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# STABILITY ANALYSIS OF SIR MATHEMATICS MODEL IN SHOPEEPAY LATER ADDICTION CASE

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***ABSTRACT***
This research discusses the case of Shopeepay Later addiction using the SIR mathematics model. Therefore, the purpose of this research is to build, analyze and find out the basic reproduction number ($R_0$) and simulation of the SIR model of Shopeepay Later addiction. The research begins by building the assumptions of the SIR model of Shopeepay Later addiction, finding the equilibrium point, analyzing the stability of the equilibrium point using the jacobian matrix, finding the basic reproduction number ($R_0$), and doing a simulation using Google Colab. The parameters used in this SIR model include the transmission rate parameter ($\alpha$), the recovery rate parameter ($\beta$) the self-control parameter ($\mu_1$), the promotion parameter ($\mu_2$), and the application stop parameter ($\mu_3$). The result of this study obtained the SIR model of Shopeepay Later addiction, one endemic equilibrium point is stable and the basic reproduction number $R_0 = -37,29$ that is, there is no transmission. Then to get expected conditions from the simulation result, namely given parameter values $\alpha = 0.5$, $\beta = 0.4$, and $\mu_1$, $\mu_2$, $\mu_3 = 0.9$.
**Keywords :** Adiccted, Mathematical Model, SIR Model, Shopeepay Later

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## PRELIMINARY

In 2022, technological developments have grown very significantly. We cannot avoid these technological developments, technology is used as a tool for working, studying, communicating and also shopping. Technological developments have had a huge impact (Agus, A., *et al*, 2020), especially on the commercial and financial sectors. One technology that makes things easier for the commercial and financial sectors is e-commerce (Sari, 2020).

Currently, fintech in Indonesia is very developed. In recent years, fintech has come up with interesting ideas, one of which is implementing a pay later system. Paylater itself is an online lending service without a credit card that allows consumers to pay for a transaction at a later date, either in one payment or in installments. (Linuwih, 2022).

The paylater function is the same as a credit card, this technology offers the advantage of registering, which is relatively easier compared to registering a credit card. Just like credit cards, paylater can also make it easier for consumers to fulfill all their needs, from shopping for primary needs to entertainment. Paylater was developed by a fintech company which then collaborated with e-commerce to provide paylater services (Asja, H. J., *et al*, 2021). One of the marketplaces that has paylater services and is very popular with the public is Shopee.

Shopeepay Later or SPaylater is intended to fulfill users' needs which can be done quickly and instantly. SPaylater provides convenience and benefits to its users by providing a payment method in installments without the need for a credit card ranging from 1 month to 12 months with an interest rate of 0% - 2.95% and provides a fairly large limit.

These conveniences and benefits can have a negative impact if users who use paylater services cannot be held responsible or users cannot pay their obligations. Users will have a consumptive lifestyle. They will experience a shopping addiction where they will no longer be able to differentiate between what they need and what they want or what can be called compulsive buying disorder (Haryani, I., & Herwanto, J, 2015).

One approach to describe cases of addiction to using Shopeepay Later is by creating a mathematical model. For this reason, the aim of this research is to build, analyze and determine the basic reproduction number ($R_0$) and simulate the SIR model for Shopeepay Later addiction.

## METHODS

The methodology used in this research is a type of literature review research, namely by reviewing literature on mathematical modeling and Shopeepay Later addiction. The library study method itself is research carried out by collecting data and information with the help of various kinds of references such as books, notes, other research results, documents and articles related to the problem object being discussed. Then the various data and information are used as material to combine several methods or methods so that research results can be formed that can answer the problem formulation created.

## RESULT AND DISCUSSION

This mathematical model divides the total population into three variables, namely individuals who are susceptible to addiction to Shopeepay Later (S), individuals who have experienced addiction to Shopeepay Later (I), and individuals who have stopped using Shopeepay Later (R). Previously in this study, there were several assumptions used to model the case of Shopeepay Later addiction.

1. The population used refers to research (Arifiyanti, 2021), namely all students of the Pamulang University Mathematics Study Program in 2021 with a total of 505 students.
2. The population is constant, that is, the number of individuals who are addicted to Shopeepay Later is equal to the number of individuals who stop using Shopeepay Later.
3. Potentially addicted individuals are individuals who access the Shopee app.
4. Each individual in the population is assumed to have the means to access the Shopee application (smartphone).
5. Every individual in the population is assumed to access the Shopee app.

Based on these assumptions, the SIR model for the Shopeepay Later addiction case is obtained as follows.

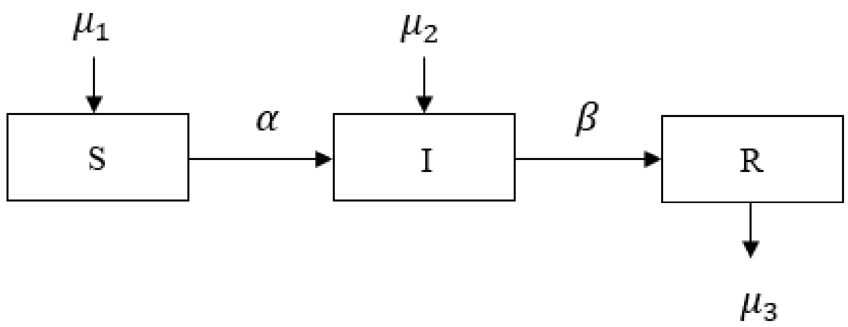


**Figure 1. Flow Chart of Mathematical Model Assumptions**

The description of the flow chart of the mathematical model can be seen in Table 1 below.

**Table 1**. **Description of Symbol**

| Symbol | Description | Unit |
|---|---|---|
| S | The population of students who use the Shopee application so that they are vulnerable to Shopeepay Later addiction | Individuals |
| I | Population that is already addicted to Shopeepay Later | Individuals |
| R | Population who are no longer addicted to Shopeepay Later | Individuals |
| $\alpha$ | Transmission rate | Individuals |
| $\beta$ | Healing rate | Individuals |
| $\mu_1$ | The rate of students who have self-control | Individuals |
| $\mu_2$ | The pace that influences students to keep using Shopeepay Later is promotion | Individuals |
| $\mu_3$ | The rate of students who have stopped using the Shopee app | Individuals |

Based on the assumptions and mathematical model charts in Figure 1, the mathematical model of Shopeepay Later addiction can be shown in the following differential equations:

$$\frac{dS}{dt} = -\alpha SI + \mu_1 \quad (1.1)$$

$$\frac{dI}{dt} = \alpha SI - \beta IR + \mu_2 I \quad (1.2)$$

$$\frac{dR}{dt} = \beta IR - \mu_3 R \quad (1.3)$$

**Equilibrium Point**

To get the equilibrium point, the differential equation in the model is equal to zero.

**Definition.** Points $\bar{x} = (\bar{x}_1, \ \bar{x}_2, \bar{x}_3, \ldots, \bar{x}_n) \in \mathbb{R}^n$ is called the equilibrium point of a system if $f_i(\bar{x}_i) = 0$ for $i = 1,2,3, \ldots, n$ (Perko, L., 1991).

From this equation, three equilibrium points are obtained, namely:

1. Equilibrium point $E_0 = (0,0,0)$
2. Equilibrium point $E_1 = \left(0, \frac{\mu_3}{\beta}, \frac{\mu_2}{\beta}\right)$
3. Equilibrium point $E_2 = \left(-\frac{\mu_2}{\alpha}, \frac{\mu_1}{\alpha}, 0\right)$

Since the equilibrium point $E_2$ has a negative value in the variable S and it is impossible if the population is negative, the equilibrium point cannot be used.

**Analysis of Equilibrium Point Stability**

To carry out an analysis of the stability of the equilibrium point, it is done by looking for the Jacobian matrix of each equilibrium point.

**Definition.** If there is a function $f = (f_1, f_2, f_3, \ldots, f_n)$ on a system where $i = 1,2,3, \ldots, n$. Then the Jacobian matrix of $f$ applies at the point $\bar{x}$,

$$J\,f(x) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \frac{\partial f_3}{\partial x_1} & \frac{\partial f_3}{\partial x_2} & \cdots & \frac{\partial f_3}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix}.$$

(Kocak, H. and Hole, J. K. , 1991).

The jacobian matrix of the equilibrium point $E_0$ is as follows.

$$JE_0 = \begin{bmatrix} \mu_1 & 0 & 0 \\ 0 & \mu_2 & 0 \\ 0 & 0 & -\mu_3 \end{bmatrix}$$

And for the jacobian matrix of the point $E_1$ is as follows.

$$JE_1 = \begin{bmatrix} -\frac{\alpha\mu_3}{\beta} + \mu_1 & 0 & 0 \\ \frac{\alpha\mu_3}{\beta} & 0 & -\mu_3 \\ 0 & \mu_2 & 0 \end{bmatrix}$$

To determine the stability of the equilibrium point $E_0$ and $E_1$ then first find the eigenvalue, namely

a) $E_0 = \partial(0,0,0)$

$\det|\lambda I - J| = 0$

$$\det\left|\lambda\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} \mu_1 & 0 & 0 \\ 0 & \mu_2 & 0 \\ 0 & 0 & -\mu_3 \end{bmatrix}\right| = 0$$

$$\det\left|\begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} \mu_1 & 0 & 0 \\ 0 & \mu_2 & 0 \\ 0 & 0 & -\mu_3 \end{bmatrix}\right| = 0$$

$$\det\begin{vmatrix} \lambda - \mu_1 & 0 & 0 \\ 0 & \lambda - \mu_2 & 0 \\ 0 & 0 & \lambda + \mu_3 \end{vmatrix} = 0$$

$\lambda - \mu_1 = 0$ or $\lambda - \mu_2 = 0$ or $\lambda + \mu_3 = 0$

So that the eigenvalue is obtained as follows.

$\lambda_1 = \mu_1$

$\lambda_2 = \mu_2$

$\lambda_3 = -\mu_3$

A point is said to be stable if every real eigenvalue is negative ($\lambda_i < 0$ to all $i$). Because $\lambda_1$ and $\lambda_2$ is positive, then the equilibrium point $E_0 = (0,0,0)$ is unstable.

b) $E_1 = \partial\left(0, \frac{\mu_3}{\beta}, \frac{\mu_2}{\beta}\right)$

$\det|\lambda I - J| = 0$

$$\det\left|\lambda\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}-\begin{bmatrix}-\frac{\alpha\mu_3}{\beta}+\mu_1&0&0\\\frac{\alpha\mu_3}{\beta}&0&-\mu_3\\0&\mu_2&0\end{bmatrix}\right|=0$$

$$\det\left|\begin{bmatrix}\lambda&0&0\\0&\lambda&0\\0&0&\lambda\end{bmatrix}-\begin{bmatrix}-\frac{\alpha\mu_3}{\beta}+\mu_1&0&0\\\frac{\alpha\mu_3}{\beta}&0&-\mu_3\\0&\mu_2&0\end{bmatrix}\right|=0$$

$$\det\begin{vmatrix}\lambda+\frac{\alpha\mu_3}{\beta}-\mu_1&0&0\\-\frac{\alpha\mu_3}{\beta}&\lambda&\mu_3\\0&-\mu_2&\lambda\end{vmatrix}=0$$

$$\left(\left(\lambda+\frac{\alpha\mu_3}{\beta}-\mu_1\right)(\lambda)(\lambda)+0+0\right)-\left(0+\left(\lambda+\frac{\alpha\mu_3}{\beta}-\mu_1\right)(\mu_3)(-\mu_2)\right)$$

$$\left(\left(\lambda+\frac{\alpha\mu_3}{\beta}-\mu_1\right)(\lambda^2)\right)-\left(\left(\lambda+\frac{\alpha\mu_3}{\beta}-\mu_1\right)(-\mu_2\mu_3)\right)$$

$$\lambda^3-\mu_1\lambda^2+\frac{\alpha\mu_3\lambda^2}{\beta}+\lambda\mu_2\mu_3-\mu_1\mu_2\mu_3+\frac{\mu_2\alpha\mu_3{}^2}{\beta}$$

$$\left(\lambda+\frac{\alpha\mu_3}{\beta}-\mu_1\right)(\lambda^2+\mu_2\mu_3)$$

$\lambda=-\frac{\alpha\mu_3}{\beta}+\mu_1$ or $\lambda=\pm\sqrt{-\mu_2\mu_3}$

So that the eigenvalue is obtained as follows.

$$\lambda_1=-\frac{\alpha\mu_3}{\beta}+\mu_1$$

$$\lambda_2=-\sqrt{-\mu_2\mu_3}$$

$$\lambda_3=\sqrt{-\mu_2\mu_3}$$

A point is said to be stable if every real eigenvalue is negative ($\lambda_i<0$ to all $i$) and each component of the real part of the complex eigenvalue is smaller or equal to zero ($Re(\lambda_i)\le 0$ to all *i*). Because $\lambda_1$ and $\lambda_2$ is negative and $\lambda_3$ component of the real part of the complex eigenvalue is less than zero, then the equilibrium point $E_1=\partial\left(0,\frac{\mu_3}{\beta},\frac{\mu_2}{\beta}\right)$ is stable.

**Basic Reproduction Number**

To find out if addiction will become a pandemic, the basic reproduction number value is calculated $(R_0)$. An epidemic requires that R(t)>1, so that the prevalence of infection increases because more than one new infection arises from the average infected person before that person is 'lost' from the infected population (White, P., J., 2017) (Ariesy, D. E, 2018) (Basir, Choirul and Aden, 2022). The basic reproduction number $(R_0)$ can be found using the next generation matrix obtained from variable I in the system of equations (1.2), namely

$$\frac{dI}{dt} = \alpha SI - \beta IR + \mu_2 I$$

As a result, it is obtained $\varphi = [\alpha SI]$ and $\psi = [-\beta IR + \mu_2]$, so that the results of $\varphi$ and $\psi$ respectively are $F = [\alpha S]$ and $V = [-\beta R + \mu_2]$, then the next generation matrix is obtained as follows:

$$K = FV^{-1} = \alpha S \frac{1}{-\beta R + \mu_2} = \frac{\alpha S}{-\beta R + \mu_2}$$

Thus the basic reproduction number is obtained $R_0 = \frac{\alpha S}{-\beta R+\mu_2}$.

**SIR Model Simulations Using Google Colabs**

Simulations for the SIR model for the Shopeepay Later addiction case use the original data. Referring to research (Arifiyanti, 2021), the number of students in the Department Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Pamulang University is 505 students. Then the initial data as follows the table 2.

**Table 2. Initial Data**

| Variable | Amount | Description |
|---|---|---|
| S(0) | 440 | Result of survey |
| I(0) | 48 | Result of survey |
| R(0) | 17 | Result of survey |
| Total | 505 | (Arifiyanti, 2021) |

The parameter values used in the simulation are parameter values that will describe the conditions following the equilibriums point obtained in the mathematical model.

a) Simulation 1

Simulations were conducted using assumed parameter values such as $\alpha = 0.51, \beta = 0.04$, and $\mu_1$ up to $\mu_3$ retrieved from 0.96. So obtained $R_0 = 801{,}43 > 1$ indicates the transmission of addiction from one individual to another. Then with the initial data presented in Table 2 and 30 months, the following simulation results are obtained.

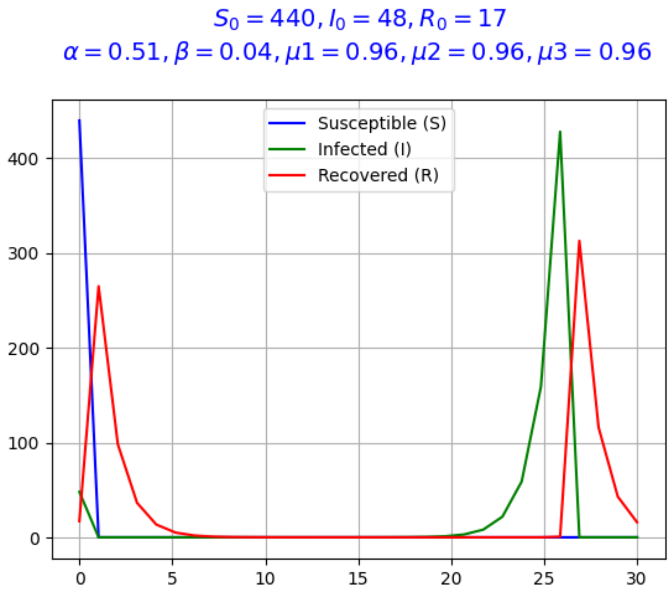


**Figure 2. First Simulation**

It can be seen that in the susceptible variable, the number of students who are potentially addicted to Shopeepay Later takes only 1 month to touch the lowest number, which is up to 0 students out of 505 students.

Then in the infected variable, the number of students who are addicted to Shopeepay Later takes approximately 25 months to touch the highest number, which is around 400 more students out of 505 students and will experience a decrease until it reaches its lowest number.

Finally, in the recovered variable, the number of students who are no longer addicted to Shopeepay Later has increased and also decreased and it takes approximately 27 months to touch the highest number, which is around 300 more students out of 505 students and begins to decline again in the following months.

b) Simulation 2

Furthermore, in simulation 2 using the assumption of parameter values $\alpha = 0.51, \beta = 0.04$, and $\mu_1$ up to $\mu_3$ retrieved from 0.2 obtained $R_0 = -467{,}51 < 1$ indicates that there is no transmission of addiction from one individual to another. Then with the initial data presented in Table.2 and a time of 30 months, the following simulation results are obtained.

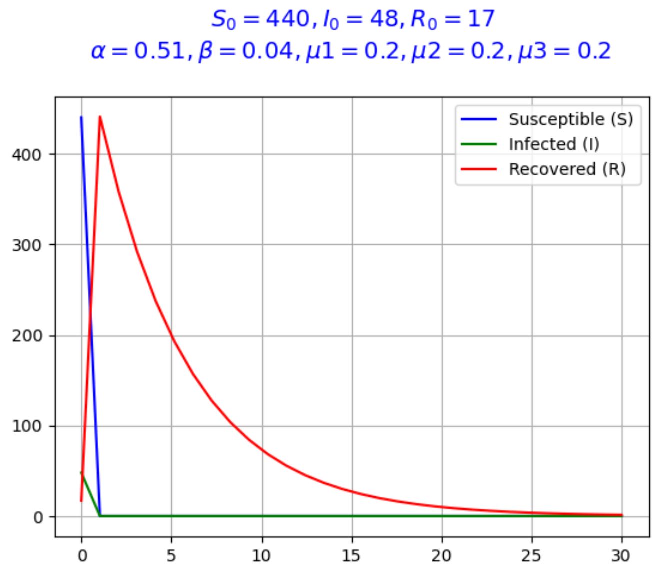


**Figure 3. Second Simulation**

It can be seen that there are only recovered variables that experience a decrease every month and it takes about 26 months to reach the lowest number until all populations total 0. From the first month to the 30th month there is no increase and only a decrease in number (constant).

c) Simulation 3

For simulation 3, parameter value assumptions were used $\alpha = 0.5, \beta = 0.4$, and $\mu_1$ up to $\mu_3$ retrieved from 0.9, so obtained $R_0 = -37{,}29 < 1$ shows that there is no transmission of addiction from one individual to another. Then with the initial data presented in Table 2 and a time of 30 months, the following simulation results are obtained.

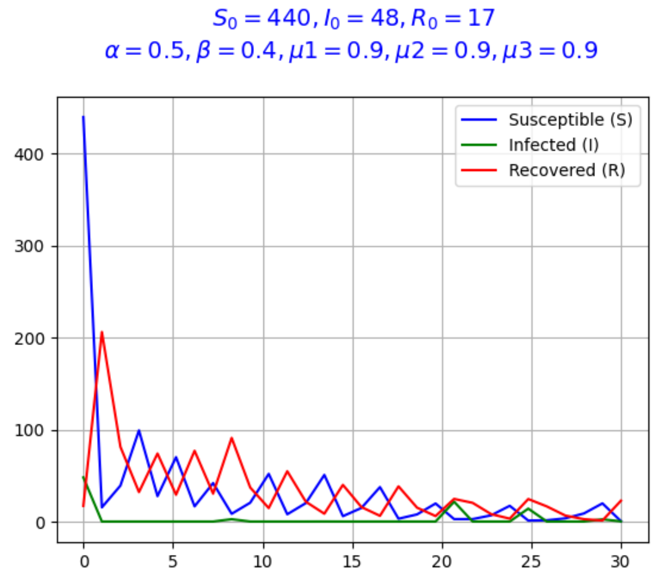


**Figure 4. Third Simulation**

It can be seen that in the susceptible variable, the number of students who are susceptible to Shopeepay Later addiction increases and decreases every month, and around the 3rd month it reaches its highest number of 100 students. Then, in the 18th month to the 30th month, there was only a slight increase and decrease. For the infected variable, the number of students addicted to Shopeepay Later only experienced a slight increase, which only reached its highest number in the 21st month.

Just like the susceptible variable, in the recovered variable the number of students who are no longer addicted to Shopeepay Later also increases and decreases every month, and reaches its highest number around month 2, which is more than 200 students. Then in the 18th month to the 30th month it also experienced a slight increase and decrease. This condition is an expected condition because only a small number of infected and a large number of recovered.

## CONCLUSION

From the results and discussions that have been carried out, the following conclusions can be obtained.

1. The formulation of the SIR mathematical model of Shopeepay Later addiction in the form of differential equations is as follows.

$$\frac{dS}{dt} = -\alpha SI + \mu_1 \tag{1.1}$$

$$\frac{dI}{dt} = \alpha SI - \beta IR + \mu_2 I \tag{1.2}$$

$$\frac{dR}{dt} = \beta IR - \mu_3 R \tag{1.3}$$

2. The equilibrium form and stability point of the SIR mathematical model of Shopeepay Later addiction are obtained as follows.
   a. Equilibrium point $E_0 = (0,0,0)$ is unstable
   b. Equilibrium point $E_1 = \left(0, \frac{\mu_3}{\beta}, \frac{\mu_2}{\beta}\right)$ is stable
   c. Equilibrium point $E_2 = \left(-\frac{\mu_2}{\alpha}, \frac{\mu_1}{\alpha}, 0\right)$ is unusable because it has a negative value in the susceptible parameter

3. Basic reproduction number obtained $R_0 = \frac{\alpha S}{-\beta R + \mu_2}$
4. To get the expected conditions from the simulation results, namely by using parameter values $\alpha$ = 0.5, $\beta$ = 0.4, and $\mu_1$ up to $\mu_3$ retrieved from 0.9

Based on the conclusions that have been obtained, the researcher suggests that further research should develop the SIR mathematical model by adding new variables, parameters or compartments and other assumptions so that it can further refine this research and find out various other conditions with different assumptions. Finally, this research is a closed population, it is hoped that in future studies it can be an open population.

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