Volume 9 Nomor 1, February 2024, 273-284

APPLICATION OF T² HOTELING CONTROL DIAGRAM TO ANALYZE DRINKING WATER QUALITY CONTROL IN PERUMDA TIRTAULI PEMATANG SIANTAR CITY

Elviana Sari^{1*}, Rina Widyasari²

^{1,2}Mathematics Study Program, Universitas Islam Negeri Sumatera Utara, North Sumatra Province, Indonesia

*Correspondence: elviana0703192026@uinsu.ac.id

ABSTRACT

Water is the most important element in supporting human life and other living creatures, where the role of water cannot be replaced by other elements. Various human activities always require large amounts of water to meet their daily needs. People really need good water quality because it has a big impact on health. However, many people complain that the water quality at PDAM Tirtauli Pematang Siantar, the water production they get is less than satisfactory, meaning that the water production at PDAM Tirtauli Pematang Siantar is of poor quality because they have not been able to distribute water production in accordance with the wishes of the community. This study aims to determine the process of controlling the quality of clean water production at PDAM Tirtauli Pematang Siantar with the Hotelling T² control diagram. T² hoteling is a multivariate analysis technique used to compare two groups of samples using an average vector, each consisting of two or more variates. The data used in this study is drinking water production data during June-July 2022. Based on the analysis that has been done, it is found that on the T² hoteling control map, the results show that the drinking water production process at PerumdaTirtauli, Pematang Siantar city is statistically controlled because there are no points that cross the upper control limit or the lower control limit on the T² Hotelling control map.

Keywords: Control Diagram, Hotelling T², Control, Quality, Drinking Water

How to Cite: Sari, E & Widyasari, R. (2024). Application of Hotelling T^2 Control Diagram to Analyze Drinking Water Quality Control at Perumda Tirtauli Pematang Siantar City. *Mathline: Jurnal Matematika dan Pendidikan Matematika*, 9(1), 273-284. http://doi.org/10.31943/mathline.v9i1.578

PRELIMINARY

Water is an essential element in human life and other living things, the role of water cannot be replaced with other elements. Humans need large amounts of water to fulfill their life needs (Islamiyati et al., 2022). Many people complain because the quality of water obtained is not satisfactory because it has not been able to distribute water production in accordance with the wishes of the community (Austin et al., 2020).

Good water quality is needed by the community because it greatly affects health, as in PDAM Tirtauli Pematang Siantar in producing clean water using chemical and biological quality control, but in quality control it is also necessary to analyze statistically.

274 Application of Hotelling T² Control Diagram to Analyze Drinking Water Quality Control at Perumda Tirtauli Pematang Siantar City

(Putro & Sa'adah, 2021). To see the quality of water, it is necessary to pay attention to the results of laboratory tests every day. Where what is checked every day is Ph Water and Chlorine Residual(Chesnaye, et al, 2023). From the research to be carried out using several variables so that the authors use the Hotelling T-Control Diagram to implement Quality Control.(Utami, A et al., 2021).

Previous research conducted by Austin, et al (2020) was entitled Designing Multivariate Control Charts Using the T² Hotelling Method for GPA and Length of Study for IT Students at Taruma Negara University. By using a Multivariate Control Chart with the T² Hotelling Method, it is concluded that Pp and Ppk will show how good the teaching and learning process is, if both values are high or ≥ 1 then the quality of the course is good. If the score is low then there must be an analysis of the cause of the low score. The calculation results of the five courses in each year get Pp and Ppk scores of less than 1, so that the five subjects must be reviewed both from the students, the study schedule and the way the lecturer teaches. Furthermore, there is research from Puja Lestari Marulu, et al (2022) entitled Application of the T² Hotelling Control Map Fast Minimum Covariance Determinant Algorithm in Controlling the Quality of Shallots of the Palu Valley Variety. It concludes that from the results of the average vector estimation (x_mcd) and matrix estimation variancecovariance (s_mcd) shows that the use of Hotelling's T^2 control chart is more effective in detecting data that contains outliers. From the calculation results in the previous chapter, the T^2 Hotelling control chart with the fast-MCD algorithm has observations >BPA, namely 93 observations. This shows that the T^2 Hotelling control chart with the fast-MCD algorithm detects more out of control observations than the ordinary T^2 Hotelling control chart. As for the results of the multivariate process capability analysis, the Cp value obtained was 0.07, which means the value is <1 so the process is said to be uncapable or the process produces many products that do not comply with specifications.

Multivariate quality control often occurs because many cases require controlling more than two quality characteristics simultaneously. Control maps and process capability analysis are two of the instruments utilized in quality control. (Ramdhania et al., 2022). Hotelling's T^2 control map is a control chart that uses the sample mean vector and covariance matrix to find shifts in the process mean. Process capability analysis is a statistical method that looks at variability against specifications to see if it can be used to reduce variability. (Mehmood et al., 2021). Based on previous research which obtained good results, the research is interested in analyzing drinking water quality control using the application of the T^2 Hotelling control diagram to compare the results when using other research variables.

METHODS

The strategy utilized in this exploration is Hotelling. The information utilized is quantitative information. This study obtained data in the form of drinking water production data Perumda Tirtauli Pematang Siantar City during June-July 2022, the variables used in this study were Ph Water and Chlorine Residual.

T² Hotelling

 T^2 Hotelling is one of the multivariate analysis techniques used to compare two groups of samples using the mean vector, each group consisting of two or more variates. (Sugiyono, 2019). The two samples to be tested must meet the assumptions, namely: the sample has a minimum interval data scale, the sample is normally distributed, and the sample has a homogeneous data covariance. Hotelling's T^2 serves to see the difference between two experimental groups, each of which consists of two or more variates, and statistical analysis will be carried out on these variatessimultaneously. (Sasaei, 2021).

The steps used in completing the T² Hotelling control diagram are as follows :

1. The mean or average is symbolized by x topped with a line (\bar{x}) or called x bar. To determine the average value we can use the equation

$$\bar{x}_{jk} = \frac{1}{n} \sum_{i=1}^{n} x_{ijk}$$
 $i = 1, 2, \cdots, n$

Description:

 x_{iik} = i-th observation data

n = Number of data

 Determine the variance and covariance of drinking water production data. Variance is a measure of the spread of data, low variance indicates data that is clustered close to each other, while high variance indicates data that is more spread out. (Sukparungsee, et al., 2018). Determining the variance value of the two characteristics can use the equation (Trihandini, 2022).

$$S_{jk}^{2} = \frac{1}{n-1} \sum_{i=1}^{n} \left(x_{ijk} - \bar{x}_{jk} \right)^{2} \begin{cases} k = 1, 2, 3, \cdots, p \\ k = 1, 2, \cdots, m \end{cases}$$
(7)

Description:

 $S_{jk}^2 =$ Variance

 x_{ijk} = i-th observation data

 \bar{x}_{jk} = average of the sample

- n = Number of data
- 3. Furthermore, covariance is a measure that expresses the shared variance of two random variables. Covariance between two discrete random variables X and Y.To determine the value of covariance using the equation:

$$S_{jhk} = \frac{1}{n-1} \sum_{i=1}^{n} (x_{ijk} - \bar{x}_{jk}) (x_{ihk} - \bar{x}_{hk}); \quad \begin{cases} k = 1, 2, 3, \cdots m \\ j \neq h \end{cases}$$
(8)

Description:

 S_{ihk} = Covariance

 x_{ijk} = i-th Observation Data of Water Ph

 \bar{x}_{ik} = sample average of Ph Water

 x_{ihk} = i-th Observation Data of Residual Chlorine

- \bar{x}_{hk} = average of the remaining Chlorine samples
- n = Number of data
- Determining the T² Value Hotelling. After determining the mean, variance and covariance of the two characteristics, then determine the Hotelling T-value using the equation, namely :

$$T_1^2 = \frac{n}{S_1^2 S_2^2 - S_{12}^2} \left[S_2^2 (\bar{x}_1 - \bar{\bar{x}}_1)^2 + S_1^2 (\bar{x}_2 - \bar{\bar{x}}_2)^2 - 2S_{12} (\bar{x}_1 - \bar{\bar{x}}_1) (\bar{x}_2 - \bar{\bar{x}}_2) \right]$$
(9)

Description:

 $T_1^2 = T \text{ value}^2 \text{Hotelling}$

 S_1^2 = Mean Variance of the samples of Ph Air

 S_2^2 = Mean Variance of the sample of Residual Chlorine

 S_{12}^2 = Average Covariance

 \bar{x}_1 = Average value of Ph Water

 \bar{x}_1 = Average of Ph Water samples

 \bar{x}_2 = Average value of Residual Chlorine

 \bar{x}_2 = Average of the remaining Chlorine samples

n = Number of data

If the hypothesis is true

$$F = \frac{N-P}{(N-1)p}T^2 \tag{2}$$

 \mathbf{T}^{2} Nomor Sampel $\mathbf{UCL} = \mathbf{T}^{2}_{\alpha \in 2:n-1}$

Has an F distribution with P and N - p degrees of fredoom (Joshi & Patil, 2022).

Figure 1. Hotelling T^2 Control Diagram for p = 2 Quality Characteristics

5. These controller diagrams store long queues of information, so you can look into coincidence or other non-random patterns. To apply these findings to quality-related pre-characteristic conditions, they were controlled together. It is assumed that the fair dissemination of the factors is a probability distribution with quality characteristics. The method requires calculating the average illustration for each ndimensional quality characteristics (Tiryaki & Aydın, 2022).

The upper control limit for the T² Hotelling control map is:

$$UCL = \frac{P(M+1)(N-1)}{MN - M - P + 1} F_{a,p,mn - m - p + 1}$$
(3)

Where p denotes the number of quality characteristics, n and m denote the sample size and F is the distribution used. The lower control breaking point will be zero assuming the worth acquired is under nothing (Noor, Ika&Fidia, 2019).

LCL = 0.

RESULT AND DISCUSSION

The following is the data of Drinking Water Production of Perumda Tirtauli Pematang Siantar City during June-July 2022 :

Table 1. Drinking Water Production Data for the Time Span Between June-July 2022

No.	Ph of Water			R	esidu	al Ch	lorin	e		
1	6,6	6,6	6,6	6,6	6.6	0.,2	0,2	0,2	0,2	0,2
2	6,6	6,5	6,5	6,5	6.5	0,2	0,2	0,2	0,2	0,2
3	6,6	6,7	6,6	6,6	6.5	0,2	0,2	0,2	0,2	0,2
4	6,5	6,5	6,5	6,5	6.5	0,2	0,2	0,2	0,2	0,2
5	6,6	6,5	6,6	6,6	6.6	0,2	0,2	0,2	0,2	0,2
6	6,6	6,5	6,5	6,6	6.8	0,2	0,2	0,2	0,2	0,2
7	6,5	6,5	6,5	6,5	6.5	0,2	0,2	0,2	0,2	0,2
	1 2 3 4 5	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	1 6,6 6,6 6,6 6,6 0.,2 2 6,6 6,5 6,5 6,5 0,2 3 6,6 6,7 6,6 6,6 6.5 0,2 4 6,5 6,5 6,5 6,5 0,2 5 6,6 6,5 6,5 6,5 0,2 6 6,6 6,5 6,5 6,5 0,2	1 6,6 6,6 6,6 6,6 0.,2 0,2 2 6,6 6,5 6,5 6,5 6.5 0,2 0,2 3 6,6 6,7 6,6 6,6 6.5 0,2 0,2 4 6,5 6,5 6,5 6,5 6,5 0,2 0,2 5 6,6 6,5 6,5 6,5 6,5 0,2 0,2 6 6,6 6,5 6,5 6,5 6,6 0,2 0,2 6 6,6 6,5 6,5 6,6 6,6 0,2 0,2	1 6,6 6,6 6,6 6,6 0.,2 0,2 0,2 2 6,6 6,5 6,5 6,5 6,5 0,2 0,2 0,2 3 6,6 6,7 6,6 6,6 6.5 0,2 0,2 0,2 4 6,5 6,5 6,5 6,5 6,5 0,2 0,2 0,2 5 6,6 6,5 6,5 6,5 6,6 0,2 0,2 0,2 6 6,6 6,5 6,5 6,5 6,6 0,2 0,2 0,2 6 6,6 6,5 6,5 6,6 6,6 0,2 0,2 0,2	1 6,6 6,6 6,6 6,6 0.2 0,2 0,2 0,2 2 6,6 6,5 6,5 6,5 6.5 0,2 0,2 0,2 0,2 3 6,6 6,7 6,6 6,6 6.5 0,2 0,2 0,2 0,2 4 6,5 6,5 6,5 6,5 6,5 0,2 0,2 0,2 0,2 5 6,6 6,5 6,5 6,6 6,6 0,2 0,2 0,2 0,2 6 6,6 6,5 6,5 6,5 6,6 0,2 0,2 0,2 0,2 6 6,6 6,5 6,5 6,6 6,8 0,2 0,2 0,2 0,2

Date	No.	Ph of Water				R	esidu	al Ch	lorin	e	
June 23, 2022	8	6,5	6,5	6,5	6,5	6.6	0,2	0,2	0,2	0,2	0,2
July 04, 2022	9	6,7	6,5	6,6	6	6.5	0,3	0,2	0,2	0,2	0,2
July 07, 2022	10	6,5	6,5	6,5	6,5	6.5	0,2	0,2	0,2	0,2	0,2
July 11, 2022	11	6,5	6,5	6,5	6,5	6.5	0,2	0,2	0,2	0,2	0,2
July 14, 2022	12	6,5	6,5	6,5	6,5	6.5	0,2	0,2	0,2	0,2	0,2
July 18, 2022	13	6,5	6,5	6,5	6,5	6.5	0,2	0,2	0,2	0,2	0,2
July 21, 2022	14	6,5	6,5	6,5	6,5	6.5	0,2	0,2	0,2	0,2	0,2
July 25, 2022	15	6,5	6,5	6,5	6,5	6.5	0,2	0,2	0,2	0,2	0,2
July 28, 2022	16	6,5	6,5	6,6	6,5	6.5	0,2	0,2	0,2	0,2	0,2

Determining the average value , variance and covariance of sample clean water production data

The mean, variance and covariance values are shown in Table 1 below:

No.	Ave	rage	Vari	C	
110.	\bar{x}_{1k}	\bar{x}_{2k}	S_{1k}^{2}	S_{2k}^{2}	<i>S</i> _{12<i>k</i>}
1	6,6	0,2	0	0	0
2	6,5	0,2	0,002	0	0
3	6,6	0,2	0,005	0	0
4	6,5	0,2	0	0	0
5	6,6	0,2	0,002	0	0
6	6,6	0,2	0,015	0	0
7	6,5	0,2	0	0	0
8	6,5	0,2	0,002	0	0
9	6,5	0,2	0,073	0,001	0,005
10	6,5	0,2	0	0	0
11	6,5	0,2	0	0	0
12	6,5	0,2	0	0	0
13	6,5	0,2	0	0	0
14	6,5	0,2	0	0	0
15	6,5	0,2	0	0	0
16	6,5	0,2	0,002	0	0
Total	104,4	3,2	0,101	0,001	0,005
Average	6,525	0,201	0,0063	0,0001	0,0003

 Table 2. Data for mean value, variance and covariance

To control multivariate data with individual observations can be seen on the Ph air control map in Figure 2 below:

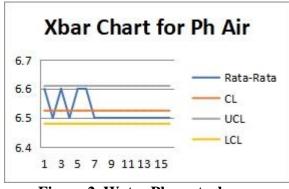


Figure 2. Water Ph control map

Based on Figure 2.there are no points that cross the control limits where the upper control limit value is 6.61 and the lower control limit is 6.48 and there are no focuses that cross the upper control cutoff and lower control limit so it very well may be affirmed that the Ph Air information is measurably controlled.

To control multivariate data with individual observations can be seen on the remaining Chlorine control map in Figure 3 below :



Figure 3. Residual chlorine control map

Based on Figure 3. there are no focuses that cross as far as possible where the upper control limit esteem is 0.204 and the lower control limit is 0.198 and there are no focuses that cross the upper control and lower control restricts so it very well may be affirmed that the leftover Chlorine information is genuinely controlled.

Determining the T² Value Hotelling

The T-square information is visible in Table 2 below:

No.	T^2
1	5.6833
2	1.0537
3	5.6833
4	1.0537
5	5.6833
6	5.6833
7	1.0537

No.	\mathbf{T}^2
8	1.0537
9	1.0537
10	1.0537
11	1.0537
12	1.0537
13	1.0537
14	1.0537
15	1.0537
16	1.0537
Total	35.3776
Average	2.21110

280 Application of Hotelling T² Control Diagram to Analyze Drinking Water Quality Control at Perumda Tirtauli Pematang Siantar City

Determining the Upper and Lower Bounds of the Control Map T² Hotelling

Next is to determine the upper and lower limits of the Hotelling Control Map T^2 Hotelling. To determine the Upper Limit (UCL/Upper *Control Limit*) If a = 0,05 then the upper limit of the T-square control map is:

$$UCL = \frac{P(M+1)(N-1)}{MN - M - P + 1} F_{a,p,mn-m-m-p+1}$$
$$UCL = \frac{2(5+1)(5-1)}{5(5) - 5 - 2 + 1} F_{a,2,5(5) - 5 - 2 + 1}$$
$$UCL = \frac{48}{19} F_{a,2,19}$$
$$UCL = \frac{48}{19} 3,52$$
$$UCL = 8,89$$
$$LCL = 0$$

The maximum furthest reaches of the T-square control map is 8.89 and the lower furthest reaches of the T-square control map is 0 (zero) because the control limit function is a quadratic function that allows no values below zero or negative.

T² Control Map Hotelling

The Hotelling T^2 control map graph can be seen in Figure 3 below:

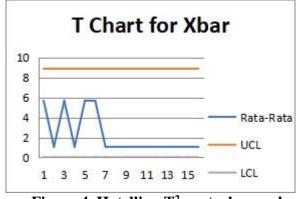


Figure 4. Hotelling T² control map chart

In view of the consequences of Figure 4. it tends to be seen that there are no points cross the upper limit or lower limit of the control map, where the upper limit of control is 8.89 and the lower limit of control is 0. So that it is possible to conclude that the production process is statistically controlled.

Based on observations made at PerumdaTirtauli Kota Pematang Siantar where the author used the Hotelling T test. Where this statistical quality control is used to determine the quality of water that has been produced by a company. In this case, control of water production is carried out in the time span between June-July 2022, with characteristics, namely water pH and Chlorine Residue. To find out the quality of clean water production can be done by looking at the control map.

Based on the Hotelling T^2 control map as shown in Figure 3, it can be seen that the pattern of points in this control map fluctuates and is irregular, and there are no points that cross the control limits where the upper control limit is 8.89 and the lower control limit value is 0. Therefore, there are no focuses outside as far as possible, and on the control map there is no typical shape of a group of points between the UCL and LCL, so the production process is stated to be in a statistically controlled state. The variability or dispersion in the clean water production process using statistical quality control shows that it is under control and runs reasonably and continuously so there is no need for any action because the water that has been studied is within the predetermined standards or in a controlled state.

CONCLUSION

Based on the research results, students with low numerical skills have indicators of mathematical expression representation. This is indicated by students creating mathematical models effectively. However, there are errors in their calculations. Students with moderate numerical skills exhibit indicators of mathematical expression and verbal

Application of Hotelling T^2 Control Diagram to Analyze Drinking Water Quality Control at Perumda Tirtauli Pematang Siantar City

representation, as shown by their use of known formulas and failure to illustrate geometric structures to solve problems. Although these students follow the correct steps, they provide incorrect answers in the final calculation stage. Students with high numerical skills demonstrate good visual representation skills, answering questions accurately and correctly.

In the context of students' responses to PISA space and shape content, there is a detected strong correlation between representation skills and the level of numerical proficiency for each student. High numerical proficiency tends to be associated with good representation skills, moderate numerical proficiency is associated with satisfactory representation skills, while low numerical proficiency is linked to less effective representation skills. Based on the results and discussion of the quality of drinking water at Perumda Tirtauli, Pematang Siantar city from June to July 2022 using the Hotelling control diagram where the Hotelling T control diagram or Hotelling T control map is a control map used if in a control process together with quality characteristics that are checked more than one. T^2 Hotelling where the Hotelling T^2 control diagram or Hotelling T^2 control map is a control map used if in a control process together with the quality characteristics that are checked more than one. Hotelling's T² control map is used when two or more characteristics technically have dependent properties or are thought to be related. Where this statistical quality control is used to determine the quality of water that has been produced by a company. So that the results obtained where the upper limit of control is 8.89 and the lower limit of control is 0. So it can be concluded that the drinking water production process at Perumda Tirtauli, Pematang Siantar city is statistically controlled because there are no points that cross the upper control limit or lower control limit on the Hotelling T^2 control map.

This research only uses water production data at PDAM Tirtauli Pematang Siantar in February 2023 with the water quality characteristics used, namely water pH and residual chlorine, so it is recommended for further research to add water quality characteristics such as turbidity and total dissolved solids.

REFERENCES

Ahsan, M., Mashuri, M., Kurwanto, H., Prastyo, D., & Dwi. (2018). Intrusion Detection System using Multivariate Control Chart Hotelling's T2 based on PCA. *International Journal on Advanced Science Engineeing Information Technology*, 8(5), 1905–1911. https://doi.org/10.18517/ijaseit.8.5.3421

Asri, G.S., Citra, F., Nisa, S., & Lestari, T.E. (2019). The bread production process using

application of the Hotelling T2 control chart. *Journal of Physics: Conference Series*, 1402(2), 1-6. https://iopscience.iop.org/article/10.1088/1742-6596/1402/2/022047/ meta

- Austin, D., Lauro, D., Manatap, Herwindiati, E., & Dyah. (2020). Perancangan Bagan Kendali Multivariate Dengan Metode T2 Hotelling Untuk IPK Dan Lama Studi Mahasiswa Ti Universitas Taruma Nagara. Jurnal Komputer Dan Informatika, 15(1), 99–296. https://journal.untar.ac.id/index.php/JKI/article/view/9446
- Chesnaye, M. K., Bell, S. L., Harte, J. M., Simonsen, L. B., Visram, A. S., Stone, M. A., Munro, K. J., & Simpson, D. M. (2023). Modified T2 Statistics for Improved Detection of Aided Cortical Auditory Evoked Potentials in Hearing-Impaired Infants. SAGE Journals, 27(1), 1-17. https://journals.sagepub.com/doi/full/10.1177/23312 165231154035
- Fathurrahmi, P., & Pratiwi, N. (2019). Pengendalian Kualitas Produksi Air Menggunakan Peta Kendali Multivariat T2 Hotelling serta Analisis Kemampuan Proses. Jurnal Statistik Industri Dan Komputasi, 4(9), 89–98. https://doi.org/10.34151/statistika.v4i01.1057
- Haddad, F., & Alsmadi, M, K. (2018). Improvement of The Hotelling's T 2 Charts Using Robust Location Winsorized One Step M-Estimator (WMOM). *Journal of Mathematic*, 50(1), 97–112. http://pu.edu.pk/images/journal/maths/PDF/Paper-9_50_1_2018.pdf
- Herwindiaty, D., & Erny. (2020). Perancangan Bagan Kendali Multivariate Dengan Metode T2 Hotelling untuk IPK dan lama Studi Mahasiswa di Universitas Tarumanegara. Jurnal Komputer Dan Informatika, 15(1), 317–325. https://journal.untar.ac.id/index.php/JKI/article/view/9446
- Islamiyati, R., Wulan, & Darwis, S. (2022). Deteksi Kerusakan Bearing Menggunakan Hotelling T2. *Bandung Conference Series*, 2(2), 234–244. https://proceedings.unisba.ac.id/index.php/BCSS/article/download/3893/1273/
- Joshi, K., & Patil, B. (2022). Multivariate statistical process monitoring and control of machining process using principal component-based Hotelling T2 charts: a machine vision approach. *International Journal of Productivity and Quality Management*, 35(1), 40–56. https://doi.org/10.1504/IJPQM.2022.120709
- Marulu, L., Puja, Junaidi, & Fadjriyani. (2022). Penerapan Peta Kendali T2 Hotelling Algoritma fast Minimum Covariance Determinant Pada Pengendalian Kualitas Bawang Merah Varietas Lembah Palu. *Journal of Probability and Statistics*, 3(2). https://doi.org/10.34312/jjps.v3i2.15522
- Mehmood, R., Riaz, M., Ali, I., & Lee, M, H. (2021). Generalizes Hotelling T2 Control Chart Based On Bivariate Ranked Set Techniques With Runs Rules. SAGE Journals, 43(10), 2180-2195. https://journals.sagepub.com/doi/abs/10.1177/0142331221992670
- Nugroho, D., Andhika, & Suwand. (2022). Diagram Kontrol T2 Hotelling Minimum Volume Ellipsoid (Penerapan pada Pengontrolan Fluktuasi Saham). Jurnal Riset Statistika (JRS)., 2(1), 19–26. https://doi.org/10.29313/jrs.vi.718
- Noor,B, I., Ika, P.,& Fidia, D, T. A.(2019). Pengendalian Kualitas Produk Menggunakan Diagram Kontrol Multivariat p(Studi Kasus: Produksi Surat Kabar Kaltim Post). Jurnal EKSPONENSIAL, 10(1), 37-46. http://jurnal.fmipa.unmul.ac.id/index.php /exponensial/article/view/389
- Putro, B., & Sa'adah, N. (2021). Bibliogtherapy Techniques AND Self-Confidence OF Students IN Mafaza Orphange Yogyakarta. Jurnal Konseling Religi, 12(1), 55–72. DOI: 10.21043/kr.v12i1.9432
- Ramdhania, A., Rifai, N, A., & Komara. (2022). Diagram Kendali Multivariat T2

4 Application of Hotelling T² Control Diagram to Analyze Drinking Water Quality Control at Perumda Tirtauli Pematang Siantar City

Hotelling pada Instalasi Pengolahan Air Limbah. *Bandung Conference Series*, 189–196. https://proceedings.unisba.ac.id/index.php/BCSS/article/download/3765/1268/

- Sadeghi, Z., Hossainali, M., & Mashhadi. (2019). Application of the T2-Hotelling test for investigating ionospheric anomalies before large earthquakes. *Journal of Atmospheric and Solar-Terrestrial Physics*, 18(5), 7–21. https://doi.org/10.1016/j.jastp.2019.01.010
- Sasaei, M., (2021). An Application Of Multivariate T2 Control Chart For Skewed Distributed Data. 52nd Annual Iranian Mathematics Conference (AIMC), IEEE, 62-64, doi: 10.1109/AIMC54250.2021.9656977. https://ieeexplore.ieee.org/document/9656977/references

Sugiyono. (2019). Metode Penelitian Kuantitatif Kualitatif dan R&d. Alfabeta.

Sukparungsee, S., Kuvattana, S., Busababodhin, P., & Areepong, Y. (2018). Bivariate copulas on the Hotelling's T2 control chart. Communications in Statistics - Simulation and Computation, 47, 413 - 419. https://www.researchgate.net/publication

/308532598_Bivariate_Copulas_on_the_Hotelling's_T_2_Control_Chart

- Tiryaki, S., & Aydın, A. (2022). Multivariate Hotelling T2 Control Chart for Monitoring Some Quality Characteristics in Medium Density Fiberboard Manufacturing Process. Drvna industrija, 73(1), 35-46. https://doi.org/10.5552/drvind.2022.2046
- Trihandini, A.(2022). Diagram Kontrol Multivariat Berdasarkan Jarak Chi-Kuadrat (Studi Kasus: Produksi Surat Kabar Kaltim Post Tahun 2017). Jurnal Matematika dan Statistika serta Aplikasinya,10(1),68-79. https://doi.org/10.24252/msa.v10i1.20722
- Utami, A, N., Fadhilah, & Suwanda. (2021). Penggunaan Estimator Robust Reweighted Minimum Covariance Determinant pada Diagram Kontrol T2 Hotelling untuk Monitoring Penyebaran Covid-19 di Korea Selatan. Jurnal Riset Statistika (JRS)., 1(1), 63–72. https://doi.org/ 10.29313/jrs.v1i1.304
- Vicentin, D.C., Oprime, P.C., & Mergulhão, R.C. (2022). Comparative Case Study Of Finite Mixture And T2-Hotelling Control Charts For Multiple Stream Monitoring. *The International Journal of Advanced Manufacturing Technology*, 123, 3233 - 3242. https://www.springerprofessional.de/en/comparative-case-studyof-finite-mixture-and-t2-hotelling-contro/23700468

284