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COMPARATIVE ANALYSIS OF NUMERICAL INTEGRATION SOLUTIONS QUADRATURE METHOD AND NEWTON COTES METHOD USING PYTHON PROGRAMMING LANGUAGE

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ABSTRACT
Irregular areas cannot be solved by ordinary calculus formulas, so it is necessary to use numerical methods such as the Quadrature and Newton-Cotes methods. This research compares numerical integration solutions using the Quadrature method (Rectangular and Trapezoidal) and the Newton-Cotes method (Trapezoidal, Simpson 1/3, Simpson 3/8, and Weddle) with the Python programming language. Manual calculation of the first case study on integrals where the smallest error from the numerical method to the analytical method is achieved by the rectangular method of 0,017. In the second case study of tabular data for manual calculations the author only uses the Simpson 3/8 method with an absolute error for an analytical area of 1.418,583 km^2. Whereas in the Python application for the first case study the smallest error was achieved by the Simpson 1/3 and Simpson 3/8 methods with an error of 0, in the sense that these two methods are very accurate to the actual analysis results. In the second case study the smallest error was achieved by the Simpson 1/3 method of 1.039,365 km^2. The difference between manual calculations and application results is due to decimal rounding, and the linspace(a,b,n+1) function in the numpy library.
Keywords: Integral, Quadrature, Newton-Cotes, Analytical, Python

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PRELIMINARY

Mulyani (2020) analyzed the accuracy of the area of the Kalibaru Ciliwung river basin using the trapezoidal and Simpson 1/3 methods with the results of the area calculated using the Simpson 1/3 method being closer to the actual value than the calculated area using the Trapezoid method. Dhali, et al., (2019) compared the trapezoidal, Simpson 1/3 and Simpson 3/8 methods for unequal data spaces with Simpson 1/3's results better results than other numerical methods. Ermawati, et al., (2019) implemented the Simpson 1/3 method for the Hankel transformation with the result that the greater the value of the n points used, the better the approximation of the heat transferred. Karpagam & Vijayalakshmi (2018) conducted a study by comparing the results of the trapezoidal, Simpson 1/3, Simpson 3/8 and weddle methods with the conclusion that the Weddle method was more accurate than the other methods. Perbani & Rinaldy (2018) apply numerical methods to calculate the volume of ships and land topography with the result that the volume of ships calculated using Simpson's rule can represent the actual volume so that it can be applied to irregular surface areas such as to calculate the volume of land or seabed topography. The shape of the ship is taken as a half ellipsoid so that it can be compared with the actual one. Volume is calculated by applying Simpson's First Rule after the gridding process. Pandu (2019) simulated the area of irregular regions with numerical integration using the rectangular rules. Arif, et al., (2017) applying the newton-cotes method to solve systems of nonlinear equations.

Several studies have analyzed numerical integration with programming languages or applications such as using the Maple application by Qani (2022), using Matlab by Maure & Mungkasi (2021), Oktamuliani & Samsidar (2015), Tentua (2016) and Nopriani, et al., (2021). Using the Wolfram Mathematica application by Warsito & Haning (2018). Using the Visual Basic.Net programming language by Dharshinni, et al., (2020). Using the Python programming language by Suherly & Shiddiq (2020). Using the programming language C # (C-Sharp) by Yahya, et al., (2019). Using the Pascal programming language by Anggur, et al., (2019), and Herfina, et al., (2019).

From some of these studies with case study analysis and comparison of methods and utilizing applications or programming languages. So the authors have the objectives of this study, namely :

1. Analyze the comparison of numerical integration solutions using the Quadrature method and the Newton Cotes method
2. Create an application with the Python programming language

METHODS

The following table looks for solutions for solving integrals using analytical methods and numerical methods used in this study based on the book Numerical Methods by Munir (2021).

Table 1. Formulas and Terms of Analytical Methods and Numerical Methods

		Analytical Method	
1.	**Method Name**	**:**	**Integral Absolute**
	Term	:	There is a start and end limit. To be compared by numerical methods

		Analytical Method	
	Formula	:	$\int_a^b f(x)dx = F(x)\big\|_a^b = F(b) - F(a)$
		Quadrature Method	
2.	**Method Name**	**:**	**Rectangular Method**
	Term	:	The number of partitions (n) is a real number
	Formula	:	$\int_a^b f(x)dx \approx \frac{h}{2}\left(f_0 + 2\left(f_1 + f_2 + f_{n-1}\right) + f_n\right)$
3.	**Method Name**	**:**	**Trapezoidal Method**
	Term	:	The number of partitions (n) is a real number
	Formula	:	Same as the rectangular method formula. Different geometric shapes.
4.	**Method Name**	**:**	**Midpoint Method**
	Term	:	The number of partitions (n) is a real number
	Formula	:	$\int_a^b f(x)dx \approx h\left(f_{1/2} + f_{3/2} + f_{5/2} + f_{7/2} + \ldots + f_{n-1/2}\right)$
		Newton-Cotes Method	
5.	**Method Name**	**:**	**Trapezoidal Method**
	Term	:	The number of partitions (n) is a real number
	Formula	:	Same as rectangular formula. The origin of the formula is different.
6.	**Method Name**	**:**	**Simpson 1/3 Method**
	Term	:	The number of partitions (n) is an even number
	Formula	:	$\int_a^b f(x)dx \approx \frac{h}{3}\left(f_0 + 4\left(f_1 + f_3 + \ldots + f_{n-1}\right) + 2\left(f_2 + f_4 + \ldots + f_{n-2}\right) + f_n\right)$
7.	**Method Name**	**:**	**Simpson 3/8 Method**
	Term	:	The number of partitions (n) is a multiple of 3
	Formula	:	$\int_a^b f(x)dx \approx \frac{3h}{8}\left(f_0 + 3\left(f_1 + f_2 + \ldots + f_{n-1}\right) + 2\left(f_3 + f_6 + \ldots + f_{n-3}\right) + f_n\right)$
8.	**Method Name**	**:**	**Weddle Method**
	Term	:	The number of partitions (n) is a multiple of 6
	Formula	:	$\int_a^b f(x)dx \approx \frac{3h}{10}\left(\begin{matrix} f_0 + 5f_1 + f_2 + 6f_3 + f_4 + 5f_5 + \\ 2f_6 + 5f_7 + f_8 + 6f_9 + f_{10} + 5f_{11} + \\ 2f_{12} + 5f_{13} + f_{14} + 6f_{15} + f_{16} + 5f_{17} + \\ 2f_{18} + \ldots + \ldots + \ldots + \ldots + \ldots + \\ 2f_{n-6} + 5f_{n-5} + f_{n-4} + 6f_{n-3} + f_{n-2} + 5f_{n-1} + f_n \end{matrix}\right)$

RESULT AND DISCUSSION

The first case study takes an example based on research from Sumarni (2020). The second case study takes an example based on research by Fauziyah, Irmansyah and Purwani (2021).

Completion of the First Case Study :

Question :

$$\int_0^2 \left(4x^3 - 2x + 5\right) dx$$

Answer :

The True Solution of Analytical Method :

$$\int_0^2 \left(4x^3 - 2x + 5\right) dx = \frac{4x^4}{4} - \frac{2x^2}{2} + 5x\Big|_a^b$$

$$= \left|\frac{4(2)^4}{4} - \frac{2(2)^2}{2} + 5(2)\right| - \left|\frac{4(0)^4}{3} - \frac{2(0)^2}{2} + 5(0)\right|$$

$$= \left|\frac{4(2)^4}{4} - \frac{2(2)^2}{2} + 5(2)\right| - (0)$$

$$= \left|\frac{4(16)}{4} - \frac{2(4)}{2} + 10\right|$$

$$= \left(\frac{64}{4} - \frac{8}{2} + 10\right)$$

$$= (16 - 4 + 10)$$

$$= 22$$

So that the analytically true solution is 22.

Numerical Method Approximation Solution :

The author determines for n of 18, then :

$$h = \frac{b-a}{n} = \frac{2-0}{18} = \frac{2}{18} = 0{,}111$$

So the table is as follows:

Table 2. x and y values

Iteration n	x	y = f(x)
0	0	5,000
1	0,111	4,783
2	0,222	4,600
3	0,333	4,482
4	0,444	4,462
5	0,555	4,574
6	0,666	4,850
7	0,777	5,322
8	0,888	6,025
9	0,999	6,990
10	1,110	8,251

Iteration n	x	y = f(x)
11	1,221	9,839
12	1,332	11,789
13	1,443	14,133
14	1,554	16,903
15	1,665	20,133
16	1,776	23,855
17	1,887	28,103
18	1,998	32,908

Answers to the Rectangular Method :

In the rectangular method, then enter the data in table 3 into the rectangular method formula as follows :

$$\int_a^b f(x)dx \approx \frac{h}{2}\left(f_0 + 2\left(f_1 + f_2 + f_3 + f_4 + \ldots + f_{n-1}\right) + f_n\right)$$

$$\int_0^2 \left(4x^3 - 2x + 5\right)dx \approx \frac{h}{2}\left(f_0 + 2\left(f_1 + f_2 + f_3 + f_4 + \ldots + f_{17}\right) + f_{18}\right)$$

$$\approx \frac{0,111}{2}\left(5 + 2\begin{pmatrix} 4,783 + 4,600 + 4,482 + 4,462 + \\ 4,574 + 4,850 + 5,322 + 6,025 + \\ 6,990 + 8,251 + 9,839 + 11,789 + \\ 14,133 + 16,903 + 20,133 + 23,855 + \\ 28,103 \end{pmatrix} + 32,908\right)$$

$$\approx \frac{0,111}{2}\left(5 + 2(179,093) + 32,908\right)$$

$$\approx \frac{0,111}{2}(5 + 358,187 + 32,908)$$

$$\approx 0,056(396,095)$$

$$\approx 21,983$$

Absolute Error = |Analytical Method| – |Rectangular Method|

= |22| – |21,983|

= 0,017

Answers to the Simpson 1/3 Method :

In the Simpson 1/3 method, then enter the data in table 3 into the Simpson 1/3 method formula as follows :

$$\int_a^b f(x)dx \approx \frac{h}{3}\left(f_0 + 4\left(f_1 + f_3 + \ldots + f_{n-1}\right) + 2\left(f_2 + f_4 + \ldots + f_{n-2}\right) + f_n\right)$$

$$\int_0^2 \left(4x^2 - 2x + 5\right)dx \approx \frac{h}{3}\left(f_0 + 4\left(f_1 + f_3 + \ldots + f_{17}\right) + 2\left(f_2 + f_4 + \ldots + f_{16}\right) + f_{18}\right)$$

$$\int_0^2 \left(4x^2 - 2x + 5\right)dx \approx \frac{0,111}{3}\left(\begin{matrix} 5 + 4\begin{pmatrix} 4,783 + 4,482 + 4,574 + 5,322 + \\ 6,990 + 9,839 + 14,133 + 20,133 + \\ 28,103 \end{pmatrix} + \\ 2\begin{pmatrix} 4,600 + 4,462 + 4,850 + 6,025 + \\ 8,251 + 11,789 + 16,903 + 23,855 \end{pmatrix} + 32,908 \end{matrix}\right)$$

$$\approx \frac{0,111}{3}\left(5 + 4(98,359) + 2(80,734) + 32,908\right)$$

$$\approx \frac{0,111}{3}(5 + 393,436 + 161,469 + 32,908)$$

$$\approx 0,037(592,813)$$

$$\approx 21,934$$

Absolute Error = |Analytical Method| – |Simpson 1/3 Method|

= |22| – |21,934|

= 0,066

Answers to the Simpson 3/8 Method :

In the Simpson 3/8 method, then enter the data in table 3 into the Simpson 3/8 method formula as follows :

$$\int_a^b f(x)dx \approx \frac{3h}{8}\left(f_0 + 3\left(f_1 + f_2 + f_4 + \ldots + f_{n-1}\right) + 2\left(f_3 + f_6 + f_9 + \ldots + f_{n-3}\right) + f_n\right)$$

$$\int_0^2 \left(4x^2 - 2x + 5\right)dx \approx \frac{3h}{8}\left(f_0 + 3\left(f_1 + f_2 + f_4 + \ldots + f_{17}\right) + 2\left(f_3 + f_6 + f_9 + \ldots + f_{15}\right) + f_{18}\right)$$

$$\int_0^2 \left(4x^2 - 2x + 5\right)dx \approx \frac{3(0,111)}{8}\left(\begin{array}{l} 5 + 3\left(\begin{array}{l} 4,783 + 4,600 + 4,462 + \\ 4,574 + 5,322 + 6,025 + \\ 8,251 + 9,839 + 14,133 + \\ 16,903 + 23,855 + 28,103 \end{array}\right) + \\ 2\left(\begin{array}{l} 4,482 + 4,850 + 6,990 + \\ 11,789 + 20,133 \end{array}\right) + 32,908 \end{array}\right)$$

$$\begin{aligned} &\approx \frac{3(0,111)}{8}\left(5 + 3(130,850) + 2(48,243) + 32,908\right) \\ &\approx \frac{3(0,111)}{8}(5 + 392,550 + 96,487 + 32,908) \\ &\approx 0,042(526,945) \\ &\approx 21,934 \end{aligned}$$

Absolute Error = |Analytic Method| – |Simpson 3/8 Method|

= |22| – |21,934|

= 0,066

Answers to the Weddle Method :

In the Weddle method, then enter the data in table 3 into the Weddle method formula as follows :

$$\int_a^b f(x)dx \approx \frac{3h}{10}\left(\begin{array}{l} f_0 + \\ 5f_1 + f_2 + 6f_3 + f_4 + 5f_5 + \\ 2f_6 + 5f_7 + f_8 + 6f_9 + f_{10} + 5f_{11} + \\ 2f_{12} + 5f_{13} + f_{14} + 6f_{15} + f_{16} + 5f_{17} + \\ 2f_{18} + \ldots + \\ 2f_{n-6} + 5f_{n-5} + f_{n-4} + 6f_{n-3} + f_{n-2} + 5f_{n-1} + \\ f_n \end{array}\right)$$

$$\int_0^2 \left(4x^2 - 2x + 5\right)dx \approx \frac{3h}{10}\left(\begin{array}{l} f_0 + \\ 5f_1 + f_2 + 6f_3 + f_4 + 5f_5 + \\ 2f_6 + 5f_7 + f_8 + 6f_9 + f_{10} + 5f_{11} + \\ 2f_{12} + 5f_{13} + f_{14} + 6f_{15} + f_{16} + 5f_{17} + \\ f_{18} \end{array}\right)$$

$$\int_0^2 \left(4x^2 - 2x + 5\right)dx \approx \frac{3(0,111)}{10}\left(\begin{array}{l} 5 + \\ 5(4,783) + 4,600 + 6(4,482) + 4,462 + 5(4,574) + \\ 2(4,850) + 5(5,322) + 6,025 + 6(6,990) + 8,251 + 5(9,839) + \\ 2(11,789) + 5(14,133) + 16,903 + 6(20,133) + 23,855 + 5(28,103) \\ 32,908 \end{array}\right)$$

$$\begin{aligned} &\approx \frac{3(0,111)}{10}(658,681) \\ &\approx 0,33(658,681) \\ &\approx 21,934 \end{aligned}$$

Absolute Error = |Analytical Method| – |Weddle Method|

= |22| – |21,934|

= 0,066

Completion of the Second Case Study :

Question :

Based on the data the author got from research references by Fauziyah, Irmansyah and Purwani (2021). Following are the x and y coordinates of the two-dimensional Cartesian plane.

Table 3 Regions and Partitions of West Java

i	Region 1		Region 2		Region 3		Region 4	
	x_i	y_i	x_i	y_i	x_i	y_i	x_i	y_i
0	0	87,760	0	88,060	0	64,440	0	64,580
1	13,960	83,700	14,960	81,410	14,960	63,900	13,960	66,380
2	27,930	79,610	29,930	83,490	29,930	63,700	27,930	74,820
3	41,900	82,090	44,900	95,110	44,900	63,900	41,900	79,190
4	55,870	91,570	59,870	113,500	59,870	65,300	55,870	77,450
5	69,830	79,150	74,830	96,040	74,830	68,040	69,830	78,440
6	86,592	71,770	89,806	101,260	89,806	70,140	86,592	79,030
7	83,806	44,160	104,770	45,900	104,770	68,780	83,806	77,270
8	97,770	38,540	119,740	44,030	119,740	24,120	97,770	63,000
9	125,710	48,760	134,710	46,280	134,710	29,470	125,710	58,160

Answer :

The True Solution of Analytical Method :

According to the analytical method, the area of West Java is 35.377,76 km^2 by BPS West Java Province (2022).

Numerical Method Approximation Solution :

The authors determine for n equal to 9. Manual calculations the author will only calculate using the Simpson 3/8 method with n equal to 9. The results of the trapezoidal method are taken from calculations performed by Fauziyah et al in his research with a result of 41.103,262.

Determined n equal to 9, so that h :

Region 1 and 4 :

The length of the x axis for region 1 is 134,71 km, so:

$$h = \frac{b-a}{n} = \frac{125{,}71 - 0}{9} = 13{,}96$$

Region 2 and 3 :

The length of the x axis for region 2 is 125,71 km, so:

$$h = \frac{b-a}{n} = \frac{134{,}71 - 0}{9} = 14{,}96$$

Answers to the Simpson 3/8 Method :

In the Simpson 3/8 method, then enter the data in table 4 into the Simpson 3/8 method formula as follows:

Region 1 :

$$S_9(f) \approx \frac{3h}{8}\left(f_0 + 3(f_1 + f_2 + f_4 + \ldots + f_{n-1}) + 2(f_3 + f_6 + f_9 + \ldots + f_{n-3}) + f_n\right)$$

$$S_9(f) \approx \frac{3(13,96)}{8}\begin{pmatrix} 87,760 + 3\begin{pmatrix} 83,700 + 79,610 + 91,570 + \\ 79,150 + 44,160 + 38,540 \end{pmatrix} + \\ 2(82,090 + 71,770) + 48,760 \end{pmatrix}$$

$$\approx \frac{3(13,96)}{8}(87,760 + 3(416,73) + 2(153,86) + 48,760)$$

$$\approx \frac{3(13,96)}{8}(87,760 + 1.250,19 + 307,72 + 48,760)$$

$$\approx 5,235(1.694,43)$$

$$\approx 8.870,341$$

Region 2 :

$$S_9(f) \approx \frac{3h}{8}\left(f_0 + 3(f_1 + f_2 + f_4 + \ldots + f_{n-1}) + 2(f_3 + f_6 + f_9 + \ldots + f_{n-3}) + f_n\right)$$

$$S_9(f) \approx \frac{3(14,96)}{8}\begin{pmatrix} 88,060 + 3\begin{pmatrix} 81,410 + 83,490 + 113,500 + \\ 96,040 + 45,900 + 44,030 \end{pmatrix} + \\ 2(95,110 + 101,260) + 46,280 \end{pmatrix}$$

$$\approx \frac{3(14,96)}{8}(88,060 + 3(464,37) + 2(196,37) + 46,280)$$

$$\approx \frac{3(14,96)}{8}(88,060 + 1.393,11 + 392,74 + 46,280)$$

$$\approx 5,61(1.920,19)$$

$$\approx 10.772,265$$

Region 3 :

$$S_9(f) \approx \frac{3h}{8}\left(f_0 + 3(f_1 + f_2 + f_4 + \ldots + f_{n-1}) + 2(f_3 + f_6 + f_9 + \ldots + f_{n-3}) + f_n\right)$$

$$S_9(f) \approx \frac{3(14,96)}{8}\begin{pmatrix} 64,440 + 3\begin{pmatrix} 63,900 + 63,700 + 65,300 + \\ 68,040 + 68,780 + 24,120 \end{pmatrix} + \\ 2(63,900 + 70,140) + 29,470 \end{pmatrix}$$

$$\approx \frac{3(14,96)}{8}(64,440 + 3(353,84) + 2(134,04) + 29,470)$$

$$\approx \frac{3(14,96)}{8}(64,440 + 1.061,52 + 268,08 + 29,470)$$

$$\approx 5,61(1.423,51)$$

$$\approx 7.985,891$$

Region 4 :

$$S_9(f) \approx \frac{3h}{8}\left(f_0 + 3(f_1 + f_2 + f_4 + \ldots + f_{n-1}) + 2(f_3 + f_6 + f_9 + \ldots + f_{n-3}) + f_n\right)$$

$$S_9(f) \approx \frac{3(13,96)}{8}\begin{pmatrix} 64,580 + 3\begin{pmatrix} 66,380 + 74,820 + 77,450 + \\ 78,440 + 77,270 + 63,000 \end{pmatrix} + \\ 2(79,190 + 79,030) + 58,160 \end{pmatrix}$$

$$\approx \frac{3(13,96)}{8}(64,580 + 3(437,36) + 2(158,22) + 58,160)$$

$$\approx \frac{3(13,96)}{8}(64,580 + 1.312,08 + 316,44 + 58,160)$$

$$\approx 5,235(1.751,26)$$

$$\approx 9.167,846$$

Total Area = Region 1 + Region 2 + Region 3 + Region 4

= 8.870,341 + 10.772,265 + 7.985,891 + 9.167,846

= 36.796,343

Absolute Error = |Analytical Method| – |Total Area|

= |35.377,76| – |36.796,343|

= 1.418,583

Application Design

After the integral area is calculated manually, then it is applied to the Python programming language source code. The author uses Google Colab to implement the Python programming language source code. The following is the source code of this study: https://colab.research.google.com/drive/1DcpAebuWQ2DkwVuRa5isln_A28ck_t5B#scrollTo=r6M18k9uamhr

and

https://colab.research.google.com/drive/1B60HMIx41wt1na6dRDUjF5LyPeYnK4KI#scrollTo=05ca891d

CONCLUSION

The author's conclusion from this research is from the manual calculation of the first case study about integrals where the smallest error from the numerical method to the analytic method is achieved by the rectangular method of 0,017. In the second case study of tabular data questions for manual calculations the author only used the Simpson 3/8 method with an absolute error for the analytical area of 1.418,583 km^2.

Whereas in the Python application for the first case study the smallest error was achieved by the Simpson 1/3 and Simpson 3/8 methods with an error of 0, in the sense that these two methods are very accurate to the true results of the analytical method. In the second case study the smallest error was achieved by the Simpson 1/3 method of 1.039,365 km^2.

The difference between manual calculations and application results is due to decimal rounding, and the linspace function (a,b,n+1) in the numpy library. This greatly affects the conclusion of which method is best suited to different case studies with different partitions.

The author's suggestion for further research is that it is necessary to add more case studies to see the suitability of the method for this type of case study. The experiment changes the partition n so that the numerical solution results are the same as the analytical solution results. Because the application has been made in the Python programming language, all that remains is to change it directly to the source code for case study and partition experiments. Need to add the Django framework or similar so that the user interface looks web-based.

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