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ANALYSIS AND SIMULATION OF SEIR MATHEMATICAL MODEL OF STUNTING CASE IN INDONESIA

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ABSTRACT

Stunting is one of the focuses of the target for improving nutrition in the world until 2025. The government of Indonesia targets the prevalence of stunting to decrease to 14% in 2024. There are some factor affect stunting, some of them are sanitation, health care and parenting of child. In this research, we constructed a SEIR mathematical model of stunting case in Indonesia. There are four compartements in this model : the population of newborn are likely to be exposed to stunting (S), the population of children were having an early symptom stunting (E), the population of affected child stunting permanent (I) and the population of children showing symptom stunting but no caught stunting (free stunting) (R). The disease free E_0 and endemic equilibrium E_1 point are the equilibrium of the formed model. The local stability of equilibrium points have been analyzed, determined the basic reproduction number, and conducted a sensitivity analysis. If $\mathrm{R}_0 = \frac{\alpha\beta(\omega+\gamma+\delta)}{\mu^2(\mu+\omega+\gamma+\delta+\varepsilon+\varphi+\sigma)} < 1$ then E_0 is asymptotically stable If $\mathrm{R}_0 > 1$ then E_1 is asymptotically stable. Sensitivity analysis revealed that the parameters ω, γ, δ and β significantly contribute to the increase in the basic reproduction number (R_0). It means that the good sanitation, good healthcare, good parenting of child and decrease the rate of transmission indirectly from the person affected stunting are the most important thing to reduce the rate of stunting in Indonesia.

Keywords: Mathematical Model, Stunting, Sanitation, Health Care, Parenting of Child

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PRELIMINARY

Indonesia is still in the process of achieving the second Sustainable Development Goals (SDGs) which is to end hunger, attaining better food endurance and nutrients, and and support sustainable agriculture. This goals are closely linked to the third goal, ensuring healthy living and supporting welfare for all for all ages (Irhamsyah, 2019). The indicator of success of the development of a nation is nutrition and food adequacy which is one of the most important factors in improving the quality of human resources. Stunting become one of the goals of SDGs and as a long-term indicator for malnutrition in children. Stunting is one of the focuses of the target for improving nutrition in the world until 2025. Therefore, stunting deserves more attention (Kemenkes, 2018; Saraswati et al., 2022).

Energy intake, birth weight, mother's education level, family income or poverty level, parenting styles and food diversity, exclusive breastfeeding and vitamin A, protein and mineral sufficiency, meeting nutritional needs, nutrition for expectant mothers and complementary foods for breast milk (MPASI) in sufficient quantity and quality, monitoring toddler growth, access to clean water and sanitation facilities, and environmental cleanliness are all factors that affect the incidence of stunting in early childhood. (Abdul et al. 2020; Arianti et al. 2022; Aswi & Sukarna 2022; Bustami & Ampera 2020; Cameron, et al., 2021; Gibson 2002; Lukman, et al., 2022; Rahayu, et al., 2018; Sutarto & Reni, 2018; Zubedi et al. 2021). Apart from that, the percentage of babies who receive complete immunization and infectious diseases and genetics can also influence stunting. Handling stunting is not enough just in the health sector, but must also touch on socio-economic aspects, such as the mother's education level, mother's employment status, and family income (Bhutta, et al., 2020; Castro, et al., 2021; Latifa, 2018; Ngwira, 2020; Ni'mah & Nadhiroh, 2015; Siswanto, et al 2022). Based on the various factors that cause stunting, of course if there are priority factors that need more special attention by looking at the reality of the existing conditions then efforts to reduce the prevalence rate of stunting cases will be more on target with wat happens in the field. Thus, there is a need for analysis to find out the right factors or variables to find out how and to what extent existing factors influence the occurrence of stunting cases in Indonesia. This can be done through mathematical models.

Mathematical models can be used to determine the dynamics of a real phenomenon over a long period of time. One of the mathematical models used is the SI (Susceptible Infective) compartment model. There is a study formulates a mathematical model used is SEIS to analyze the co-dynamics of malaria and malnutrition in children under five years old (Tchoumi et al. 2023). This model can be expanded by adding incubation or latent (Exposed) period compartments to the SsEIR model. Stunting is not an infectious disease but the disease can be occurred as a result of the interaction indirectly to the patient other stunting. The problem solving approach that can be taken is analysis and simulation of the SEIR model which is appropriate to the phenomenon of stunting. Previous research that has been carried out relating to compartment models (Akinyemi et al. 2018; Tengger & Winarni & Sofiyati, 2022; Syrti et al. 2023) has become one of the references in developing mathematical models. The first thing to do is study literature related to the model to achieve this goal. Then determine the assumptions needed so that the model can approach real conditions. Next, form a mathematical model based on predetermined assumptions. Then determine the equilibrium point and basic reproduction number and then analyze the stability of the equilibrium point obtained. The final step is to simulate the model that has been formed using MATLAB software and interpret the results to draw conclusions.

Based on the description above, researchers are interested in conducting research on "Analysis and Simulation of the SEIR Mathematical Model in Stunting Cases in Indonesia". Previous research only examined mathematical models that only involved sanitation factors. The results of the simulation show that the higher the sanitation number then the number of stunting cases decreases faster (Pratama & Lismayani, 2023). In fact, there are other factors that cause stunting (Sumekar et al., 2019). Therefore, researchers will involve parenting of child factors, lack of nutritious food intake and lack of health services in addition to involving sanitation factors. The aim of this research is to find out the exact factors or variables that influence the occurrence of stunting cases in Indonesia. The hope is that it can reduce or reduce the prevalence of stunting cases in Indonesia in order to achieve the reduction target by 2024.

METHODS

The steps involved in this research are depicted in Figure 1. as follows.

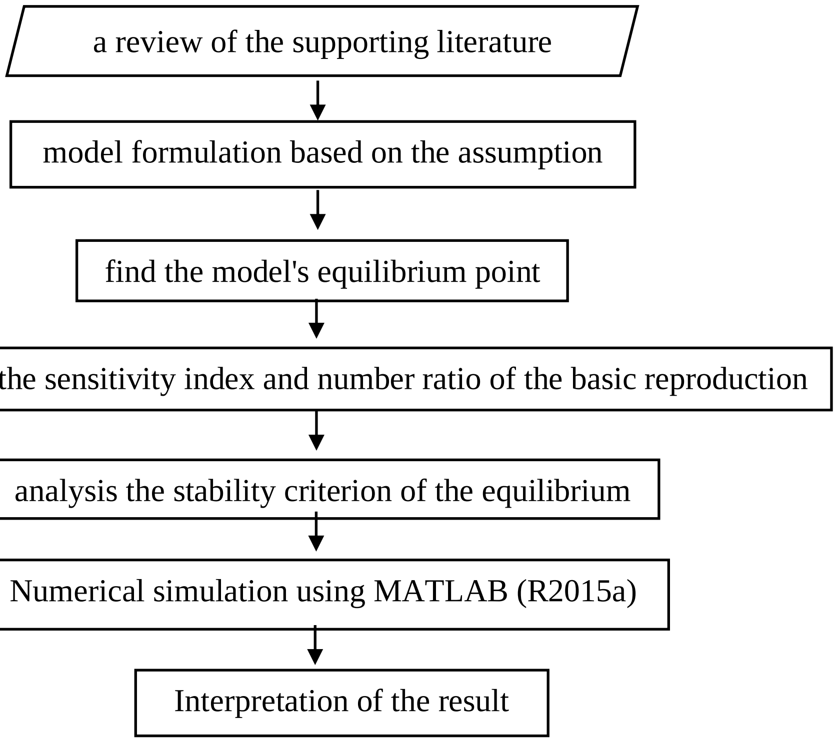

Figure 1. Flowchart of The Research

RESULT AND DISCUSSION

Model Formulation and Analysis

On SEIR mathematical modeling, newborn children are likely to be affected by stunting which is denoted Suspectible (S). Children who show symptoms of stunting are denoted as Exposed (E). Children who are affected by stunting and cannot be cured are denoted as Infected (I), and children who show symptoms of stunting but are not affected by stunting (free from stunting) because they are given special treatment for symptoms are denoted as Recovered (R). This population can be defined in mathematical form as follows.

$$N = S + E + I + R$$

In the SEIR model formulation, the spread of stunting in children, assumptions have been made to limit and clarify the model that will be formed as follows.

1. Population of children under five.
2. Stunting is not an infectious disease but the disease can be occurred as a result of the interaction indirectly to the patient other stunting.
3. Every individual born will fall into a sub-population S with a birth rate of α.
4. There are individuals who experience natural death with a death rate of μ.
5. There are individuals who are vulnerable to becoming individuals affected by the initial symptoms of stunting with a rate of change of β. Due to indirect transmission with individuals who experience permanent stunting.
6. There are individuals with early symptoms of stunting who become individuals affected by permanent stunting because they do not improve their parenting patterns with a rate of change of ω.
7. There are individuals with early symptoms of stunting who become individuals affected by permanent stunting due to lack of access to clean water and sanitation with a rate of change of γ.
8. There are individuals with early symptoms of stunting who become individuals affected by permanent stunting because they do not follow health services properly with a rate of change of δ.
9. There are individuals with early symptoms of stunting who become recovered individuals due to improving parenting patterns with a recovery rate of ε.
10. There are individuals with early symptoms of stunting who become recovered individuals because they attend health services with a recovery rate of σ.

11. There are individuals with early symptoms of stunting who become recovered individuals because they have access to clean water and sanitation with a recovery rate of φ.
12. Any individual who has been permanently infected cannot recover.

Based on the assumptions above, a model was formed with the relationship between the four subpopulations can be presented in the following figure.

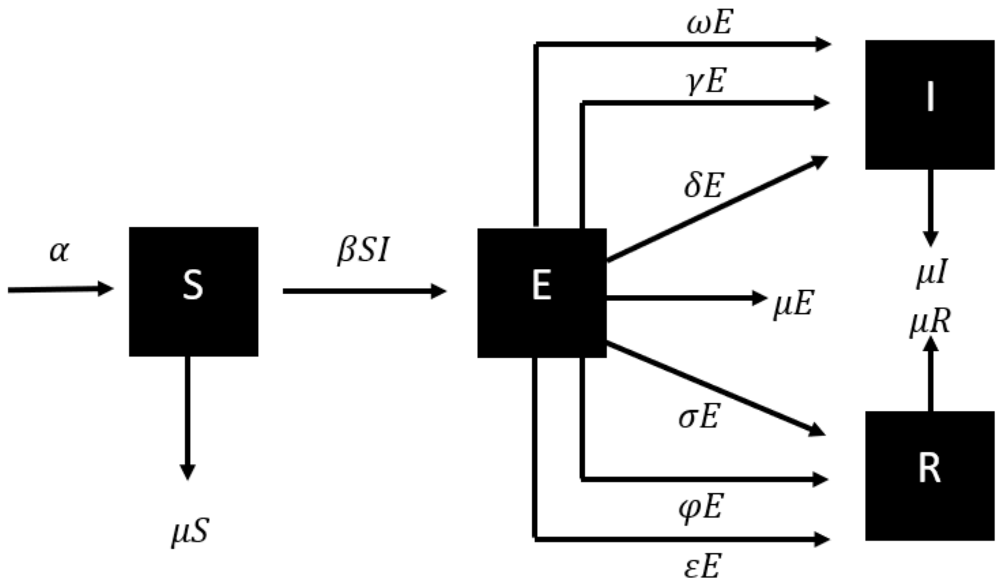

Figure 2. Compartement diagram of the SEIR model

Based on the chart above, can be formed a system of mathematical model equations as below:

$$
\begin{aligned}
\frac{dS}{dt} &= \alpha - (\beta I + \mu)S \\
\frac{dE}{dt} &= \beta SI - (\mu + \omega + \gamma + \delta + \varepsilon + \varphi + \sigma)E \\
\frac{dI}{dt} &= (\omega + \gamma + \delta)E - \mu I \\
\frac{dR}{dt} &= (\varepsilon + \varphi + \sigma)E - \mu R
\end{aligned}
\qquad (1)
$$

All variables of system (1) denote the number of subpopulation so that all of them must be non-negative for time $t \geq 0$ when the initial conditions are also non-negative. From the first equation in system (1), we get $\frac{dS}{dt} \geq -(\beta I + \mu)S$. Therefore, $S(t) \geq S(0)\exp\big(-(\beta I + \mu)\big) \geq 0$. It means that $S(t)$ remains non-negative for all times $t > 0$. Analogoys results hold for the other variables $E(t)$, $I(t)$ and $R(t)$.

Theorem 1. Solution of the system (1) with positive initial values will remain positive for all time $t \geq 0$ and the feasible region $\Omega = \left\{(S,E,I,R) \in \mathbb{R}_+^3 "S,E,I,R \geq 0, N = S+E+I+R \leq \frac{\alpha}{\mu}\right\}$ is positively invariant for the system (1).

Proof. Consider total population $= S + E + I + R$ are positive. Adding all the equations of system (1), the total population satisfies the following equations.

$$\frac{dN}{dt} = \alpha - \mu N$$

$$\leftrightarrow N(t) = \frac{\alpha}{\mu} + N(0)e^{-\mu t} - \frac{\alpha}{\mu}e^{-\mu t}$$

As $t \to \infty$ then $N(t)$ approaches $\frac{\alpha}{\mu}$ asymptotically. Hence, the feasible domain of system (1) is $\Omega = \left\{(S,E,I,R) \in \mathbb{R}_+^3 "S,E,I,R \geq 0, N = S+E+I+R \leq \frac{\alpha}{\mu}\right\}$, which is positively invariant. Thus, system (1) is well-posed.

Table 1. Variables and parameters description

Variable / parameter	Description	Domain	Unit
S	The population of newborn are likely to be exposed to stunting.	$S \geq 0$	children
E	The population of children were having an early symptom stunting	$E \geq 0$	children
I	The population of affected child stunting permanent	$I \geq 0$	children
R	The population of children showing symptom stunting but no caught stunting (free stunting)	$R \geq 0$	children
α	The number of newborn	$\alpha > 0$	${}^{\text{children}}/_{year}$
μ	The rate of natural death	$\mu > 0$	${}^{1}/_{year}$
β	The rate of change of the population from S to E due to transmission of indirectly from the person affected stunting	$\beta > 0$	$\frac{1}{\text{children} \cdot \text{year}}$
ω	The rate of change of population from E to I is due to not improving parenting patterns	$\omega > 0$	${}^{1}/_{year}$
γ	The rate of change of population from E to I due to not improving sanitation	$\gamma > 0$	${}^{1}/_{year}$
δ	The rate of change of population from E to I due to lack of health services	$\delta > 0$	${}^{1}/_{year}$
ε	The rate of recovery rate is due to improving parenting patterns	$\varepsilon > 0$	${}^{1}/_{year}$
φ	The rate of recovery rate due to improved sanitation	$\varphi > 0$	${}^{1}/_{year}$
σ	The rate of recovery is due to improvements in health services	$\sigma > 0$	${}^{1}/_{year}$

Equilibrium Analysis

Equilibrium point on the equation system (1) is determined by setting the equations on the system constant to time or $\frac{dS}{dt} = \frac{dE}{dt} = \frac{dI}{dt} = \frac{dR}{dt} = 0$. Based on system (1), it are obtained that the free stunting equilibrium point $E_0 = \left(\frac{\alpha}{\mu}, 0, 0, 0\right)$ and the exist stunting equilibrium point $E_1 = (S^*, E^*, I^*, R^*)$, where

$$S^* = \frac{\mu(\mu + \omega + \gamma + \delta + \epsilon + \phi + \sigma)}{\beta(\omega + \gamma + \delta)}$$

$$E^* = \frac{-\mu^3 - (\omega + \gamma + \delta + \epsilon + \phi + \sigma)\mu^2 + \alpha\beta(\omega + \gamma + \delta)}{\beta(\omega + \gamma + \delta)(\mu + \omega + \gamma + \delta + \epsilon + \phi + \sigma)}$$

$$I^* = \frac{-\mu^3 - (\omega + \gamma + \delta + \epsilon + \phi + \sigma)\mu^2 + \alpha\beta(\omega + \gamma + \delta)}{\beta\mu(\mu + \omega + \gamma + \delta + \epsilon + \phi + \sigma)}$$

$$R^* = \frac{(-\mu^3 - (\omega + \gamma + \delta + \epsilon + \phi + \sigma)\mu^2 + \alpha\beta(\omega + \gamma + \delta))(\epsilon + \phi + \sigma)}{\beta\mu(\omega + \gamma + \delta)(\mu + \omega + \gamma + \delta + \epsilon + \phi + \sigma)}$$

Basic Reproduction Number

The basic reproduction number, which can be denoted as R_0, is used to determine whether a disease will disappear or remain. R_0 can be said to be a ratio that shows the number of susceptible individuals who can be infected or infected due to one infected individual. The basic reproduction number is determined using the next generation matrix method ornext generation matrices (NGM)(Akinyemi et al. 2018; Ndii 2018). The next generation matrix (G) is defined as follows

$$G = FV^{-1}$$

where

$$F = \begin{bmatrix} 0 & \frac{\alpha\beta}{\mu} \\ 0 & 0 \end{bmatrix} \text{ and } V = \begin{bmatrix} \mu + \omega + \gamma + \delta + \varepsilon + \varphi + \sigma & 0 \\ -(\omega + \gamma + \delta) & \mu \end{bmatrix}$$

So that the matrix G is obtained as follows

$$G = \begin{bmatrix} \frac{\alpha\beta(\omega + \gamma + \delta)}{\mu^2(\mu + \omega + \gamma + \delta + \varepsilon + \varphi + \sigma)} & \frac{\alpha\beta}{\mu^2} \\ 0 & 0 \end{bmatrix}$$

The largest positive eigen value of the g matrix is R_0, which is

$$R_0 = \frac{\alpha\beta(\omega+\gamma+\delta)}{\mu^2(\mu+\omega+\gamma+\delta+\varepsilon+\varphi+\sigma)} \quad (2)$$

Theorem 2. Existence of Equilibrium Point

- If $R_0 < 1$ then system (1) has only one equilibrium point, which is E_0
- If $R_0 > 1$ then system (1) has two equilibrium point, E_0 and E_1.

Proof. Existence of equilibrium point is indicated by each positive element according to the conditions of the formation of the system (1).

- Note that $E_0 = \left(\frac{\alpha}{\mu}, 0, 0, 0\right)$, Because all the parameter used in system (1) are positive so obtained $\frac{\alpha}{\mu} > 0$. Next to guarante that If $\mathrm{R}_0 < 1$ then system (1) has only one equilibrium point, enough to prove that I^* at $E_1 = (S^*, E^*, I^*, R^*)$ is negative. Note that $\mathrm{R}_0 < 1$ so

$$\frac{\alpha\beta(\omega+\gamma+\delta)}{\mu^2(\mu+\omega+\gamma+\delta+\epsilon+\phi+\sigma)} < 1$$

$$\leftrightarrow \ \alpha\beta(\omega+\gamma+\delta) < \mu^2(\mu+\omega+\gamma+\delta+\epsilon+\phi+\sigma)$$
$$\leftrightarrow \ -\alpha\beta(\omega+\gamma+\delta) > -(\mu^3+(\omega+\gamma+\delta+\epsilon+\phi+\sigma)\mu^2)$$
$$\leftrightarrow \mu^3+(\omega+\gamma+\delta+\epsilon+\phi+\sigma)\mu^2-\alpha\beta(\omega+\gamma+\delta) > 0$$
$$\leftrightarrow -\mu^3-(\omega+\gamma+\delta+\epsilon+\phi+\sigma)\mu^2+\alpha\beta(\omega+\gamma+\delta) < 0$$

  Because $\beta\mu(\mu+\omega+\gamma+\delta+\epsilon+\phi+\sigma) > 0$ and $-\mu^3-(\omega+\gamma+\delta+\epsilon+\phi+\sigma)\mu^2 < 0$ so $I^* < 0$. It's proven that if $\mathrm{R}_0 < 1$ then $I^* < 0$, so $E_1 = (S^*, E^*, I^*, R^*)$ doesn't exist. In the other word, if $\mathrm{R}_0 < 1$ then system (1) has only one equilibrium point (E_0).

- This only needs to be proven that the equilibrium point E_1 exist if $R_0 > 1$. Note that $E_1 = (S^*, E^*, I^*, R^*)$, because of all the parameter used in system (1) are positive then S^* is positive and E^*, I^*, R^* would be positive if $-\mu^3-(\omega+\gamma+\delta+\epsilon+\phi+\sigma)\mu^2+\alpha\beta(\omega+\gamma+\delta) > 0$. Because $\alpha\beta(\omega+\gamma+\delta) > 0$ should be $-\mu^3-(\omega+\gamma+\delta+\epsilon+\phi+\sigma)\mu^2 > 0$ so

$$-\mu^3-(\omega+\gamma+\delta+\epsilon+\phi+\sigma)\mu^2+\alpha\beta(\omega+\gamma+\delta) > 0$$
$$\leftrightarrow \mu^3+(\omega+\gamma+\delta+\epsilon+\phi+\sigma)\mu^2-\alpha\beta(\omega+\gamma+\delta) < 0$$
$$\leftrightarrow \ -\alpha\beta(\omega+\gamma+\delta) < -(\mu^3+(\omega+\gamma+\delta+\epsilon+\phi+\sigma)\mu^2)$$
$$\leftrightarrow \ \alpha\beta(\omega+\gamma+\delta) > \mu^2(\mu+\omega+\gamma+\delta+\epsilon+\phi+\sigma)$$
$$\leftrightarrow \ \frac{\alpha\beta(\omega+\gamma+\delta)}{\mu^2(\mu+\omega+\gamma+\delta+\epsilon+\phi+\sigma)} > 1 \qquad (3)$$

  Based on equation (2), equation (3) can be write $R_0 > 1$. Therefore $E^*, I^*, R^* > 0$ if $R_0 > 1$. In the other word, if $\mathrm{R}_0 > 1$ then system (1) has only two equilibrium point (E_0 and E_1).

Stability Analysis

The stability of the equilibrium point of system (1) can be determined by observing the sign of the real part of the eigenvalues of the jacobian matrix. If all eigenvalues of the coeficient matrix of the Jacobian Matrix (linearization) at this equilibrium have negative real part, then the equilibrium is local asymptotically stable (Brauer et al. 2019). The Jacobian Matrix of system (1) is

$$J = \begin{bmatrix} -\beta I - \mu & 0 & -\beta S & 0 \\ \beta I & -(\mu + \omega + \gamma + \delta + \varepsilon + \varphi + \sigma) & \beta S & 0 \\ 0 & \omega + \gamma + \delta & -\mu & 0 \\ 0 & \varepsilon + \varphi + \sigma & 0 & -\mu \end{bmatrix} \quad (4)$$

Theorem 3. If $\mathrm{R}_0 < 1$ then the equilibrium point E_0 is locally asymptotically stable.

Proof. System (1) is linearized at the equilibrium point E_0. The eigenvalues derived from the Jacobian Matrix (4) evaluated at equilibrium point E_0. So that the obtained characteristic equation of the Jacobian Matrix (4) evaluated at the point E_0 as follows,

$$(\lambda + \mu)^2(\lambda^2 + a_1\lambda + a_2) = 0 \quad (5)$$

with

$$a_1 = 2\mu + \omega + \gamma + \delta + \varepsilon + \varphi + \sigma \quad (6)$$

$$a_2 = \frac{\mu^3 + \mu^2(\omega + \gamma + \delta + \varepsilon + \varphi + \sigma) - \alpha\beta(\omega + \gamma + \delta)}{\mu}$$

Based on equation (5) can be obtained $\lambda_{1,2} = -\mu$, Because the value of μ is positive, then the real part of eigen values is negative. Furthermore, the sign of the real part of λ_3 and λ_4 can be known through Routh Hurwitz criterion. According to the equation (5) can be obtained Routh Hurwitz matrix $H_1 = 1$ with $Det(H_1) = 1 > 0$ and $H_2 = \begin{bmatrix} a_1 & 1 \\ 0 & a_2 \end{bmatrix}$. If $R_0 < 1$, then notice that

$R_0 < 1$

$$\leftrightarrow \frac{\alpha\beta(\omega + \gamma + \delta)}{\mu^2(\mu + \omega + \gamma + \delta + \varepsilon + \varphi + \sigma)} < 1$$

$$\leftrightarrow \alpha\beta(\omega + \gamma + \delta) < \mu^2(\mu + \omega + \gamma + \delta + \varepsilon + \varphi + \sigma)$$

$$\leftrightarrow \mu^2(\mu + \omega + \gamma + \delta + \varepsilon + \varphi + \sigma) - \alpha\beta(\omega + \gamma + \delta) > 0$$

$$\leftrightarrow \mu^2(\mu + \omega + \gamma + \delta + \varepsilon + \varphi + \sigma) - \alpha\beta(\omega + \gamma + \delta) > 0 \quad (7)$$

Inequality (7) causes $a_2 > 0$, so it can be obtained $a_1, a_2 > 0$ and $Det(H_2) = a_1a_2 > 0$. Therefore, the real part of λ_3 and λ_4 are negative. Thus the real part of characteristic equation (5) is negative and it can be said that the equilibrium point is locally asymptotically stable if $R_0 < 1$.

Theorem 4. If $\mathrm{R}_0 > 1$ then the equilibrium point E_1 is locally asymptotically stable.

Proof. System (1) is linearized at equilibrium point E_1 . The eigenvalues derived from the Jacobian Matrix (4) evaluated at equilibrium point E_1 . So that the obtained characteristic equation of the Jacobian Matrix (4) evaluated at the point E_0 as follows,

$$(\lambda+\mu)^2(\lambda^2+b_1\lambda+b_2)=0 \tag{8}$$

with

$$b_1 = \beta \mathrm{I}^* + 2\mu + \omega + \gamma + \delta + \varepsilon + \varphi + \sigma \tag{9}$$

$$b_2 = \beta(\mu I^* + (\omega+\gamma+\delta+\varepsilon+\varphi+\sigma)\mathrm{I}^* - (\omega+\gamma+\delta)S^*) + \mu(\mu+\omega+\gamma+\delta+\varepsilon+\varphi+\sigma)$$

Based on equation (8) can be obtained $\lambda_{1,2} = -\mu$, Because the value of μ is positive, then the real part of eigen values is negative. Furthermore, the sign of the real part of λ_3 and λ_4 can be known through Routh Hurwitz criterion. According to the equation (9) can be obtained Routh Hurwitz matrix $H_1 = 1$ with $Det(H_1) = 1 > 0$ and $H_2 = \begin{bmatrix} b_1 & 1 \\ 0 & b_2 \end{bmatrix}$. Then the values of S^* and I^* subtitutes to b_2 so it can be obtained

$$b_2 = \frac{-\mu^3 - \mu^2(\omega+\gamma+\delta+\varepsilon+\varphi+\sigma) + \alpha\beta(\omega+\gamma+\delta)}{\mu} \tag{10}$$

Note that $b_2 > 0$ if

$$-\mu^3 - \mu^2(\omega+\gamma+\delta+\varepsilon+\varphi+\sigma) + \alpha\beta(\omega+\gamma+\delta) > 0$$

$$\leftrightarrow \alpha\beta(\omega+\gamma+\delta) > \mu^3 + \mu^2(\omega+\gamma+\delta+\varepsilon+\varphi+\sigma)$$

$$\leftrightarrow \alpha\beta(\omega+\gamma+\delta) > \mu^2(\mu+\omega+\gamma+\delta+\varepsilon+\varphi+\sigma)$$

$$\leftrightarrow \frac{\alpha\beta(\omega+\gamma+\delta)}{\mu^2(\mu+\omega+\gamma+\delta+\varepsilon+\varphi+\sigma)} > 1 \tag{11}$$

If $R_0 = \frac{\alpha\beta(\omega+\gamma+\delta)}{\mu^2(\mu+\omega+\gamma+\delta+\varepsilon+\varphi+\sigma)} > 1$ then inequality (11) is satisfied. So it can be obtained $b_1, b_2 > 0$ and $Det(H_2) = b_1 b_2 > 0$. Therefore, the real part of λ_3 and λ_4 are negative. Thus the real part of characteristic equation (8) is negative and it can be said that the equilibrium point is locally asymptotically stable if $R_0 > 1$.

Numerical Simulation

MATLAB (R2015a) software is used for numerical simulation of this mathematical model, which summarises the analytical outcomes from the previous discussion.$\omega, \gamma, \delta, \epsilon, \varphi$ and σ are the parameters that are changed in the simulation model because they have a significant impact on variations in the value of R_0. The simulation process was run using the parameter values listed in Table 2 for the simulation.

Table 2. Parameter Estimation Values

Parameter	Unit	Simulation 1 ($R_0 < 1$)	Simulation 2 ($R_0 > 1$)
α	*people*	100	100
β	*1/(people.year)*	0.00002	0.00002
μ	*1/year*	0.01	0.01
ω	*1/year*	0.004	0.04
γ	*1/year*	0.003	0.03
δ	*1/year*	0.001	0.01
ε	*1/year*	0.03	0.03
φ	*1/year*	0.02	0.02
σ	*1/year*	0.15	0.015

If used parameters of Simulation 1 in Table 1 then obtained the values of $R_0 = 0.74 < 1$. According to Theorem 2, the disease-free equilibrium point E_0 is asymptotically stable. If used parameters of Simulation 2 in Table 1 then obtained the values of $R_0 = 10.32 > 1$. According to Theorem 2, the endemic equilibrium point E_1 is asymptotically stable. The following graphic are the dynamic simulation of Simulation 1 and Simulation 2 with the initial values $S(0) = 5000$, $E(0) = 4000$, $I(0) = 3000$ and $R(0) = 2000$.

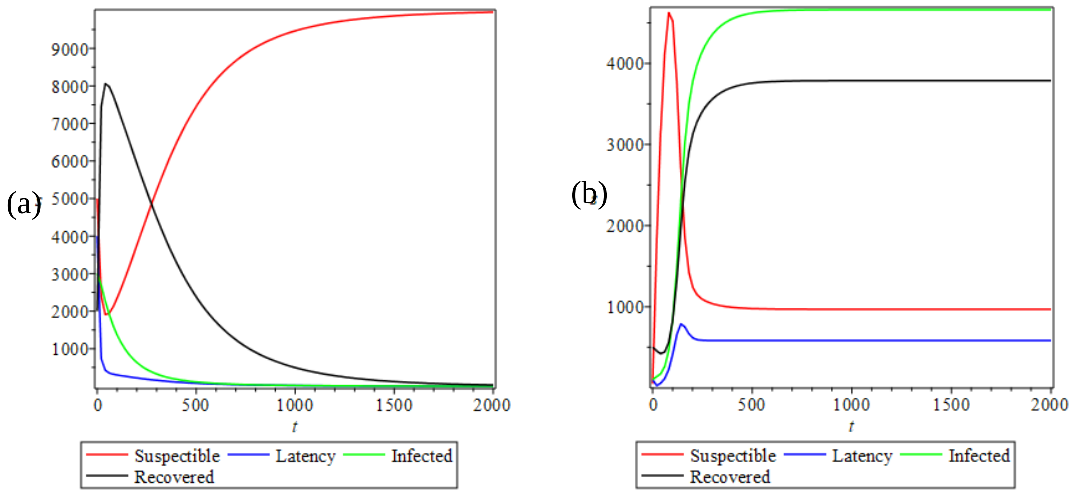

Figure 3. (a) Graph of System (1) when $R_0 < 1$ (b) Graph of System (1) when $R_0 > 1$

According to the Figure 2 (a) above obtained that if $R_0 < 1$ then the equilibrium point $E_0 = (10000,0,0,0)$ is locally asymptotic stable. Thus, it can be concluded that the illness will vanish from the populace. Meanwhile, based on Figure 2 (b), if $R_0 > 1$ then the equilibrium point $E_1 = (968,582,4661,3787)$ is locally asymptotic stable. Thus, it can be said that the disease continues to affect the population.

Sensitivity Analysis

The technique of mathematical modeling that evaluates the effects of changes in input variables on the output of a model is called sensitivity analysis. The normalize sensitivity indeks is determined by normalizing the effect of changing the parameter values on the basic reproduction number (R_0).

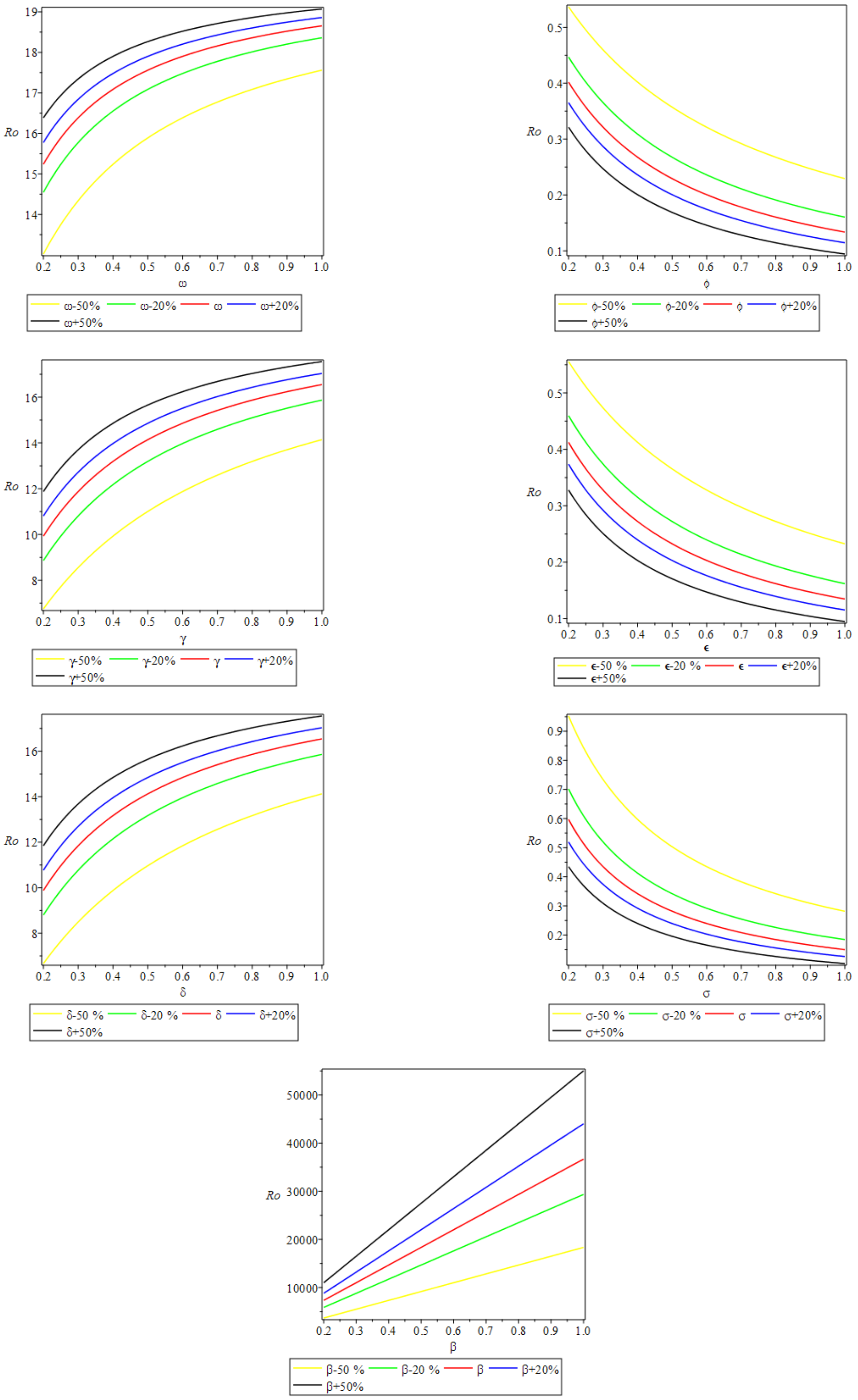

Figure 4. Parameter sensitivity index to Basic Reproduction Number (R_0)

The sensitivity simulation using simulation 2 values in Table 2 and initial condition $S(0) = 5000, E(0) = 4000, I(0) = 3000, R(0) = 2000$ are shown in Figure 3. The graphic in Figure 3 showed how the value of R_0 will increase as the rate of change of the population from S to E due to transmission of indirectly from the person affected stunting (β), the rate of change of population from E to I is due to not improving parenting patterns (ω), the rate of change of population from E to I due to not improving sanitation (γ) and the rate of change of population from E to I due to lack of health services (δ) each rise. Meanwhile, the value of R_0 will decreasing as the rate of recovery rate is due to improving parenting patterns (ε), the rate of recovery rate due to improved sanitation (φ) and the rate of recovery is due to improvements in health services (σ) each of them decreased.

CONCLUSION

In this research, we develop a SEIR mathematical model of stunting case in Indonesia. There are four compartements in this model : the population of newborn are likely to be exposed to stunting (S), the population of children were having an early symptom stunting (E), the population of affected child stunting permanent (I) and the population of children showing symptom stunting but no caught stunting (free stunting) (R). There are the equilibrium E_0 and E_1 of the formed model. If $R_0 < 1$ then system (1) has only one equilibrium point (E_0) and the stability of E_0 is locally asymptotically stable. Meanwhile, If $R_0 > 1$ then system (1) has two equilibrium point, the equilibrium point E_0 and E_1 and the stability of E_1 is locally asymptotically stable. Sensitivity analysis revealed that the parameters ω,γ,δ and β significantly contribute to the increase in the basic reproduction number (R_0). It means that the good sanitation, good healthcare, good parenting of child and decrease the rate of transmission indirectly from the person affected stunting are the most important thing to reduce the rate of stunting in Indonesia. This model is still limited by not involve the action of treatment. The next study that can be done involves medical factors or treatment compartments for stunting sufferers.

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