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Hypothetical Learning Trajectory: To Determine The Solution of Linear Equation System in Elementary Row Operation

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ABSTRACT
The background of this research is the learning obstacles that arise on the topic of determining the solution of a system of linear equations (SLE) in applying elementary row operations (ERO). From these findings, it is necessary to formulate a hypothetical learning trajectory (HLT) that can guide the flow of thinking in designing didactic situations in the learning process. HLT expect to anticipate learning obstacles (LO) that arise. This research is a preliminary design of a series of didactical design research. The steps taken in compiling the HLT are 1) reviewing the lesson plan, the methods used to convey the material to determine SST solutions, and the textbooks used; 2) reviewing students' thinking processes conceptually and procedurally regarding theoretically determining SLE solutions; 3) reviewing design plans didactic. HLT is formulated into five types of student responses. It integrates into didactic situations by explaining the relevance of the elimination method in high school with three operations on ERO and then by explaining matrix material before SPL material. From this didactic situation, it can be formulated in teaching material that can accommodate emerging LO.
Keywords : Hypothetical Learning Trajectory, Elementary Row Operations, Linear Equation System, Didactical Situation.

ABSTRAK
Penelitian ini dilatarbelakangi dari hambatan belajar (*learning obstacles*) yang muncul pada topik menentukan solusi sistem persamaan linier (SPL) dalam mengaplikasikan operasi baris elementer (OBE). Dari temuan ini perlu dirumuskan *hypothetical learning trajectory* (HLT) yang bisa menjadi pedoman alur berpikir dalam merancang situasi didaktis dalam proses pembelajaran. Hal ini diharapkan dapat mengantisipasi hambatan belajar yang muncul. Penelitian ini merupakan *preliminary design* dari rangkaian *didactical design research*. Tahapan yang dilakukan dalam menyusun HLT adalah 1) mengkaji rencana pembelajaran semester (RPS), metode yang digunakan untuk menyampaikan materi menentukan solusi SPL, dan buku ajar yang digunakan, 2) mengkaji proses berpikir mahasiswa secara konsep dan prosedur mengenai penentuan solusi SPL secara teoritis, 3) mengkaji rancangan desain didaktis. HLT dirumuskan ke dalam 5 tipe respon mahasiswa kemudian integrasinya terhadap situasi didaktis adalah dengan menjelaskan relevansi metode eliminasi di SMA dengan tiga operasi pada OBE kemudian dengan menjelaskan materi matriks terlebih dahulu sebelum materi SPL. Dari situasi didaktis ini bisa dirumuskan dalam suatu bahan ajar yang bisa mengakomodir *learning obstacles* (LO) yang muncul.
Kata kunci: Dugaan Lintasan Berpikir, Operasi Baris Elementer, Sistem Persamaan Linier, Situasi Didaktis.

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PRELIMINARY

Linear equation system (LES) is one of the subject matters in mathematics that is always exists at every level of formal schooling. At the higher education level, this material is increasingly complex and applicable. What is learned is to determine the solution of an LES. In research conducted by (Siahaan et al., 2023) for mathematics education students at the Universitas Timor, there are several findings on learning obstacles experienced by students in solving an LES. The first obstacle is that respondents (student college) do not comprehend the concept and procedure of elementary row operation (ERO). This is indicated by students' inability to execute what ERO will be used in each step of row reduction work in the augmented matrix. The second obstacle is the classification of reduced row echelon matrices (RREM). To find the LES solution, we can implement the Gauss-Jordan method, where the reduced matrix will eventually form the RREM. The problem faced is that students need to understand that the form of RREM is just identity matrix, even though RREM is not necessarily an identity matrix.

According to the findings of the previous study, the learning obstacles urgently need to be minimized therefore learning difficulties in conceptual and operational errors can be avoided. For this reason, teachers need to understand student learning trajectories in this context. Learning trajectory is a description of students' thoughts and learning processes in certain mathematical domains. These are then implemented through situations designed to lead to the development of students' thinking levels. The learning trajectory is able to provide opportunities for teachers to focus on the audience's thinking process (Fuadiah, 2017). This method is carried out with the expectation of improving the learning design as a form of evaluation from the teacher regarding the learning process (Rezky & Jais, 2020).

Hypothetical Learning Trajectory (HLT) is an assumption of a catena of learning activities, including the flow of thought or students' responses in those activities (Sarvita & Syarifuddin, 2020). The HLT formulation is based on learning obstacles (LO) that emerged in the previous learning process and the learning tools involved during learning. If we refer to more familiar terms, we can analogize HLT as an itinerary and routes in a trip. If someone who goes on a trip thoroughly understands the routes in the itinerary prepared, then that person will be able to achieve all travel targets according to the itinerary (Wijaya, 2009).

The teaching plan design will apply is based on the student's thinking flow and is an anticipation of student learning that may occur in the learning process based on knowledge of the level of understanding that is part of the lesson design. The primary function of lesson design is to give students the opportunity to respond to learning; therefore, the teacher must plan what might happen during the learning process (Hendrik et al., 2020).

Based on the explanation that has been presented, the focus of this research is how HLT students determine LES solutions using ERO. By determining the HLT, a description of the flow of students' thinking will be clearly illustrated, and didactic anticipation can be well-designed.

METHODS

This research belongs to the qualitative approach and is part of the didactical design research which focuses on the preliminary design stage. The subjects of this study were 38 students of the Mathematics Education study program who took Elementary Linear Algebra courses. The instruments used in this study are those used in research (Siahaan et al., 2023).

This study will arrange HLT based on the learning obstacles described in the study of (Siahaan et al., 2023). The stages involved in compiling the HLT by the researcher were 1) examining the lesson plan, the methods used to convey the material for determining the LES solution, and the textbooks used; 2) examining the student's thinking processes conceptually and procedurally regarding the determination of the LES solution theoretically, 3) reviewing the arranged didactic design; prediction of student responses and didactic and pedagogical anticipation of these responses based on emerging learning obstacles.

RESULTS AND DISCUSSION

This section will explain two main topics, A) HLT in determining LES solutions and B) didactic designs based on HLT. In part A, the LO that appears will be reviewed again and then analyzed in order to produce HLT. Then part B will be explained based on the resulting HLT.

A. HLT in determining the solution of LES

The initial step is to re-examine the lesson plan relating to the learning objectives of the LES material.

Table 1. **Syllabus of LES in Elementary Linear Algebra Course**

Course Learning Outcomes (CLO)	Study Material	Class Activity	Achievement Indicator
Students comprehend the concept of linear equation system	LESs and Matrices	• Using student-based learning methods with interactive lecture techniques and group discussions • Having discussions between lecturers and students regarding the general form and types of LESs solutions	Able to classify the general form of LES
			Able to determine the solutions and the kinds of solutions of LES
Students can determine LES solutions using the Gauss elimination method and the Gauss-Jordan elimination method by utilizing elementary row operations (ERO).		• Using student-based learning methods with interactive lecture techniques and group discussions • Having a discussion between lecturer and students regarding the implementation of ERO in Gauss elimination method and Gauss-Jordan method • Having discussion between lecturer and students regarding the identification of matrices in representation of solution of LESs	Able to implement ERO using Gauss elimination method and Gauss-Jordan method
			Able to implement the concept of LES solution in analysing matrices behaviour as the representation of LES

In the textbook used in the class, the material for Elementary Linear Algebra lectures begins with LES, and the next material is matrices. In lesson plan, material for determining LES solutions is included in LES study material. However, in the learning process, LES is closely related to the matrix. This is one of the learning obstacles faced by students and will be discussed further in the review of learning obstacles.

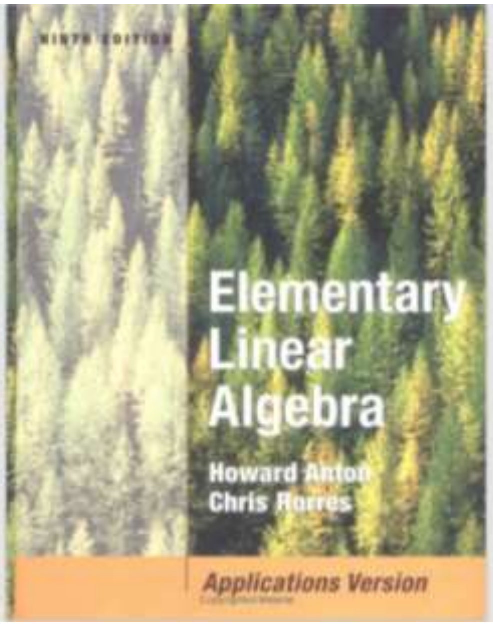
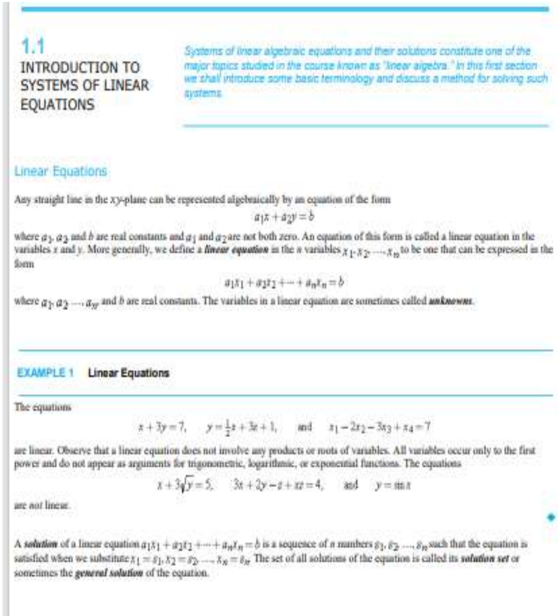


Figure 1. The Textbook



1.1
INTRODUCTION TO SYSTEMS OF LINEAR EQUATIONS

Systems of linear algebraic equations and their solutions constitute one of the major topics studied in the course known as "linear algebra." In this first section we shall introduce some basic terminology and discuss a method for solving such systems.

Linear Equations

Any straight line in the xy-plane can be represented algebraically by an equation of the form

$$a_1x + a_2y = b$$

where a_1, a_2 and b are real constants and a_1 and a_2 are not both zero. An equation of this form is called a linear equation in the variables x and y. More generally, we define a ***linear equation*** in the n variables $x_1, x_2, \ldots, x_n$ to be one that can be expressed in the form

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n = b$$

where $a_1, a_2, \ldots, a_n$ and b are real constants. The variables in a linear equation are sometimes called ***unknowns***.

EXAMPLE 1 **Linear Equations**

The equations

$$x + 3y = 7, \quad y = \frac{1}{2}x + 3z + 1, \quad \text{and} \quad x_1 - 2x_2 - 3x_3 + x_4 = 7$$

are linear. Observe that a linear equation does not involve any products or roots of variables. All variables occur only to the first power and do not appear as arguments for trigonometric, logarithmic, or exponential functions. The equations

$$x + 3\sqrt{y} = 5, \quad 3x + 2y - z + xz = 4, \quad \text{and} \quad y = \sin x$$

are *not* linear.

A ***solution*** of a linear equation $a_1x_1 + a_2x_2 + \cdots + a_nx_n = b$ is a sequence of n numbers $s_1, s_2, \ldots, s_n$ such that the equation is satisfied when we substitute $x_1 = s_1, x_2 = s_2, \ldots, x_n = s_n$. The set of all solutions of the equation is called its ***solution set*** or sometimes the ***general solution*** of the equation.

Figure 2. The First Material in The Textbook

The second step will be discussed regarding the students' thinking processes in terms of concepts and procedures. Conceptual understanding is defined as an understanding that goes beyond the ability to memorize a definition or certain material or the ability to answer questions and problems correctly (Klau et al., 2020). The situation experienced by students who can solve math problems correctly could be based on their experience working on routine problems, including implementing the same procedures as those carried out in class with the teacher's assistance (Putri, 2018).

As a result, students' thinking abilities seem to be regulated to work on a problem with a procedural approach even though for the level of student thinking ideally it is already at the ability to think abstractly. So that students' conceptual understanding must be addressed because it has a big impact on the student's thinking system as a whole (Suryabayu et al., 2022).

This is relevant to LO's findings which serve as guidelines in this article for compiling HLT. The first LO is a student's understanding that is not firm about ERO, which is indicated by the inability to execute what type of ERO is used in the row reduction steps. This becomes an obstacle experienced by students in the formation of conceptual understanding regarding the implementation of ERO in row reduction. Even though the row reduction process is equivalent to the two variables-LES elimination process was learned in high school (Anton et al., 2005).

For this reason, we need an explanation of why the ERO operations multiply a row by a non-zero number, add the multiple of a row to another, and swap any two rows. This could be applied by explaining why and how ERO includes these three operations. In addition, an explanation of how relevant this is to the elimination method in two variables-LES, is the simplest example.

The second LO finding is that students' ability to classify RREM is not comprehensive yet. To find LES solutions, students are introduced to the Gauss-Jordan method, where the reduced matrix will form RREM. The problem faced is that students understand that the form of RREM is only an identity matrix, even though not all forms of RREM are identity matrices (Siahaan et al., 2023).

The compiled HLT contains learning objectives, learning activities, hypothetical learning processes. Learning objectives will focus on determining LES with unique solutions, multiple solutions, and LES with no solutions

Table 2. **Hypothetical Learning Trajectory**

Learning Goal	Learning Activity	Hypothetical Learning Trajectory
Students can determine a unique solution from an LES	Students work on problems by determining the solution of an LES using the Gauss-Jordan method. The steps are 1. To create an augmentation matrix 2. To reduce the rows from the augmentation matrix to produce RREM 3. To interpret RREM into a unique LES solution	**Type 1.** Students can create augmented matrices, reduce rows and generate RREM, and correctly interpret RREM into unique LES solutions. **Type 2.** Students can create augmented matrices, reduce rows and produce RREM, but are not able to interpret RREM into unique LES solutions. **Type 3.** Students can create augmented matrices, reduce rows but not up to RREM. **Type 4.** Students can create augmented matrices and make mistakes in reducing rows. The mistakes made were miscalculations and misunderstandings in using ERO.

Learning Goal	Learning Activity	Hypothetical Learning Trajectory
		Type 5. Students are not able to create an augmented matrix from an LES.
Students can determine many solutions from LES	Students work on problems by determining the solution of an LES using the Gauss-Jordan method. The steps are 1. To create an augmented matrix 2. To reduce the rows from the augmented matrix to produce RREM 3. To interpret RREM into LES with many solution 4. To convert the solution into parameter form	**Type 1.** Students can create augmented matrices, reduce rows and generate RREM, and interpret RREM into multiple solutions of an LES, and make solutions in the form of parameters correctly. **Type 2.** Students can create augmented matrices, reduce rows and produce RREM and interpret RREM into multiple LES solutions but are unable to make solutions in parameter form. **Type 3.** Students can create augmented matrices, reduce rows and generate RREM, but are unable to interpret RREM into many solutions. **Type 4.** Students can create augmented matrices and make mistakes in reducing rows. The mistakes made were miscalculations and misunderstandings in using ERO. **Type 5.** Students are not able to create an augmented matrix from an LES
Students can determine no solution from an LES	Students work on problems by determining the solution of an LES using the Gauss-Jordan method. The steps are 1. To create an augmentation matrix 2. To reduce the rows from the augmentation matrix to produce RREM 3. To interpret RREM into LES with no solution	**Type 1.** Students can create augmented matrices, reduce rows and generate RREM, and correctly interpret RREM into LES with no solutions. **Type 2.** Students can create augmented matrices, reduce rows and produce RREM but unable to interpret RREM into LES with no solution; **Type 3.** Students can create augmented matrices, reduce rows but not up to RREM. **Type 4.** Students can create augmented matrices and make mistakes in reducing rows. The mistakes made were miscalculations and misunderstandings in using ERO.

Learning Goal	Learning Activity	Hypothetical Learning Trajectory
		Type 5. Students are not able to create an augmented matrix from an LES

B. Didactical design based on HLT

The didactical situation was prepared based on the HLT which had been formulated by considering the LO. The first thing to do is when explaining the use of ERO in reducing rows, the teacher must first explain the essence of the elimination method which is usually done in high school which is carrying out a transformation activity to obtain the simplest LES. Then the transformation made to the LES does not change the form of the LES. For example, known SPL $\begin{cases} 3x = 9 \\ x + 3y = 8 \end{cases}$ then we apply the transformation to be $\begin{cases} (3x)^2 = 9^2 \\ x + 3y = 8 \end{cases}$. The values of x which satisfy $(3x)^2 = 9^2$ are -3 or 3 while we refer to initial equation $x = -3$ could not satisfy $3x = 9$. It is clear that the squaring transformation changes the form of the LES which affects the value of the LES solution. Therefore, we cannot use the squaring transformation to get the simplest LES.

By starting with the case examples above, the teacher can explain why the three operations on ERO do not change the LES so that it becomes the ERO we are now using in reducing matrices. In this part of the explanation, we will see its relevance to the elimination method that was usually done in high school. So that from this didactical situation it is expected that it can minimize the inability of students to execute which ERO must be carried out in each stage of reducing the row.

The second thing that can be done is to change the order of the materials. Handbooks related to Elementary Linear Algebra will usually start the material with LES. Whereas in the process of determining the LES solution, it is necessary to create an augmentation matrix, in the Gauss method it is necessary to make it into a row echelon matrix (REM), and in the Gauss-Jordan method it is necessary to make it into a reduced row echelon matrix (RREM). In LO's findings, students assume that RREM is an identity matrix even though RREM is not only an identity matrix. Then in looking at the characteristics of the matrix which represents the three types of LES solutions, an understanding of the matrix concept is needed so that the matrix material becomes the first material before LES material becomes a consideration in designing didactic situations in LES material.

CONCLUSION

Based on the findings of learning obstacles, HLT was formulated through five types of student responses based on learning objectives, namely determining a unique LES solution, an LES with many solutions, and an LES with no solutions. Then an LES learning can be designed that explains the relevance of the two variables-SLE elimination method with ERO, ERO which does not change LES so that it retains the solution with three transformations from ERO, and changes the order of material to matrix material as a precursor to LES material. From this didactical situation, it can be formulated in a handout or student module that can accommodate LO that arise.

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