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DETERMINATION OF CRITICAL NODE IN THE JAVA SUMATRA KALIMANTAN SUBMARINE CABLE COMMUNICATION SYSTEM

Haliza Suci Rachmadini[1*], Amril Aman[2], Bib Paruhum Silalahi[3]
[1,2,3]Departement of Mathematics, IPB University, West Java Province, Indonesia

*Correspondence: halizasrhaliza@apps.ipb.ac.id

ABSTRACT

Disruption of the Java Sumatra Kalimantan (Jasuka) submarine cable communication system significantly impacted the smooth flow of communications. To reduce the impact, the detection of critical nodes in the network uses the critical node detection method to identify the most important nodes in the Jasuka network. This study aims to apply the critical node detection method as integer linear programming on the Jasuka network to obtain critical nodes by minimizing the number of paired connections on the nodes. The data in this research comes from the Jasuka network, represented as nodes and edges, and then analyzed using Python 3.11 software. The results showed that the critical node of the Jasuka submarine cable communication system is located at index 5 and 14 or the landing point Dumai, Riau and Palembang Jambi. The critical node on the Jasuka network can be a reference for Telkom Indonesia to pay special attention to the landing point because the damage will impact the entire network.
Keywords: Critical Node Detection, Integer Linear Programming, Submarine Cable

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PRELIMINARY

Indonesia is the largest archipelagic country in the world. According to data (Badan Pusat Statistik, 2022), Indonesia has more than 17,000 islands and a long coastline. Geographical conditions like these make Indonesia need a reliable communication system to connect islands and remote areas to other regions, both within and outside the country (Tarayana et al., 2021). One effective solution that can be used to overcome these geographical constraints is a submarine cable communication system. This system is also known as a submarine cable network, a cable network located under the sea that is used to transmit data and connect two or more points between islands and countries that cross oceans and seas (Mamatsopoulos et al., 2020). This submarine cable network has a crucial role in global communications. Most data transmitted via the Internet uses a submarine cable network because of its high speed (Nugraha et al., 2019). Besides that, internet users in Indonesia have increased yearly; in 2023, it will reach 215 million people, an increase of 1.17% compared to the previous period (Asosiasi Penyelenggara Jasa Internet Indonesia, 2023).

Communication between islands and remote areas becomes more accessible and faster with submarine cables. In addition, this submarine cable also enables reliable and high-quality Internet and telephone access throughout Indonesia, including in the regions that are difficult to reach by land telecommunication networks. According to (Iswara & Afriansyah, 2022), underwater cables were first introduced in Indonesia in 1990, and since then, submarine cables have continued to develop into critical infrastructure in Indonesian telecommunications. Indonesia already has many submarine cable networks connecting with countries such as Singapore, Malaysia, Australia, the United States and Europe. In Indonesia, many submarine cables have been built, including the Jasuka cable owned by Telkom Indonesia. Jasuka is a submarine cable connecting three major islands in Indonesia: Java, Sumatra and Kalimantan.

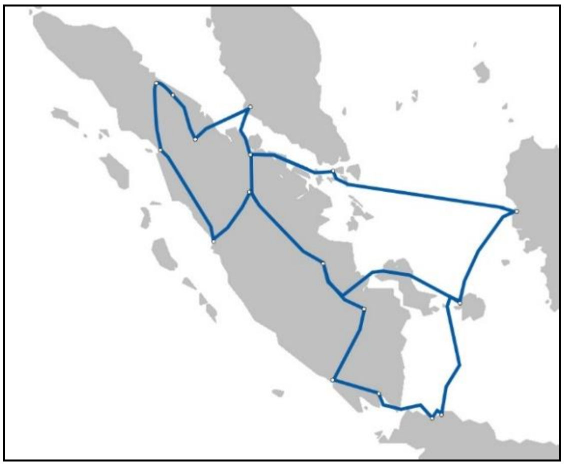

Figure 1. Jasuka Submarine Cable Communication System

However, submarine cable networks are vulnerable to threats like earthquakes, tsunamis, sea storms, and human activities. If one of the cables is cut, data exchange cannot occur (Haryanto et al., 2019), and if there is damage to the submarine cable network, it will require a lot of repair costs (Gasica et al., 2021). Therefore, it is necessary to monitor and maintain submarine cable networks periodically.

The existence of several nodes in the network is essential to maintain connectivity and continuity of network functions. Therefore, identifying these nodes as critical nodes in the network is very important to understand how the network functions and optimize its performance so that prevention and recovery measures can be carried out more effectively and efficiently. One of the methods used to identify influential nodes in this network is to perform critical node detection.

The problem in determining critical node detection is identifying nodes with a crucial role in a network. In this context, a network can refer to various things, such as a biological network (X. Liu et al., 2019), a disaster Network (Hassan et al., 2020), a security network (W. Zhang et al., 2020), network forest landscapes (Yemshanov et al., 2021), social networks (Ajam & Seyedaghaee, 2021), transportation networks (Wang et al., 2021), power grids (Y. Liu et al., 2022), communication Networks (Atat et al., 2022), and so on.

Recently, many search methods have been proposed to identify influential critical points in networks. Such as detecting critical nodes in complex networks with a cascade model with a multi-objective approach (L. Zhang et al., 2020). Determining critical nodes with a heuristic approach genetic algorithm (Alozie et al., 2021). Utilizes centrality measures to identify essential nodes in complex networks (Ugurlu, 2022). Using combinatorial and heuristic optimization techniques detects critical nodes in in-flight networks (H. Zhang et al., 2022). Finding critical nodes by comparing the heuristic and integer linear programming methods results in integer linear programming being more effective than the heuristic method (Faramondi et al., 2018).

In this study, the problem of determining critical nodes will be modelled as integer linear programming (ILP) and then applied to the Jasuka submarine cable communication network and solved using Python software version 3.11. The ILP model produces optimal critical points expected to become a reference for Telkom Indonesia to give special attention to these critical points.

METHODS

The data used in this study came from secondary data obtained from Submarine Cable Map pages. The data is a map of the Jasuka submarine cable communication network owned by Telkom Indonesia. The landing point on the Jasuka submarine cable network is represented as a node and connected with an optical fibre cable that is presented as an edge and then transformed into a graph shape. The stages of this study are as follows:

1. Describe the problem of the Jasuka submarine cable communication system.
2. Formulate critical node detection mathematical models in the form of Integer Linear Programming. According to (Winston, 2014), ILP is a linear programming problem that allows some or all decision variables to be integer values. In this problem, the Jasuka submarine cable communication system, the decision variable is 0-1.

3. Perform compatibility testing of the critical node detection mathematical model with several test cases whose solutions are known using the help of Python software.
4. Apply the critical node detection mathematical model to the Java Sumatra Kalimantan submarine cable communication system. These steps can be seen in Figure 2.

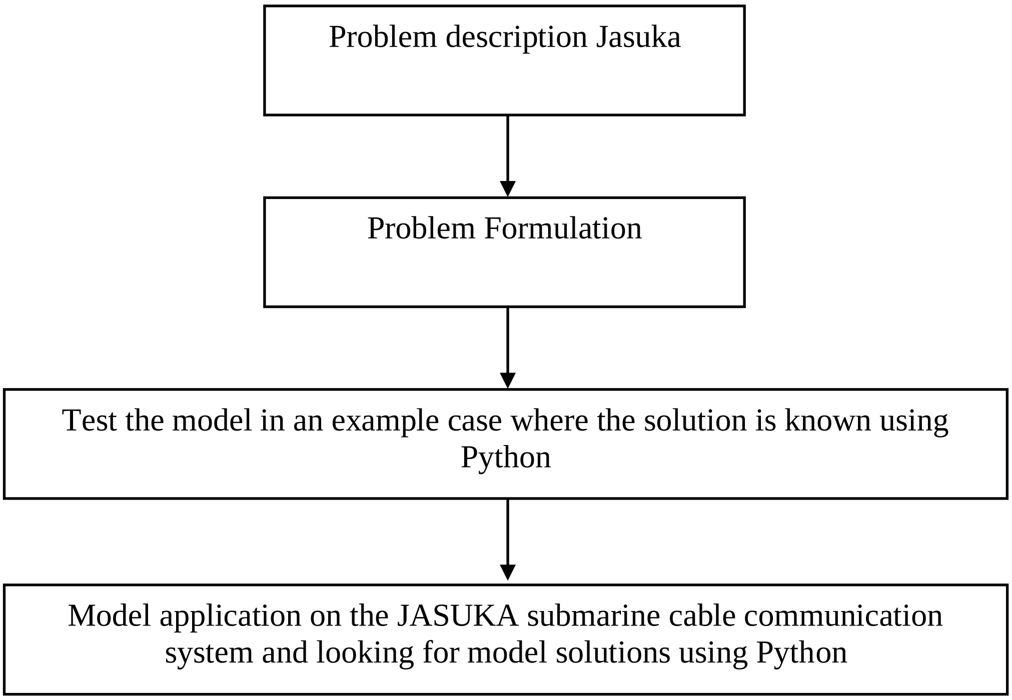

Figure 2. Research Procedure Chart

RESULT AND DISCUSSION

Problem Description of the Jasuka Cable Communication System

After experiencing several problems, most recently, on 21 September 2021, the Telkomsel and Indihome networks experienced disruptions caused by a disturbance in the Jasuka submarine cable system. This disturbance causes the flow of communication to be not smooth. To minimize disruption to the entire Jasuka system, Telkom Indonesia needs to anticipate by maintaining critical nodes so that damage does not have too much impact on the whole network. A mathematical model is formed to determine critical nodes in the network. Critical node detection is based on the landing point that connects the network. Therefore, the mathematical model constructed aims to identify critical nodes by minimizing the number of paired connections on network nodes that significantly influence the network.

Suppose there is a submarine cable communication network. The network has several submarine cable landing points, represented as nodes. Each landing point will be connected to other landing points via fibre optic cables represented as edges. Thus, the submarine cable communication network, which consists of a collection of nodes and edges, will form a graph. The following presumptions are used to restrict the challenge of identifying critical nodes in the Jasuka network:

1. The landing point is predetermined
2. There is one additional node between the nodes on the Palembang landing point and Jambi landing points

Determining critical nodes in the Jasuka network using the critical node detection method will be formed into integer linear programming to identify the most crucial landing points in the Jasuka network. Critical node detection on the Jasuka network aims to determine critical nodes that can be used as a reference for PT. Telkom Indonesia is giving special attention to the landing point to minimize disruption to the entire Jasuka network.

Problem Formulation

Based on the problem already described, it is possible to formulate the problem of critical node detection on the Jasuka submarine cable communication system as an ILP. Here is the formulation of the problem:

Index

i, j = landing point; $i, j = 1,2, \dots, P$

Table 1. Landing Point Index

i	**Landing point**
1	Medan, Sumatera Utara
2	Tebing Tinggi, Sumatera Utara
3	Rantau Prapat, Sumatera Utara
4	Bandar Bukit Tinggi, Malaysia
5	Dumai, Riau
6	Batam, Kepulauan Riau
7	Pontianak, Kalimantan Barat
8	Tanjung Pandan, Bangka Belitung
9	Tanjung Pakis, Karawang, Jawa Barat
10	Jakarta
11	Bandar Lampung, Lampung
12	Baturaja
13	Pelembang, Sumatera Selatan

i	**Landing point**
14	Palembang-Jambi
15	Jambi
16	Pekanbaru, Riau
17	Padang, Sumatera Barat
18	Sibolga, Sumatera Utara

Set

$\mathcal{V}$ = set of nodes in the network

$\mathcal{E}$ = set of edges in the network

Parameter

B = the number of critical nodes to be identified

Decision Variable

$$v_i = \begin{cases} 1; & \text{If node } i \text{ is deleted in the optimal solution} \\ 0; & \text{otherwise} \end{cases}$$

$$u_{ij} = \begin{cases} 1; & \text{If node } i \text{ and } j \text{ are in the same component} \\ 0; & \text{otherwise} \end{cases}$$

Objective functions

The objective function of this problem is to minimize the number of node-pairing connections, which is used to identify the critical nodes that play an essential role in the network.

$$Min \sum_{i,j \in \mathcal{V}} u_{ij}$$

Constraints

The constraints used in this problem are as follows:

1. Ensure that if an edge connects two nodes and both nodes are not critical, the two must be on the same component.

$$u_{ij} + v_i + v_j \geq 1 \qquad (i,j) \in \mathcal{E}$$

2. Triangle inequality, ensuring there is connectivity on the graph (If node i is connected to node j and node j is connected to the node l, then node i must also be connected to node l)

$$u_{ij} + u_{jl} - u_{il} \leq 1 \qquad i,j,l \in \mathcal{V}$$

$u_{ij} - u_{jl} + u_{il} \leq 1 \qquad i, j, l \in \mathcal{V}$

$-u_{ij} + u_{jl} + u_{il} \leq 1 \qquad i, j, l \in \mathcal{V}$

3. Critical Node Limits

$$\sum_{i, \in \mathcal{V}} v_i \leq B$$

4. Binary Variables

$v_i \in \{0,1\} \qquad i \in \mathcal{V}$

$u_{ij} \in \{0,1\} \qquad i, j \in \mathcal{V}$

Test Models

A model test is carried out to ensure that the formulation of the mathematical problem above is fulfilled. Test the model using a simple network example where the solution or critical point is known. Each network has nodes which are landing points and edges connecting landing points. Critical nodes significantly influence the connections between other nodes in the network. If the node is removed from the network, the connection between the nodes will be lost. The network is tested to test the model's suitability with a parameter value of B=1.

The test will be performed on a star network. Figure 3 shows a network of star network graphs whose edges are connected only to one node. In this case, because the network is relatively small, the critical node is located at node one because all the edges are connected to node 1.

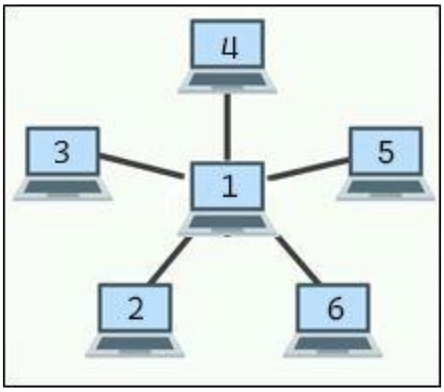

Figure 3. Star Network

Furthermore, Figure 3 will be tested using Python software.

1. Enter data, data in the form of index nodes connected from the star network in Figure 3, and node i is connected to node j.

Table 2. Data Input Star Network

i	j
1	2
1	3
1	4
1	5
1	6

2. Enter the command to display the resulting graph from the input data. The output results can be seen in Figure 4.

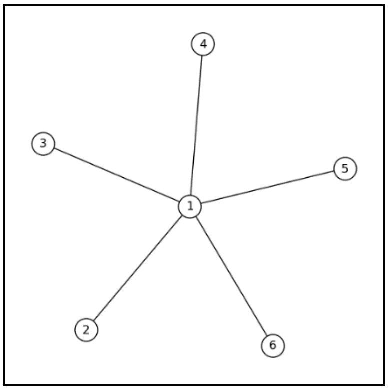

Figure 4. Star Network Graph

3. Then a search for critical nodes is carried out using the Python programming language, and the results are represented in Figure 5. From this figure, node one is selected as a critical node. It can be seen if node one is used as a critical node.

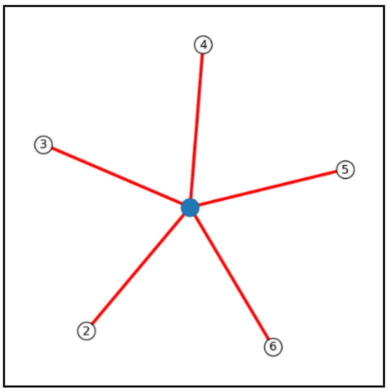

Figure 5. Critical Node in Star Network

Because the results obtained in the model test with examples of cases where solutions are known using Python, the results are the same as available solutions. The model and program are undoubtedly correct. Therefore, the following model can be applied to the Jasuka cable sea communication system.

Model Applications on Java Sumatra Kalimantan Submarine Cable Communication System

Based on data from the Jasuka in Table 1 and the formulation of the problem described then, the model will be applied to the Jasuka submarine cable communication system using Python software. A graph is obtained, which can be seen in Figure 6.

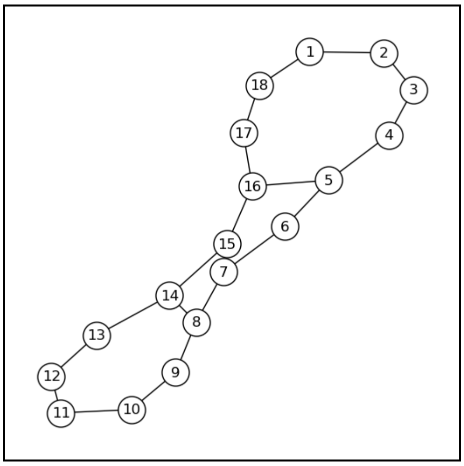

Figure 6. Jasuka Submarine Communication Network Graph

Then a search for critical nodes in the Jasuka communication network was carried out using the Python programming language as in the model test, and the results are represented in Figure 7. With B=1, the network is still connected as one component.

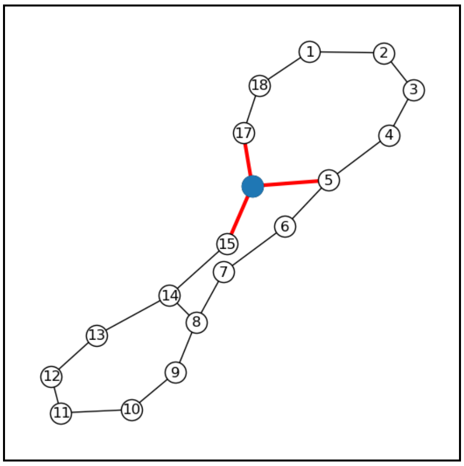

Figure 7. Critical Node if B=1 in Jasuka Submarine Communication Network

Then a search for critical nodes was carried out by changing the parameter B = 2. The critical nodes were found at indexes 5 and 14, landing points in Dumai, Riau, and the Palembang-Jambi Section in Figure 8. The two critical nodes divide the network into two components.

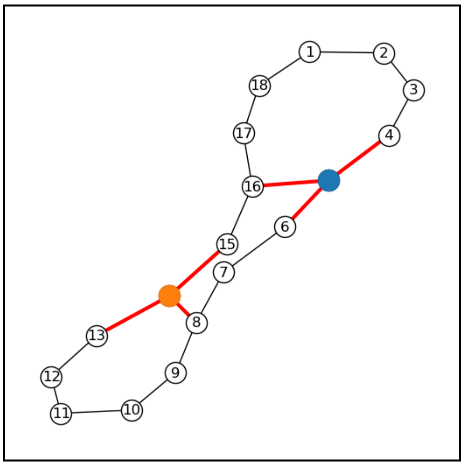

Figure 8. Critical Node in Jasuka Submarine Communication Network

CONCLUSION

According to the findings of this study, it would be possible to determine the critical nodes in submarine communication systems by creating mathematical models in the form of linear integer programming. With this model, the critical nodes on the network can be identified so that they can be kept connected and the network remains connected. The model application results on the Jasuka submarine cable communication network showed that the critical node is located on index number 5 and 14, which means that the critical node on the Jasuka network is located at the landing point Dumai, Riau and Palembang-Jambi. Therefore, protection should be provided to such critical nodes to ensure the communication system continues functioning correctly and efficiently.

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