARY

Residents are people who are in an area and are controlled by applicable rules and interact with each other continuously or continuously. (Pandu, 2020). According to sociology, population is a collection of people who occupy a certain geographic area and space. According to BPS data in 2020, Indonesia is the fifth most populous country in the world. Based on BPS data rankings for 2020, Indonesia is in fourth place with a population of 267,026,366 after China, India and the United States. Meanwhile, in Southeast Asia, Indonesia has the first largest population, and the third largest in the Asian continent (Wulandari, 2020).

The topic of population has become one of the most discussed topics, both through the mass media and in conversation (Basuki,2021). This is an indication of the growing awareness of the world community that the demographic crisis has become a situation that is getting worse so that it becomes one of the challenges and threats to life as a whole on our earth. In the current era of globalization, the times are progressing rapidly (Wirasasmita, 2020). In addition, population growth is also increasing rapidly. The monetary crisis and the moral crisis that occurred also contributed to the decline of the Indonesian nation. This condition causes a lot of unemployment and crime. The lack of investment, high unemployment makes the Indonesian people poorer. In addition to poverty, the problem faced by the Indonesian people today is population density (Helvira, 2020).

If population growth increases this will cause many problems, including: poverty rates, unemployment, dropout rates and increasing crime rates in each region. Population growth can be affected by reproductive changes in birth and death rates. Therefore, it is necessary to have a handler from the government in an even distribution of the population, in order to reduce the negative impact on every resident (Janiar,2017). The occurrence of population growth affects the development process and the development of activities in an area and increases the need for space/land (Firdaus,2018). The increasing population of an area requires the provision of living necessities, both physical needs such as housing, facilities and infrastructure, as well as non-physical needs such as education, economy, and recreation areas (Fejriani, 2020).

Aceh province is a province that is ranked 13th with the largest population of 34 provinces in Indonesia. In 2020, in 2020 the population of Aceh province reached 5,459,891 people, while in 2019 the population of Aceh province reached 5,371,532 people. 88,369 Souls (Bps, 2021). In the province of Aceh, due to several factors, namely the rise of early marriages carried out by teenagers, families who do not follow family planning procedures and at the same time migration also plays a role in increasing and reducing the population.

The population problem can be categorized as a major national problem which requires immediate problem solving. The population of an area must be balanced with the number of sources of income, so that an increase in national income can be obtained. This shows that the rate of population growth also affects planning in the economic sector (Sunandar, 2020). In anticipating the problem of increasing the population of Aceh Province, information about the population is needed. Therefore, this study will discuss several applications of mathematical methods in estimating the population of Aceh Province in the coming year using the Euler method and the Runge Kutta Order method 4 Aceh Province.

METHODS

The type of research used in this research is quantitative research. The data used is from the Website of the Central Statistics Agency for Aceh Province. The method used is the Euler method and the Runge Kutta method.

Euler Method

At $x = 0$, rated $y = y_0$. So $x = 0$ sebagai x_0 slope on $x = x_0$ is,

$$\text{Slope } = \frac{y_1 - y_0}{x_1 - x_0} = f(x_0, y_0)$$

Wherein, $y_1 = y_0 + f(x_0, y_0)(x_1 - x_0)$

$x_1 - x_0$ step size *h*, get

$$y_1 = y_0 + f(x_0, y_0)h$$

Used value y_1 (on the approximate y and $x = x_1$) at y_2, will be the predictive value of x_2.

Wherein,

$$y_2 = y_1 + f(x_1, y_1)h$$

$$x_2 = x_1 + h$$

Based on the above equation, it is known that the value of $y = y_i$ at x_i,

$$y_{i+1} = y_i + f(x_i, y_i)h$$

Runge-Kutta Method

According to writing (Setiawan dan Mungkasi, 2021) that the runge-kutta method is one of the most important sets of methods for solving differential equations with initial conditions

$y' = f(x,y)$

In general, the "step length" depends on the value of r, but for convenience it is constant, it is *h*. The general form of the order-n runge-kutta method is:

$$y_{r+1} = y_r + a_1k_1 + a_2k_2 + \ ... + \ a_nk_n$$

With $a_1, a_2, \dots, a_n$ is a constant and

$$k_1 = hf(x_r, y_r)$$

$$k_2 = hf(x_r + p_1h, y_r + q_{11}k_1)$$

$$\dots$$

$$k_n = hf(x_r + p_{n-1}h, y_r + q_{n-1,1}k_1 + q_{n-1,2}k_2 + \dots + q_{n-1,n-1}k_{n-1}$$

Score a_i, p_i, q_{ij} chosen in such a way as to minimize the error per step, and equation (4) will be equivalent to the Taylor series method of the highest order possible.

With, Error per step runge-kutta method of order -n : $O(h^{n+1})$

Runge-kutta method error of order -n : $O(h^n)$

Order method = n

The One Order Runge-Kutta Method

The level one runge-kutta method takes the form of.

$$k_1 = hf\ (x_r, y_r)$$

$$y_{r+1} = \ y_r + (a_1k_1)$$

The Two Order Runge-Kutta Method

The level two runge-kutta method takes the form of.

$$k_2 = hf(x_1 + \ p_1h, y_r + q_{11}k_1)$$

$$y_{r+1} = \ y_r + (a_1k_1 + a_2k_2)$$

The Third Order Runge-Kutta Method

The third-order Runge-Kutta method takes the form of

$$k_3 = hf(x_r + h, y_r - k_1 + 2k_2)$$

$$y_{r+1} = y_r + \left(\frac{k_1+4k_2+k_3}{6}\right)$$

The Fourth Order Runge-Kutta Method

The fourth-order runge-kutta method takes the form of

$$k_4 = hf(x_r + h, y_r + k_3)$$

$$y_{r+1} = y_r + \left(\frac{k_1+2k_2+2k_3+k_4}{6}\right)$$

With,

y_r = Initial solution on iteration r

x_r = years r

Verhulst Models

The Verhulst model is a modification of the Malthusian model. The inspiration for the emergence of the Verhulst model is because the solution in the Malthus model is not realistic, that is, it increases or decreases exponentially (Zeng *et al*., 2020). If the maximum population size that can be maintained is $\frac{a}{b}$, that $\frac{a}{b} - N$ will give an indication of how many additional individuals the environment can accommodate, and $\left(\frac{\frac{a}{b}-N}{\frac{a}{b}} = \frac{a-bN}{a}\right)$ give an indication of how many fractions $\frac{a}{b}$ available for population growth.

The modified equation using the new terms is (Mohamed dkk, 2018) :

$$\frac{dN}{dt} = aN\left(\frac{a-bN}{a}\right) = \frac{a^2N-abN^2}{a} = aN - bN^2$$

$$\frac{dN}{dt} = aN - bN^2$$

This model is a nonlinear differential equation which has a solution :

$$\frac{1}{a}\left(lnN - ln(a-bN)\right) = t + c$$

It is known that the current population$t = 0 = t_0$ is N_0, so :

$$c = \frac{1}{a}\left(lnN_0 ln(a - bN_0)\right)$$

By substituting the value of c, equation (5) becomes :

$$ln\frac{N(a-bN_0)}{N_0(a-bN)} = at$$

By exponentiating on both sides, we get :

$$N(t) = \frac{\frac{a}{b}}{1+\left(\frac{\frac{a}{b}}{N_0}-1\right)e^{-at}}$$

Suppose $\frac{a}{b} = k$, that

$$N(t) = \frac{K}{1+\left(\frac{K}{N_0}-1\right)e^{-at}}$$

If t towards infinity, then obtained :

$$ln\frac{N}{N_0} = k$$

This means that the population will grow asymptotically to K and t towards infinity. We will find the equilibrium point of the Verhults model, which is :

$$\frac{dN(t)}{dt} = 0$$

So as,

$$\frac{dN}{dt} = kN\left(1-\frac{N}{K}\right)$$

With,

N = Total population at time t

K = carrying capacity of an area for the population

k = Growth rate

Equation (6) is known as the Verhulst . population growth/population growth model (Hasan *et al*., 2020).

***Mean Absolute Percentage Error* (MAPE)**

Mean Absolute Percentage Error (MAPE) is calculated using the absolute error in each period divided by the actual observed value for that period. MAPE is an error measurement that calculates the size of the percentage deviation between the actual data and the forecast data. The MAPE value can be calculated by the following equation.

$$\text{MAPE}=\frac{1}{n}\sum_{i=1}^{n}\frac{|Actual-Approximation|}{|Approximation|}\times 100\%$$

The value generated through MAPE calculations, shows the approximation criteria shown in the following table (Sinaga dan Irawati, 2018).

Table 1. Criteria for approximation using MAPE

MAPE Value	Approximation Criteria
MAPE $\leq 10\%$	Very good
10% < MAPE $\leq 20\%$	Well
20% < MAPE $\leq 50\%$	Pretty good
MAPE $\leq 50\%$	Enough

.

RESULTS AND DISCUSSION

To determine the approximation of the amount of coffee production in Aceh Province, the growth rate and carrying capacity will be determined in equation :

1. Growth rate

$$k = \frac{1}{t} ln\left(\frac{N(t)}{N_0}\right)$$

$$k = \frac{1}{1} ln\left(\frac{4110100}{4074900}\right) = 0{,}01$$

2. *Carrying capacity*

Because the amount of coffee production in Aceh Province from 1998-2021 is still below 54000000 tons, it is assumed that K = 54000000.

On the interval [0,25] with many iterations $n = 25$ then,

$$h = \frac{b-a}{n} = \frac{25-0}{25} = 1$$

Determine the Verhulst model as a nonlinear differential equation..

$$\frac{dN}{dt} = k\left(1 - \frac{N}{K}\right)N, \text{ with initial value } N(t_0) = N_0$$

$$= 0{,}01\left(1 - \frac{N}{54000000}\right)N$$

The results of both methods are operated using Matlab software. The following is the Matlab program code in determining the results using the Runge Kutta method.

```
% dN/dt=0.01*N*(1-N/54000000) ; 0<=t<=25 ; y(0)=4074900;
f = @(t,N) (0.01*N*(1-N/54000000));
a = input('Enter left end ponit, a: ');
b = input('Enter right end point, b: ');
n = input('Enter no. of subintervals, n: '); % n=(b-a)/h
alpha = input('Enter the initial condition, alpha: ');
h = (b-a)/n;
t = a;
N = alpha;
fprintf(' t y\n');
fprintf('%5.3f %11.7f\n', t, N);
for i = 1:n
 k1 = h*f(t,N);
 k2 = h*f(t+h/2.0, N+k1/2.0);
 k3 = h*f(t+h/2.0, N+k2/2.0);
 k4 = h*f(t+h,N+k3);
 N = N+(k1+2.0*(k2+k3)+k4)/6.0;
 t = a+i*h;
 fprintf('%5.1f %11.9f\n', t, N);
end
```

The following is the Matlab program code in determining the results using the Euler method.

```
% dN/dt=0.01*N*(1-N/54000000) ; 0<=t<=25 ; y(0)=4074900;
f = @(t,N) (0.01*N*(1-N/54000000));
```

```
a = input('Enter left end ponit, a: ');
b = input('Enter right end point, b: ');
n = input('Enter no. of subintervals, n: ');
alpha = input('Enter the initial condition, alpha: ');
h = (b-a)/n; t = a; w = alpha;
fprintf(' t w\n');
fprintf('%5.4f %11.8f\n', t, w);
for i = 1:n
    w = w+h*f(t, w);
    t = a+i*h;
    fprintf('%5.4f %11.8f\n', t, w);
    plot(t,w,'r*'); grid on;
    xlabel('t values');
    ylabel('w values');
    hold on;
end
```

The results of the completion of the Runge Kutta method and the Euler method can be seen in the following table:

Table 2. The results of the completion of the Runge Kutta method and the Euler method

t	Years	Total population	Runge Kutta Method	Euler Method
0	1998	4074900,0000	4074900,000000000	4074900,000000000
1	1999	4110100,0000	4112734,341388110	4112574,035183330
2	2000	3929234,0000	4150890,757993340	4150567,689387970
3	2001	4114000,0000	4189371,447750960	4188883,141810700
4	2002	4166040,0000	4228178,613410450	4227522,576566630
5	2003	4218486,0000	4267314,462332090	4266488,182492420
6	2004	4075599,0000	4306781,206278880	4305782,152944870
7	2005	4031589,0000	4346581,061203670	4345406,685594940
8	2006	4153573,0000	4386716,247031450	4385363,982216960
9	2007	4223833,0000	4427188,987436760	4425656,248473110
10	2008	4293915,0000	4468001,509616190	4466285,693693090
11	2009	4363477,0000	4509156,044055880	4507254,530648970
12	2010	4494410,0000	4550654,824294050	4548564,975325080
13	2011	4597308,0000	4592500,086678410	4590219,246683000
14	2012	4693934,0000	4634694,070118520	4632219,566421570
15	2013	4791924,0000	4677239,015832970	4674568,158731800
16	2014	4906835,0000	4720137,167091340	4717267,250046780
17	2015	5001953,0000	4763390,768951000	4760319,068786440
18	2016	5096248,0000	4807002,067988580	4803725,845097140
19	2017	5189466,0000	4850973,312026130	4847489,810586100
20	2018	5281314,0000	4895306,749851920	4891613,198050530
21	2019	5371532,0000	4940004,630935900	4936098,241201530
22	2020	5459891,0000	4985069,205139640	4980947,174382660
23	2021		5030502,722420910	5026162,232283150
24	2022		5076307,432532670	5071745,649645760
25	2023		5122485,584716570	5117699,660969120

The results can be viewed using the graph in figure 1.

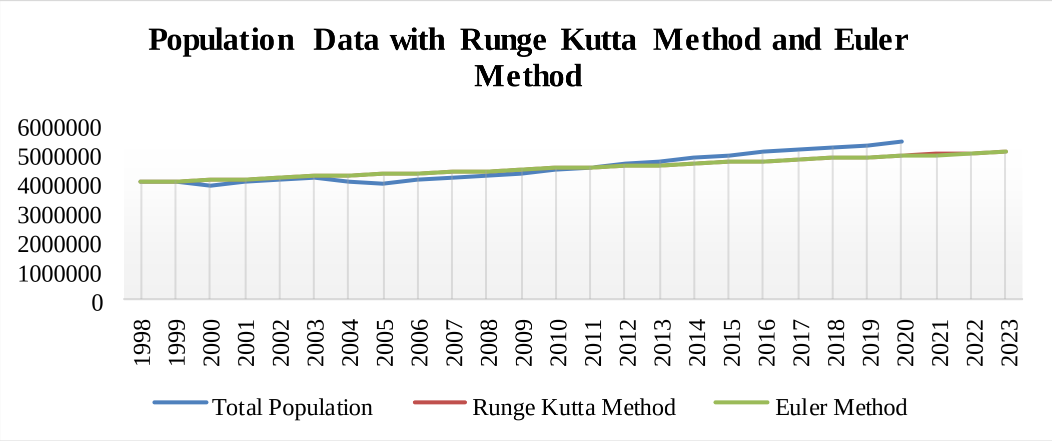

Figure 1. Population Data with Runge Kutta Method and Euler Method

The accuracy of both methods is measured by MAPE obtained from both methods, where the MAPE value obtained from the Runge Kutta method is 0.039702316% and the Euler method is 0.039839570%. Therefore the Runge Kutta method is more accurate than the Euler method. The results found for the estimated population of Aceh Province using the Runge Kutta method are 5030502.722420910 souls in 2021, 5076307,432532670 in 2022 and 5122485,584716570 in 2023 The estimation results using the Euler method are 5026162,232283150 in 2021, 5071745,649645760 in 2022 and 5117699,660969120 in 2022. And the results that can be applied to the estimated population of Aceh Province are the estimation results from the Runge Kutta method.

CONCLUSION

According to the formulation of the problem and the results obtained from this study, it can be concluded that the estimation results of the population of Aceh Province using the Runge Kutta method are 5.030.503 people in 2021, 5.076.307 in 2022, and 5.122.486 in 2023. The estimation results using the Euler method are 5.026.162 in 2021, 5.071.745 in 2022 and 5.117.700 in 2022. The conclusion in this study is that estimation using the Runge Kutta method is a method that is better applied than the Euler method on the problem. estimated population of Aceh Province. This is because the error value of the Runge Kutta method is smaller than the Euler method with an error value of about 0.04%.

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