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ANALOGICAL REASONING IN SOLVING INDIRECT PROBLEM-BASED AREA PROBLEMS

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ABSTRACT
This study aims to identify the stages of students' analogical reasoning in solving indirect problems. The type of research used is qualitative with a case study approach. The participants in this study were 25 fifth-grade students who selected research subjects using purposive techniques that represent each analogical reasoning category based on the analogical reasoning test. Data collection techniques used indirect problem tests and interviews. Data analysis techniques included data reduction, presentation, and verification. The results showed that there were two categories of students' analogical reasoning when solving indirect analogy problems: (1) Indirect problem succeeded, students who successfully solve source problems and target problems by using all stages of analogical reasoning, namely representation and mathematical modeling, structuring, mapping, applying, verifying; (2) Indirect problem failed, students who did not successfully solve source problems and target problems by using all stages of analogical reasoning. The results of this study suggest that educators, especially at the elementary level, should explore more deeply the students' ability to solve indirect analogical problems with analogical reasoning.
Keywords: Analogical Reasoning, Indirect Problem, Mathematical Area-Based Problems

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PRELIMINARY

Analogical reasoning is crucial in learning mathematics (Duit, 1991; Gentner, 1982; Gentner et al., 2001; Holyoak et al., 2004; Magdaş, 2015). Analogical reasoning allows one to solve problems that are unclear, complex, and new (Putri & Masriyah, 2022). When a problem at first glance seems complicated, by analogy, the problem becomes easily known and easier to solve. This shows that analogical reasoning benefits solving difficult and abstract mathematical problems.

Analogical reasoning also relates to the ability to describe an abstract concept to be concrete and draw conclusions based on the similarity of processes or data (Fauzi et al., 2020). Students can use analogical reasoning on patterns and properties to change mathematics so that evidence is clearly organized to explain an idea and systematic statements (Masfufah & Afriansyah, 2022).

Analogical reasoning is crucial in solving problems (Mutia et al., 2022). Reasoning skills are essential, can support the understanding of fundamental concepts in mathematics, and are interrelated between one concept and another (Rahma et al., 2023). One of the things related to learning mathematics with analogical reasoning will make it easier for students to solve problems by looking at the similarities of problems that have been solved before. If the analogical reasoning skills of students are good enough, then students will be accustomed to solving more detailed problems by looking at the similarities of the problems (Fatra & Angraini, 2020).

Related problem-solving using analogical reasoning involves an analogy component. Some researchers discuss the analogical reasoning component in solving mathematical problems (Putri et al., 2019). This is very important, so they need to be trained continuously to solve problems that they will face in everyday life and learning mathematics (Nurrochmatunnisa, 2020).

Area calculation using points is a concept in calculating the area of a field not based on length but on a point. In general, this calculation will be more straightforward than the existing one, so the calculation can also be generated entirely in the calculation (Utomo, 2019). The problem of calculating area problems is often taken for granted, especially on a plane figure, as long as it is based on the length dimension of the plane figure (Utomo, 2021). The flat area, for example, is a rectangle, so when calculating the area of a rectangle, the sum of the side lengths of the rectangle is required. If the rectangle has dimensions of length $p = a\ cm$ and the width $l = b\ cm$ then the area of the rectangle is expressed in area $L = p \times l = ab\ cm2$. Information on the length dimension that is used as the basis for calculating area, the length in question is the size of a line segment.

Indirect problems are those based on information processing theory (Kristayulita & Sucipto, 2022). When students are given an indirect problem test, the indirect problem test is seen as an external stimulus. The analogy problems students face are in the form of target problems that enter the sensory register. If the target problem does not get students' attention, students will forget it. If the target problem gets the student's attention, it will be transferred to the second stage of the memory system, short-term memory. In the second stage of the memory system, students, in solving the target problems they face, require information in the form of schemas stored in long-term memory in the form of solving source problems by retrieving information in the form of schemas that are stored in long-term memory and have similarities with the target problem at hand.

The problems faced in solving this indirect analogy problem require students to think critically and reason to find the same and related answers. The nature of certain concepts is to make comparisons (Kristayulita et al., 2019). This is called analogical reasoning, and analogical reasoning is used not only in learning but also in everyday life. Therefore, learning this analogy is crucial to forming a responsive attitude and solving problems (Santika & Sudiana, 2021). Analogical reasoning can solve problems that are clear, unclear, new, and very complex (Putri & Masriyah, 2022). This reasoning can be called a process to get a conclusion by using the similarity of structured properties and relationships between target problems and source problems.

Based on previous research, many students still experience problems in analogical reasoning. Analogical reasoning is an ability that students learning mathematics can master, and students should use analogical reasoning skills to solve problems, including mathematical problems (Ratnaningsih et al., 2022). Several studies discuss students' difficulties in solving indirect analogy problems. One often encountered problem is that students cannot solve indirect problems in learning and everyday life (Kristayulita et al., 2019; Kristayulita, 2018). The difficulties that students often experience when solving indirect analogy problems can be overcome by using analogical reasoning to make it easier to solve them (Kristayulita, 2021).

A more in-depth identification of analogical reasoning stages in solving problems is needed. Several researchers have conducted analogical reasoning research on indirect analogy problem-solving (Kristayulita et al., 2019; Kristayulita & Sucipto, 2022). Previous research focused analogical reasoning on indirect analogy problems but not on elementary school students and area problems (Kristayulita et al., 2019; Kristayulita & Sucipto, 2022). This research focuses on the stages of indirect analogical reasoning and problem areas in elementary schools. This study aimed to identify the stages of students' analogical reasoning in solving indirect analogy problems.

METHODS

This research method uses qualitative with a case study approach. Qualitative methods involve detailed descriptions of settings or individuals and are followed by data analysis for a theme or problem (Creswell, 2017). The participants of this study were 25 fifth-grade students of SDN Sidokepung 2 Sidoarjo, East Java, Indonesia. Research subjects were selected using a purposive technique, with 3 students selected as research

subjects. The purposive criterion in this study is to see the results of students' indirect problem test answers that can represent each category.

Based on the results of previous research, there are four components of analogical reasoning (Ruppert, 2013): structuring, mapping, applying, and verifying. In addition to Ruppert's four stages (Kristayulita et al., 2019), indirect analogical reasoning was developed into five stages: representation and mathematical modeling, structuring, mapping, applying, and verifying (see Table 1).

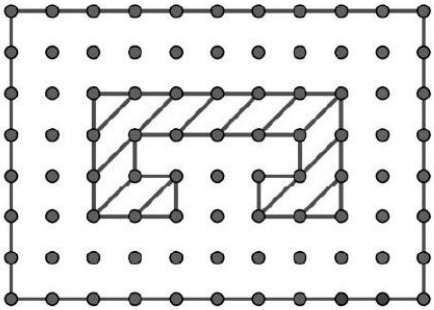
Table 1. Stages of Analogical Reasoning Indirect Problems

Components	Descriptors
Representation and mathematical modeling	Students complete the analogical reasoning test using the formula coherently at this stage. Students can use symbolic representation, with mathematical modeling referred to as a behavioral representation of real objects with the completeness of the terms used in mathematics.
Structuring	Students can use the same steps as solving source problems, so target problems are similar to source problems.
Mapping	Students perform a mapping from target problems to source problems.
Applying	Students have done from target problems to source problems. Students will recognize the similarity of a concept between target problems and source problems. Students also use the same processes and procedures to solve source problems and target problems.
Verifying	Students carry out the process of justifying and responding to a solution to source and target problems.

The instruments used were indirect problem tests and interview guidelines. The indirect problem test was in the form of 1 source problem and 1 target problem on the problem area material. The indirect problem area problems test adapts from (Clarke & Roche, 2018) by changing the pattern of the sides of the flat shape and modifying the material adapted to the area problem material in grade five elementary school (see Figure 1).

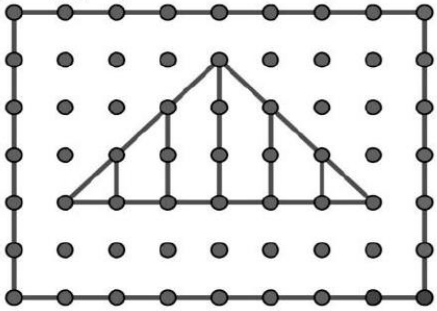
Masalah Sumber

Tentukan luas bangun yang diarsir pada bidang persegi yang berukuran 1 cm × 1 cm.



Masalah Target

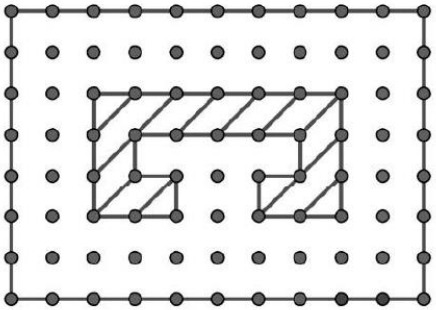
Dengan menggunakan cara yang mirip dengan masalah sebelumnya tentukan luas bangun yang diarsir pada bidang persegi yang berukuran 1 cm × 1 cm.



In English

Source Problem

Find the area of the shape shaded by the 1 cm × 1 cm square



Target Problem

Using a similar method to the previous problem, determine the area of the shaded area of the 1 cm × 1 cm square

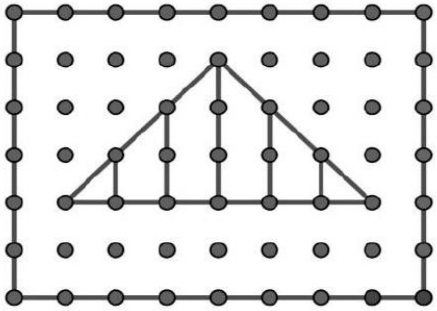


Figure 1. The Indirect Problem Test Instrument

The data collection techniques used are: 1) Test, the test used is the indirect problem test. The purpose of giving the indirect problem test is to identify the stages of students' analogical reasoning in solving indirect problems; 2) Interviews, interviews were conducted on selected research subjects to explore more in-depth information about students' analogical reasoning when solving indirect problems, in this case, there were three research subjects interviewed.

Data analysis techniques in this study were carried out by collecting, reducing, making conclusions, or verifying data. The steps of data collection are: (1) document the results of student answers; (2) reduce data by making abstractions; (3) make coding of each thinking process carried out by students; (4) make an analysis of what happened during the research; and (5) draw conclusions.

RESULT AND DISCUSSION

Referring to theory (Kristayulita et al., 2019), students' indirect problem tests identified two findings of students' analogical reasoning categories when solving indirect problems. The findings of the two categories in this study are presented in Table 2.

Table 2. Classification Results of Analogical Reasoning Indirect Problem

Category analogical reasoning indirect problem	n	Subjects
Indirect problem succeeded	1	S1
Indirect problem failed	24	S2 and S3

Description:
n = the number of students in each category
S1 - S3 = Research subjects in each category

Table 2 shows that there is 1 student who falls into the indirect problem succeeded category, and 24 students fall into the indirect problem failed category. Based on this classification, the indirect problem succeeded category is only represented by 1 subject because the results show that there is only 1 student who managed to solve the indirect problem perfectly, namely subject 1 (S1), and the indirect problem failed category is represented by subject 2 (S2) and subject 3 (S3).

Indirect Problem Succeeded

In the indirect problem succeeded category, students solved indirect analogy problems consisting of source and target problems. This shows that students successfully solve indirect analogy problems by using five stages of indirect analogical reasoning: representation and mathematical modeling, structuring, mapping, applying, and verifying. In this category, there was only 1 student who managed to finish well. The indirect problem test results presented from the results of subject 1 (S1) are shown in Figure 2.

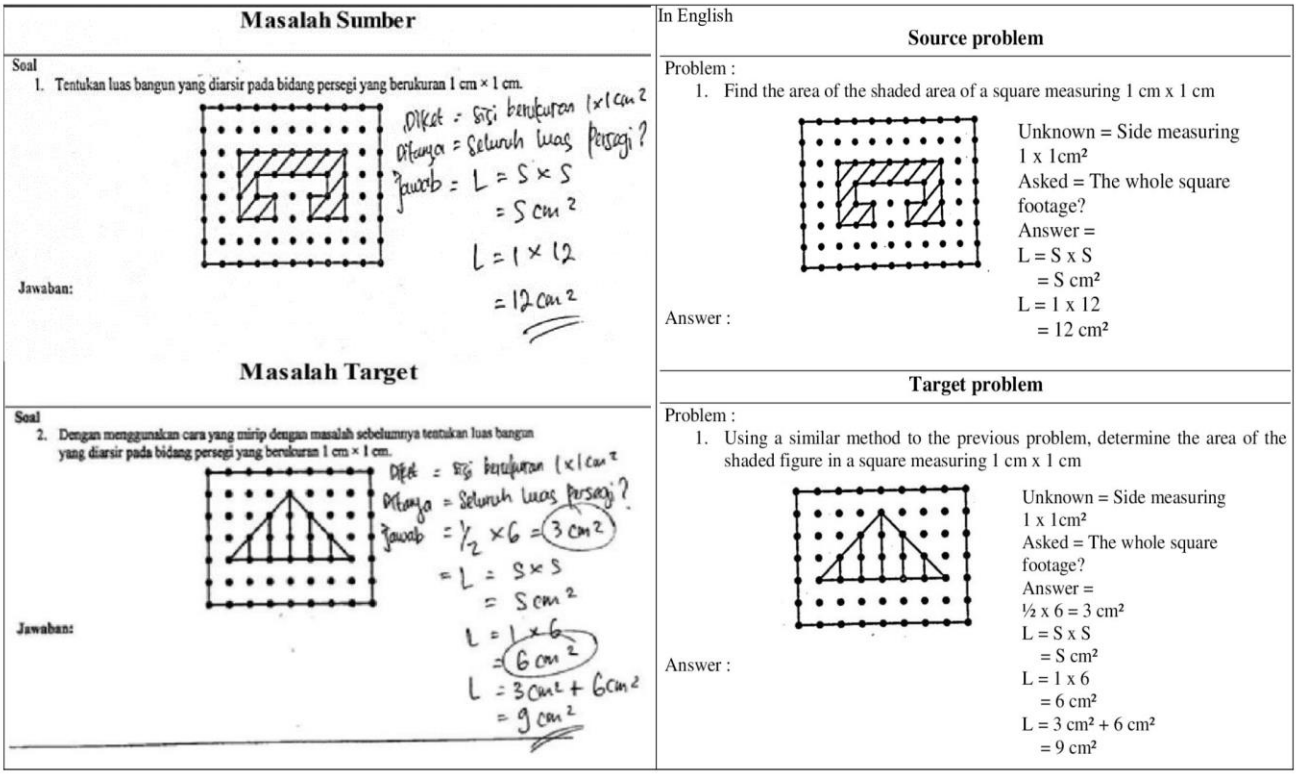


Figure 2. Results of Indirect Problem Answer by S1

The answer results in Figure 2 show the process of analogical stages in solving indirect analogy problems can be explained in the following stages of analogical reasoning.

Representation and Mathematical Modeling

Students can solve the indirect problem test consisting of source and target problems at this stage. The indirect problem test can be solved correctly and using the formula coherently by students. Source problems are solved correctly by students, so students have an idea to solve the target problems even though the target problems differ slightly from the source problems. Students use symbolic representations from L = s x s = s^2 to L = s^2. Then, the mathematical modeling can be called a behavioral representation of real objects with its completeness in the terms used in mathematics.

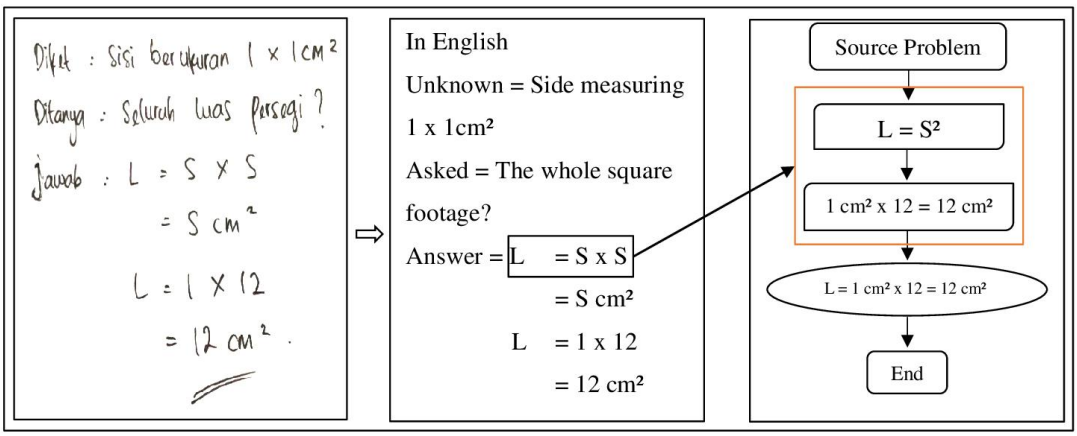


Figure 3. Representation and Mathematical Modeling Stage

Structuring

At this stage, students can perform mathematical models on target problems, as seen in Figure 4. Students start by identifying a new mathematical model that has provided a statement that a mathematical model has been obtained with several formulas. Solving target problems uses the same steps as solving source problems.

The structuring stage is similar in meaning to retrieval and access. Retrieval is a reminder of some topic in the working storage, which makes one recall a previous analogous situation in the long-term storage. Access is retrieving a familiar analogy back to the source (schema or rule) from storage for the target problem. This stage is a process of identifying problems in the form of mathematical objects in source problems with characteristics with a target problem to conclude a very identical relationship between source problems and target problems, as Kristayulita et al. (2019).

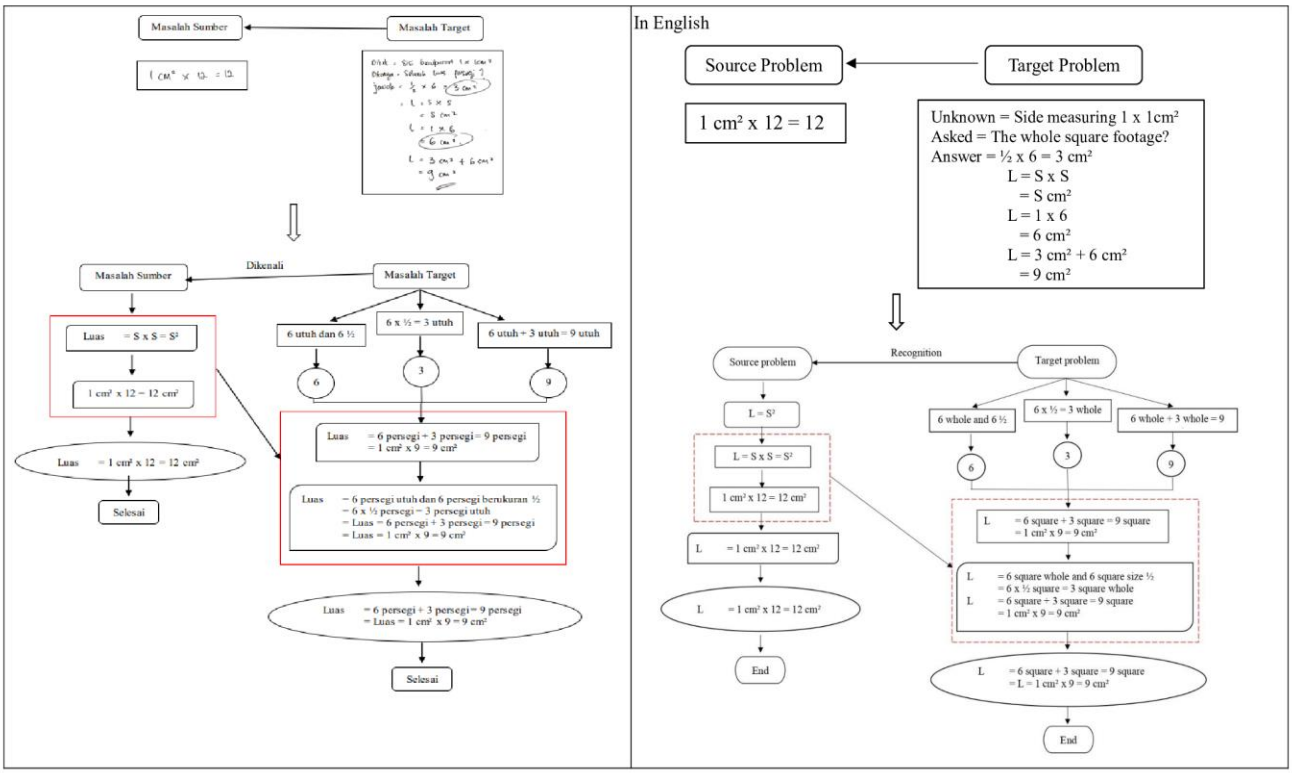


Figure 4. Structuring Stage

Mapping

At this stage, students do a mapping of target problems and source problems. Students have successfully identified the same things between source problems and target problems, such as making conclusions from source problems and target problems are problems looking for area problems, while the solution method for both problems is the same, namely by using the square area formula (L = S x S). Students systematically

connect one-to-one between source problems and target problems. Students perform excellent analogical reasoning, can see a similarity on the dominant surface, and are considered prominent for recalling information.

Applying

At the applying stage, S1 could do the applying stage correctly and precisely (see Figure 5). S1 used the same process and solution procedure in the source problem to solve the target problem. S1 solved the source problem by using the solution procedure L = S x S = S^2, L = 1 x 12 = 12 cm^2 and managed to find the correct answer, which is 12 cm^2. S1 solved the target problem by using the same initial solution procedure when solving the source problem. This is L = S x S = S^2, 1 x 6 = 6 cm, then continued calculating the next completion step until he found the correct answer, 9 cm^2.

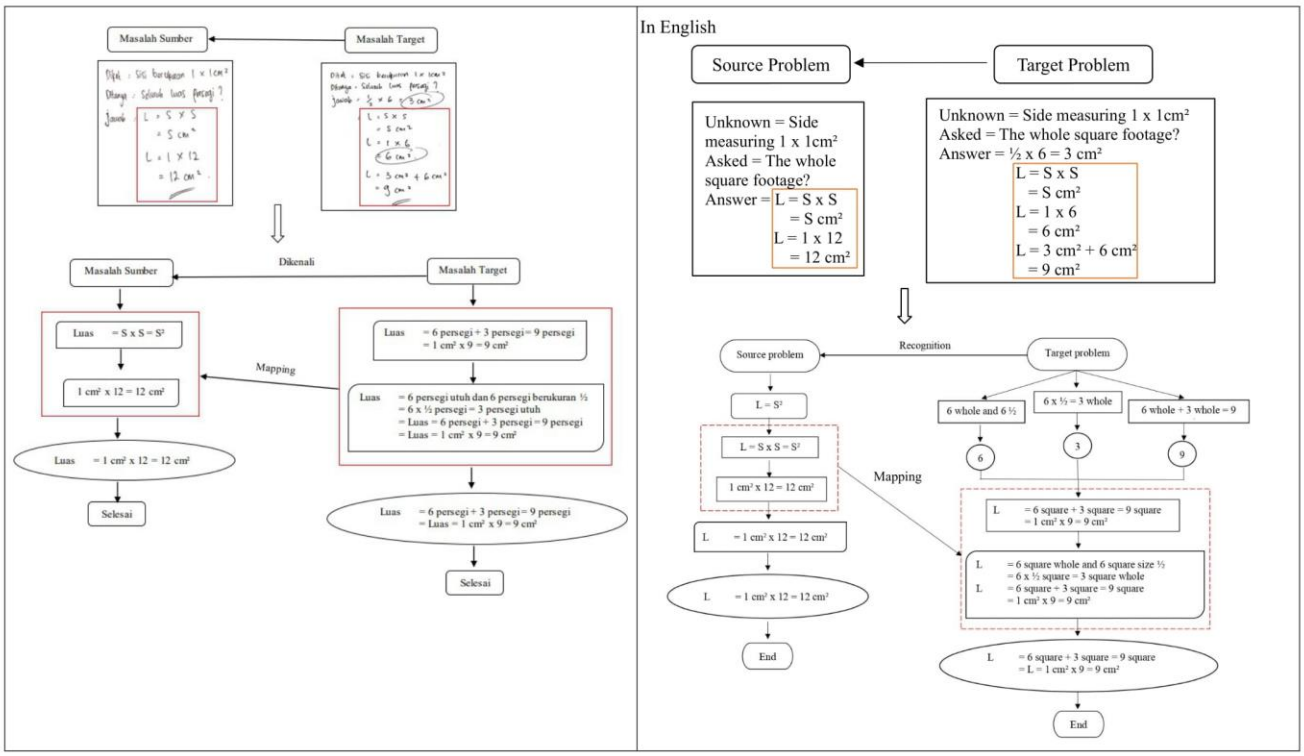


Figure 5. Applying Stage

Verifying

At this stage, students can find the final answer to the solution of source and target problems (see Figure 6). This stage in an analogical reasoning process is when students justify and respond to a solution to source and target problems. In this verifying stage, justification and response occur. At this stage, there is a process of sending or transferring in analogical reasoning, which has described a process of forming a conclusion about the target problem based on mapping this source problem. The process of mapping and

sending, which has repeatedly consisted of mapping similarities, sending information, checking similarities, and identifying components that are not the same, all of which have been limited by a context and purpose, this transferring process makes students able to send new information that was previously unknown (Pratitisari, 2021).

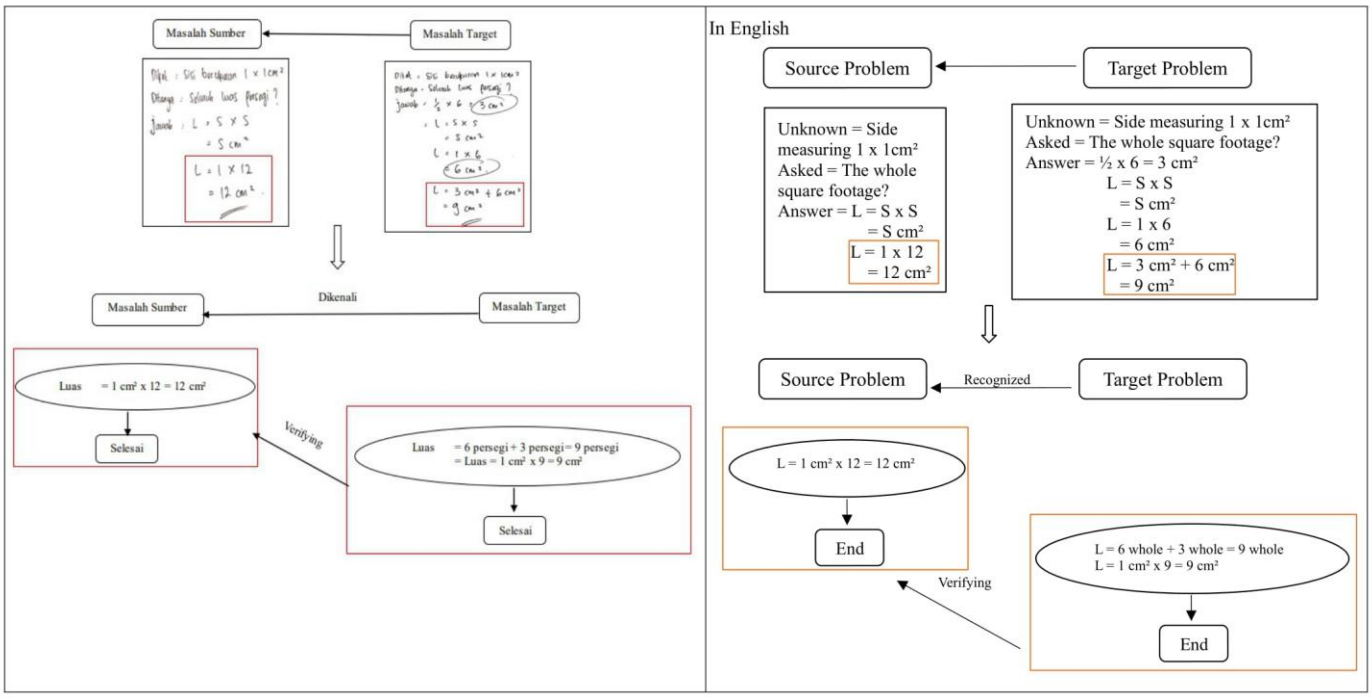
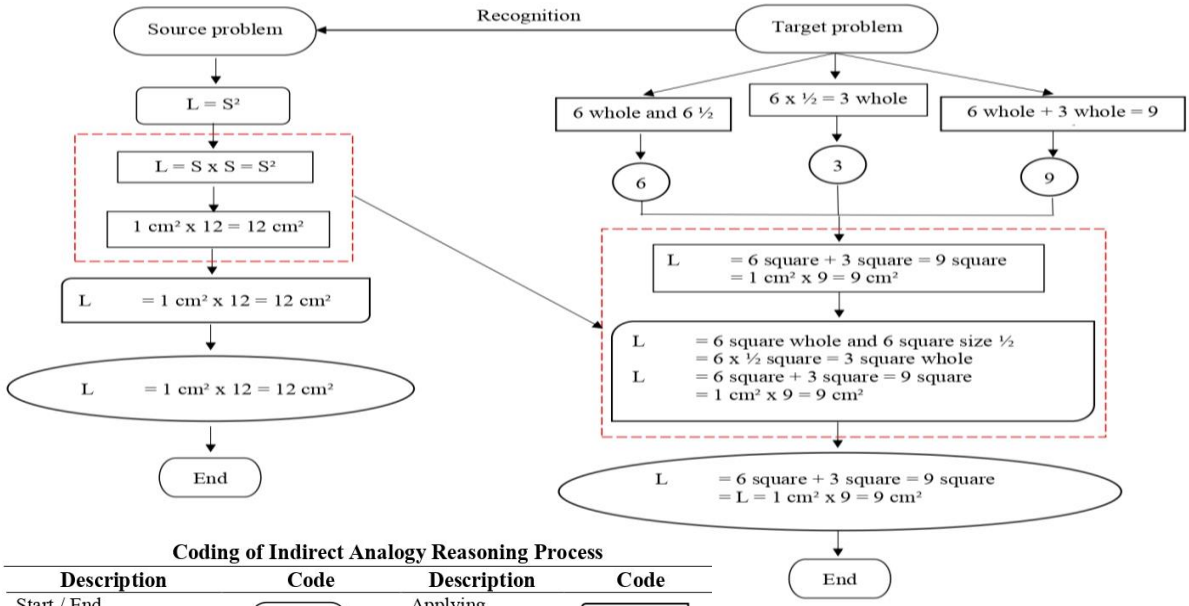


Figure 6. Verifying Stage



Coding of Indirect Analogy Reasoning Process

Description	Code	Description	Code
Start / End		Applying	
Representation and mathematical modeling		Verifying	
Structuring		Process activity	
Mapping			

Figure 7. Process Stages of Analogical Reasoning Indirect Problem Succeed by S1

Indirect Problem Failed

In the indirect problem failed category, students did not correctly solve indirect analogy problems consisting of source and target problems. This shows that students fail to solve indirect analogy problems using the five stages of indirect analogical reasoning: representation and mathematical modeling, structuring, mapping, applying, and verifying. In this category, 24 students did not manage to solve well and only indirectly brought up 2 to 3 stages of analogical reasoning. The indirect problem test results presented by subject 2 (S2) results are shown in Figure 8; meanwhile, the results of subject 3 (S3) are shown in Figure 9.

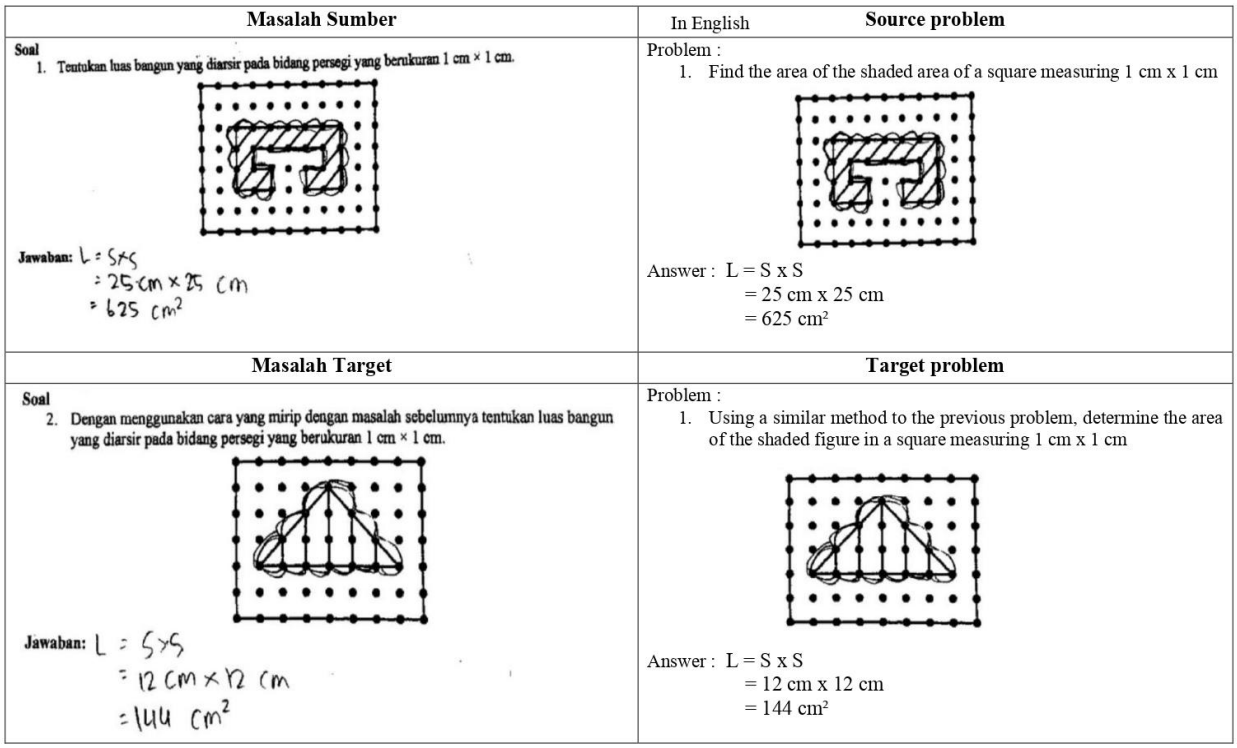


Figure 8. Result of Indirect Problem Answer by S2

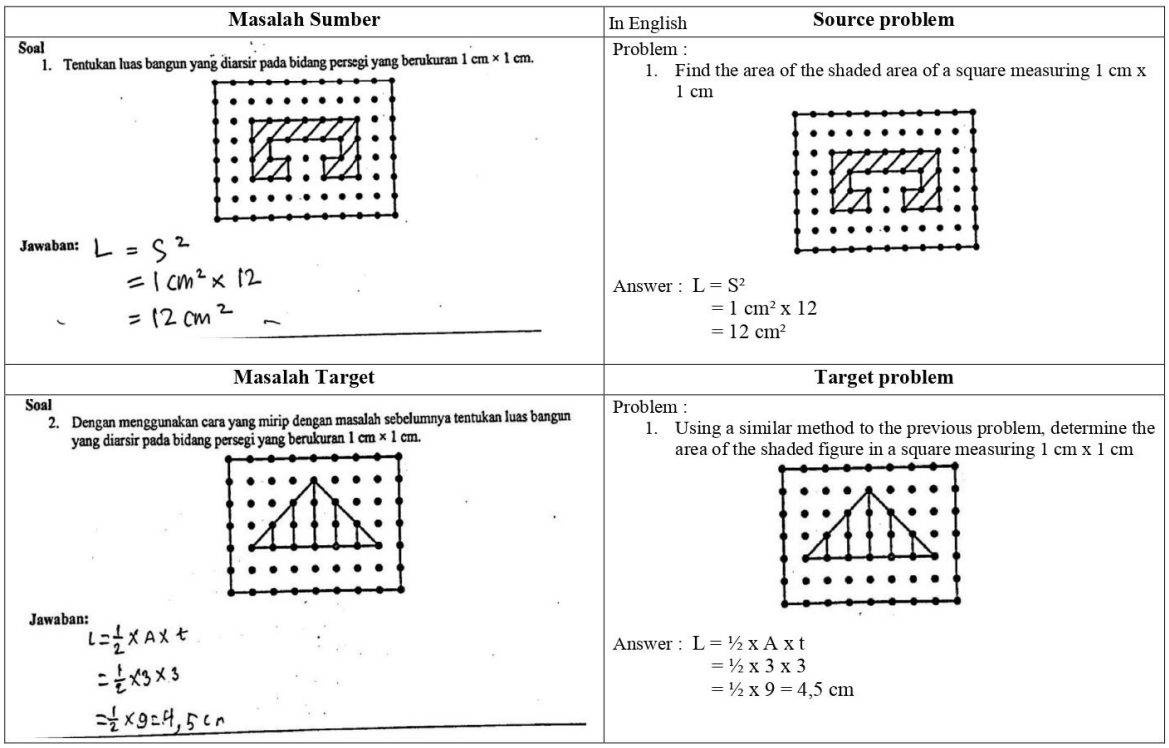


Figure 9. Results of Indirect Problem Answers by S3

Based on Figure 8 and Figure 9, S2 and S3 were unsuccessful in the analogical reasoning process because they did not bring up the five stages of the analogy process in solving indirect analogy problems, namely representation and mathematical modeling, structuring, mapping, applying, and verifying. At the representation and mathematical modeling stage, S2 and S3 should solve using the formula coherently and symbolic representation, but S2 and S3 cannot solve analogy problems. At the **structuring** stage, S2 and S3 cannot answer the source problems correctly. This makes the target problems not resolved correctly. At the **mapping** stage, S2 and S3 should map from target problems to source problems, but students cannot identify them because the source problems are not solved correctly by S2 and S3. At the **applying** stage, S2 and S3 did not use the same process and solution procedures in the source problems to solve the target problems. This is because S2 and S3 experienced errors in solving the source problems using the area formula. The correct formula should be L = S x S; they could not solve the source problems correctly. This also happened when S2 and S3 solved the target problems, experienced errors in using formulas, and could not solve the target problems correctly. At the **verifying** stage, S2 and S3 could not find the correct final answer. At this stage, there is a process of justifying and responding to a solution to the source and target problems. Still, the source and target problems are not resolved correctly, so S2 and S3 carry out no verifying stage.

CONCLUSION

Based on the results of this study, it can be concluded that there are two findings of students' analogical reasoning categories when solving indirect analogy problems, namely: (1) Indirect problem succeeded, students successfully solve source problems and target problems by using all stages of analogical reasoning, namely: representation and mathematical modeling, structuring, mapping, applying, verifying; (2) Indirect problem failed, students did not succeed in solving source problems and target problems by using all stages of analogical reasoning. This proves that students' analogical reasoning can be stimulated by using indirect analogy problems in solving target problems.

These findings provide implications for future research. Practitioners in psychology and mathematics can be used as guidelines in identifying students' analogical reasoning using indirect problems. Educators at the elementary level should deepen students' problem-solving skills first so that students can solve indirect problems appropriately. Indirect problems can be solved using analogical reasoning. Educators need to facilitate problem-based learning that can construct the knowledge they already have to be used as a solution in solving new problems.

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