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POPULATION PROJECTION WITH THE APPLICATION OF THE DIFFERENTIAL EQUATION OF THE LOGISTIC AND EXPONENTIAL MODEL (Case Study: Yogyakarta Special Region Province)

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ABSTRACT

The calculation of population projections in Indonesia at the provincial level is carried out by BPS using the component method, while at the district level it is done using the geometric method with the basic reference of calculating compound interest. From this, it is necessary to take another approach in calculating the projected population in the province of the Special Region of Yogyakarta. This study aims to determine population projections in the Special Region of Yogyakarta using exponential and logistic models based on growth rate and carrying capacity. The data used in this study is secondary data from the Central Bureau of Statistics of the Special Region of Yogyakarta in 2017–2022. The research method used is to determine the research subject, collect data, analyze the data, and draw conclusions. The result of this research is that the carrying capacity of the Special Region of Yogyakarta is 10,652,814 and the population growth rate for the logistic model is 0,02048.

Keywords: Carrying Capacity, Exponential Model, Logistic Model, Population Projection

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PRELIMINARY

Projection of population is an important issue that is the focus of studies in several countries in the world even today. The determining factors in the variables that make global changes in the future include the projected trend of population, population age, growth rate and migration. Economics, education, social, politics and culture greatly affect the rate of population growth in a country (Anggreini, 2018).

The population projection is calculated based on the past and future population using certain assumptions of changes in the rate of population growth and its components. According to calculations. The Province of the Special Region of Yogyakarta in 2020 has a population of 3.668.719 people, with a population density of 1.227 people/km2 and administratively divided into 5 districts. Talking about the population, the Special Region of Yogyakarta is one of the provinces with the most population in Indonesia. With such a large population, the Special Region of Yogyakarta is the fourth most populous area in Indonesia (BPS, 2019).

Differential equations can be represented on problems that occur in real life. Modeling differential equations with continuous population models such as exponential models and logistic models is a mathematical application that is often applied to everyday life (Myhammad Ahsar Karim, 2019). Continuous is defined as a model in which the population is directly dependent on time.(Di & Xu, 2021). According to Malthus, the exponential growth model describes an unlimited population in the environment (Ma, 2020). Anggreini (2020) argues that In 2030 the population of East Java Province amounted to 42.552.016 calculated by the exponential model. The logistics model is 41.444.035. It can be concluded that the most accurate calculation uses the smallest V MAPE Logistics model.

The exponential growth model is a population growth rate proportional to the existing population (Ullah et al., 2019). Logistical growth is growth that is limited by biological situations and certain resources. Logistics model approach is a model that is considered closest to the actual population, this model produces the maximum number of populations (Naurah zahwa & Nurviana, 2021). Malthus' theory also argues that the logistic model is a model that is close to maximum growth, but it is different from Verhulst's theory which connects the concept of maximum population with population equations (Rozikin et al., 2021).

Some relevant studies that support and strengthen this research are as follows.(Barbosa & Rothwell, 2021) and (Uddin et al., 2019). The research concluded that the logistic model is a good calculation tool in estimating the population, and the results obtained are the environmental carrying capacity of 2.2 billion. Venkatesha et al., (2019) and Anggreini, (2020) discuss population growth using logistic and exponential models in calculating the population.Based on the explanation above, a projection with the model will be carried out on the future population growth of Yogyakarta City with reference to known population data. From the literature review, no one has discussed the projection of exponential and logistic calculations in Yogyakarta City.

METHODS

The approach in this study uses a descriptive qualitative approach. The research data uses secondary data types. The first stage of the research method is to determine the research subject, the population of Yogyakarta in 2017 to 2022. Second, collecting research data from secondary data from the Yogyakarta Central Statistics Agency, analyzing the data and then making conclusions. The variables observed in this study were the rate of intrinsic growth and carrying capacity. The two models used in this study are the exponential and logistic models. Based on the assumptions, a population growth model is derived which is known as the logistic model. The data analysis technique used by the researcher was the first to construct the exponential and logistic differential equation model, looking for a solution from

$\frac{dN}{dt} = rN(t)$ dan $\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)$,

The next step is to determine the projected population in Yogyakarta in 2035 using Excel and MAPLE applications.

The exponential model is expressed as follows:

$$\frac{dN}{dt} = rN(t) \quad \text{atau} \quad \frac{dN_t}{N_t\,dt} = rN(t) \tag{1}$$

The logistic model is used because of the fact in nature that the size of the population depends on the density of the population, so that the birth and death rates are not constant (Karim et al., 2022).

$$\frac{1}{N}\frac{dN}{dt} = k\left(1 - \frac{N}{K}\right) \tag{2}$$

$$N = \frac{K}{e^{-rt}\left(\frac{K}{N_0} - 1\right) + 1} \tag{3}$$

Anggreini (2020), explained that the value of *K* was obtained from the limit N(*t*), namely:

$$\text{N}(t)\ \text{max} = \lim_{t\to\infty} N(t) = \frac{a}{b} = \frac{N_1(2N_0N_2 - N_2N_1 - N_0N_1)}{N_0N_2 - {N_1}^2} \tag{4}$$

The techniques used in analyzing the form and model of research are:

1. Constructing the Verhulst/logistic differential equation model
2. Finding the solution of the differential equation $\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)$
3. Determining the measured time and the number of the initial population and the population in the following year
4. Determining the carrying capacity
5. Find the population growth rate using the solution of the logistic modal equation.
6. Calculating the total population of Yogyakarta Province with the logistic equation

7. Determining the population projection of Yogyakarta Province with modal logistic growth whose value is close enough to the census results
8. Determine the population projection of Yogyakarta Special Region Province using Maple and Microsoft Excel applications.

RESULT AND DISCUSSION

The following is data on the population in Yogyakarta from 2017-2022:

Table 1. Population Data in Yogyakarta

Time	Years	Population	Years (N)
t=0	2017	3.768.235	N0
t=1	2018	3.818.266	N1
t=2	2019	3.868.588	N2
t=3	2020	3.919.197	N3
t=4	2021	3.970.220	N4
t=5	2022	4.021.816	N5

It is assumed that time (t) is measured in years and suppose $t = 0$ in 2017 then the initial condition is $N(0) = 3,768,235$. Determine the carrying capacity value, namely:

$$K = \underset{max}{N(t)} \quad \lim_{t\to\infty} N(t) = \frac{a}{b} = \frac{N_1(2N_0N_2 - N_2N_1 - N_0N_1)}{N_0N_2 - N_1{}^2}$$

Value N_0, N_1, N_2 substituted by the equation to find the Carrying Capacity value in Yogyakarta, namely $N(t) = \frac{a}{b}$. The calculation is as follows:

$$N(t)_{max} = \lim_{t\to\infty} N(t) = \frac{a}{b} = \frac{N_1(2N_0N_2 - N_2N_1 - N_0N_1)}{N_0N_2 - N_1^2} \tag{5}$$

$$= \frac{3.818.266\ (2\ x\ 3.768.235\ x\ 3.868.588 - 3.868.588\ x\ 3.818.266 - 3.768.235\ x\ 3.836.3195)}{3.768.235\ x\ 3.868.588 - (3.818.266)^2}$$

$K = 10.652.813$

value $KdanN_0$ distributed to the logistics model so that obtained

$$N = \frac{K}{e^{-rt}\left(\frac{k}{N_0} - 1\right) + 1},$$

$$N = \frac{10.652.813}{e^{-rt}\,(1{,}827) + 1}, \tag{6}$$

Equation (6) will then look for a logistic model that represents the population growth rate of Yogyakarta for t=1 in 2018 then $N(1)$=3.818.266, if distributed into equation (6) we get

$$3.818.266 = \frac{10.652.813}{e^{-r}\ (1{,}827) + 1},$$

$r = 0{,}0204833,$

value r then substituted to (6) so obtained

$$N = \frac{10.652.813}{(1{,}827)e^{-(0{,}0204833)t} + 1},$$

The following table.2 contains the results of the population based on five logistical models from 2017-2022.

Tabel .2 Comparison between Census Results and Logistics Model

Years	Census Results	M 1	M 2	M 3	M 4	M 5
2017	3.768.235	3.768.235	3.768.235	3.768.235	3.768.235	3.768.235
2018	3.818.266	3.818.266	3.818.266	3.818.267	3.818.302	3.818.383
2019	3.868.588	3.868.588	3.868.588	3.868.591	3.868.661	3.868.823
2020	3.919.197	3.919.193	3.919.193	3.919.197	3.919.303	3.919.548
2021	3.970.220	3.970.072	3.970.072	3.970.078	3.970.220	3.970.548
2022	4.021.816	4.021.218	4.021.218	4.021.225	4.021.403	4.021.816

Tabel 3. Comparison of Census Results with Exponential Model

Years	Census Result	MI	M2	M3	M4	M5
2.017	3.768.235	3.768.235	3.768.235	3.768.235	3.768.235	3.768.235
2.018	3.818.266	3.818.266	3.818.082	3.817.747	3.809.360	3.809.360
2019	3.868.588	3.868.961	3.868.588	3.867.909	3.850.934	3.850.934
2.020	3.919.197	3.920.330	3.919.762	3.918.731	3.892.961	3.892.961
2.021	3.970.220	3.972.380	3.971.614	3.970.220	3.935.448	3.935.448
2.022	4.021.816	4.025.121	4.024.151	4.022.386	3.978.397	3.978.397

Galat (Error)

The Galat value is calculated by calculating the MAPE (Mean Absolute Percentage Error) formula, which is obtained from:

$$\text{MAPE}=\frac{1}{n}\sum\frac{|x_i-F_i|}{x_i}x100\%.$$

Analyzing errors is very important in calculations that use numerical methods. An error is associated with how close the approximate solution is to the true solution. The smaller the error, the more precise the numerical solution obtained. Error can be applied to several calculation methods to determine the best projection based on the smallest error value in the several calculation methodsused.Before determining the best model used to calculate population projections, the magnitude of the error from each model will be calculated exponential and logistic models using the MAPE formula (Terano, 2018).

Tabel 4. Galat Eksponensial Model

Years	MI	MII	MIII	MIV	MV
2017	0	0	0	0	0
2018	0	0,0048239	0,0135965	0,2332476	0,2332476
2019	0,009649	0	0,0175452	0,4563476	0,4563476
2020	0,028899	0,0144237	0,0118967	0,6694148	0,6694148
2021	0,054404	0,0350996	0	0,8758327	0,8758327
2022	0,082187	0,05805	0,0141674	1,0795773	1,0795773
∑ Galat /n	0,022	0,014	0,007	0,414	0,414

Tabel 5. Galat Logistik Model

Years	M I	M II	M III	M IV	M V
2017	0	0	0	0	0
2018	0	0	3,54035E-05	0,00095066	0,003065486
2019	0	0	7,02857E-05	0,001887325	0,006085868
2020	0,000104642	0,000104642	0	0,002705231	0,00895611
2021	0,003718061	0,003718061	0,003579597	0	0,008271289
2022	0,014870403	0,014870403	0,014698667	0,010258904	0
∑ Galat /n	0,002336638	0,002336638	0,002297994	0,001975265	0,003297344

Based on the previous explanation, the MAPE value from the Logistics IV model is 0.0019. So the Logistics IV model with Equation $N = \frac{10.652.813}{(1,827)e^{-(0,02050)t}+1}$

predicting the total population is taken t=13 substituting into the logistic model IV we get:

$$N = \frac{10.652.813}{(1,827)e^{-(0,02050)13}+1} = N = 4.439.373$$

Based on these calculations, the total population in Yogyakarta resulting from the logistic model in 2035 is 4.439.373. To see in more detail the graphic description of the population of Yogyakarta between the results of the census and the results of the model exponential III and logistical models IV can be seen in figure 1 below:

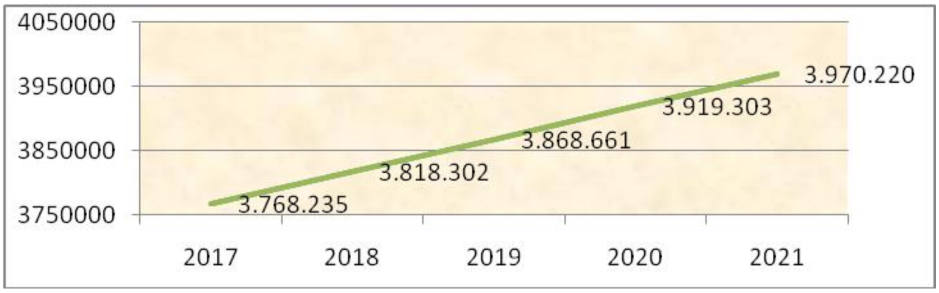

Figure 1. Comparison of Census Population with Exponential Model III

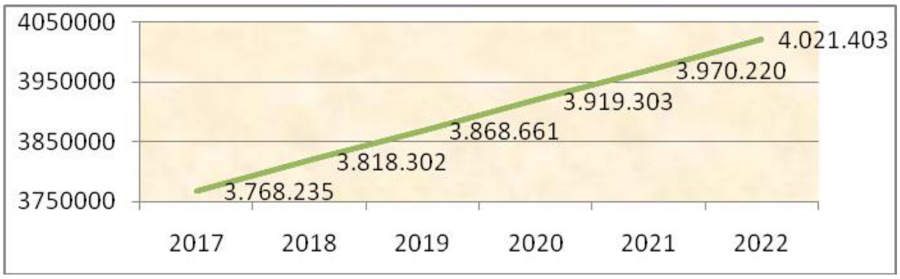

Figure 2.Comparison of Census Population with Logistic Model IV

Figure 1 and Figure 2 show that the exponential model and the logistic model are both close to the actual data so that they can be used to project the population in Yogyakarta in the future.The following will be given a projection image for the future population in Yogyakarta using the exponential model with a time taking t = 0…18. Graphs 3 and 4 show that population growth in Yogyakarta is increasing from year to year.

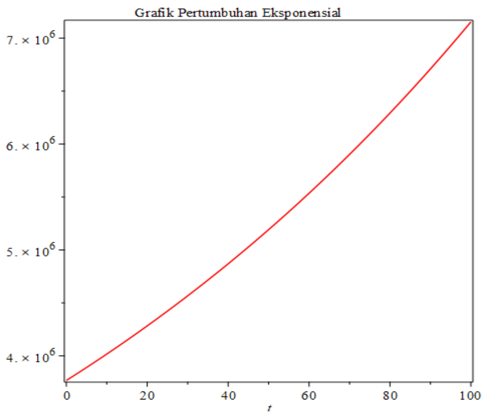

Figure 3. Projection of Population Growth in Yogyakarta (exponential) t = 0…18

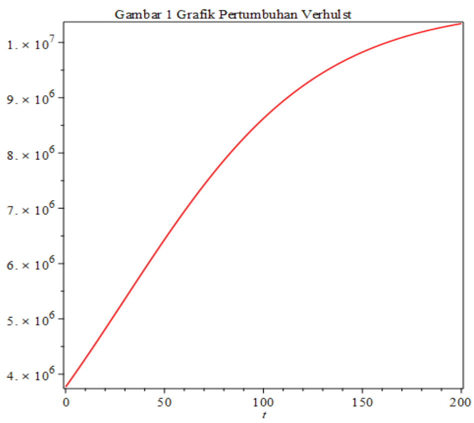

Figure 4 . Projection of Population Growth in Yogyakarta (*Verhulst*) t = 0..300

Based on figure 4 it can be seen that population growth in Yogyakarta is increasing in addition to that the number of residents will never exceed the carrying capacity so that the population in Yogyakarta will approach the value of K = 10.652.813. The smallest error using MAPE lies in the logistics model, namely the logistics model IV of 0.0019. The smallest value becomes the best value because the error value is smaller. This is in accordance with the opinion expressed by (Venkatesha et al., 2019) MAPE which has a value of less than 10% is an accurate projection for predicting the population. So that the Logistics IV model is appropriate to use to predict the population in Yogyakarta in 2035 with a population of 4.439.373 and a population growth rate of r = 0.02050. The results of this study are also in accordance with research (Mondol et al., 2018) which concluded that the logistic model has a very low absolute error percentage (MAPE) compared to the exponential model. Therefore, the Logistics model is better at predicting the population for long-term predictions.

The exponential and logistic models are almost identical to the actual data. Logistics IV model produces a more accurate value in describing the population because for the future the logistic model gives results that are closer to the census results than the exponential model (Putri et al., 2022). This is in accordance with the results of research (Bhadula et al., 2021) which states that although the exponential and logistic models both have the smallest error values in projecting the population, the logistic model is more accurate and has a smaller error. It can be said that the logistic model is the best model for projecting the population in Yogyakarta.

This finding is in line with Naurah zahwa & Nurviana (2021), Rozikin et al., (2021) Ullah. MS et al., (2019), Nurmadhani & Faisol,(2022) and Rosiyanti et al., (2022) also defining that the results of population projections for the future period are based on calculations using a logistic model close to the results of the population census. The projected population growth of Yogyakarta in the long term will experience an increase in population which will approach the carrying capacity value. So for the future the population in Yogyakarta will approach the value of K = 10.652.813. In line with (Venkatesha et al., 2019), Rozikin et al., (2021), Silvia et al., (2022) and Rosiyanti et al., (2022). Values limit population growth in an area or country (Mondol et al., 2018). So the limit value of the asymptotic population growth of Yogyakarta is 10.652.813.

CONCLUSION

Based on the results of the analysis, it can be concluded that: The population growth rate (r) in the Special Region of Yogyakarta using the exponential model is 0.002048 with (Nt) =3.768.235$e^{(0,002048)t}$, while using the logistic model the growth rate (r) is equal to 0.0019. The total population of the Special Region of Yogyakata in 2035 with an estimate of the exponential model III is 4.467.461 while using the logistic model IV it is

4.439.373. Carrying capacity value = 10.652.813. The most accurate model for estimating the population of the Special Region of Yogyakarta in 2035 is the logistic model IV based on the smallest MAPE value, which is 0.0019. The projection of the population of the Special Region of Yogyakata based on the exponential model graph shows that population growth has increased. The projected population growth of the logistic model will not exceed the carrying capacity value of 10.652.813. So it can be ascertained that the logistic model is closer to the actual population conditions. Other researchers can develop this research to look for local stability analysis of the logistic equation model for the future. This is because this research has not discussed the stability of the model.Finding the stability of the system of each model so that it can determine when population growth in an area can be stabilized.

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