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APPLICATION OF THE ADAMS BASHFORTH-MOULTON METHOD TO COFFEE PRODUCTION QUANTITY APPROACH

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ABSTRACT

Coffee is a type of plantation plant that has long been cultivated so that it has high economic value. Therefore, the authors are interested in approximating the amount of coffee production using the Adams Basforth Moulton method. The Adams Bashforth Moulton method is a way of finding a numerical solution at a certain point of a non-linear differential equation with a known initial value. The differential equation is first solved using the fourth-order Runge Kutta method to obtain four initial solutions which are then substituted into the fourth order Adams Bashforth predictor equation. Furthermore, the approximation value is corrected using the fourth-order Adams Moulton corrector equation. If the relative error is less than the stopping criterion 5×10^{-7}, then the iteration is stopped. The iterations are performed 25 times with an interval of [0,25]. The results of the approximation using the Adams Bashforth Moulton method on the Verhults equation in the approximation show that every year the amount of production increases. The results of the total coffee production in Aceh Province in 2022 will reach 71468 tons and in 2023 it will reach 72966 tons and in 2024 it will reach 74465 tons.

Keywords: Adam Basforth Moulton, Approximation, Coffe.

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PRELIMINARY

Coffee plants grown in Indonesia include robusta, arabica and liberica coffee. Coffee plays an important role not only as a source of foreign exchange but also as a source of income for Indonesian coffe producers (Ervina, 2021). Coffee contains caffeine which can affect one's body coordination (Martauli, 2018). Caffeine can stimulate the central nervous system so that it encourages nervous work which can have an effect on increasing morale, thinking faster, concentrating and reducing mental fatigue (Zarwinda, 2019). When after drinking coffee, individuals can usually feel psychological freshness, therefore, this caffeine can be beneficial for one's body health if it is still consumed within reasonable limits, namely not exceeding 100 mg (Fajriana, 2018).

Aceh is one of the Arabica coffee growing regions in Indonesia. Aceh largest coffee growing regions are Aceh Tengah and Bener Meriah. Gayo arabica coffee is one of Indonesia's main export products, which is well known in the domestic and international

markets (Bagio, 2021). This type of coffee is well-known in the world market because it has a distinctive taste such as a strong aroma and body (Munira et al, 2020). Arabica coffee is very suitable for growing in the Gayo Highlands. Geographically it is located between 3º45`0"–4º59`0" North Latitude and 96º16`10"–97º55`10" East Longitude (Munira, 2020). This is because coffe trees play an important role as a source of local income, contributing to the local economy and absorbing labor and contributing to famers income (Eskak, 2016).

Aceh coffee is one of the varieties of Sumatran coffee that has the finest texture and has the best texture and the strongest, sweetest and purest taste. Aceh coffee is the standard for aging coffee beans before purchasing by consumers around the world. Aceh coffee beans give off a unique spice flavor and are one of the most sought-after coffees in the world with a cupping test score of 86 – 90 (Majidah, 2021). There are several factors that affect the taste of coffee, namely the altitude and soil type, coffee varieties and post-harvest processing methods. Some of these factors become characteristics and standards for consumers in buying coffee (Kasimin, 2021). Aceh coffee has received a Fairtrade Certified Certificate from an International Organization on May 27, 2010 and Aceh coffee received an IG certificate (Geographical Indication) and Aceh coffee is the best organic coffee (Zuniyanto, 2019).

The amount of coffee production has increased every year, in increasing the amount of coffee production there are several obstacles that occur, namely, constraints in cooperation due to the low share of product profits to farmers. Lack of capital for farmers and collectors as well as constraints on meeting quality export standards (Ruaida, 2020). Therefore, the authors are interested in approximating the amount of coffee production using the Adams Basforth Moulton (ABM) method. In principle, the Adams-Bashforth-Moulton method provides a fairly accurate solution in solving approximation (Badan Pusat Stasistik Provinsi Aceh, 2020). The method used to find the initial solution is the Runge-Kutta method (Siregar, 2022). The approximation of the amount of coffee production can assist the government in estimating the amount of assistance or efforts to develop coffee production that will be given to coffee farmers and collectors. Based on the problems stated above, the formulation of the problem is how is the result of applying the Adams Bashforth Moulton method to the amount of coffee production. By utilizing the ABM method, this research not only fills the gap in the use of more dynamic and sophisticated prediction methods but also offers practical solutions that can be applied in the context of coffee production in Aceh. It is expected to provide deeper and more relevant insights for stakeholders in the agricultural sector.

METHODS

This type of research is applied research. Applied research is a type of research whose results can be directly applied to solve the problems faced. In this study, the Adams-Bashforth-Moulton method is applied to solve the problem of approximating the coffee production quantity in Aceh Province. The data used in this study is secondary data. Secondary data is primary data that has been further processed and presented in the form of tables or diagrams. The data was obtained from the publications of the BPS (Central Statistics Agency) of Aceh Province, consisting of the coffee production quantities from 1996 to 2020 in Aceh Province. The procedure conducted in this study is as follows:

1. Determine the data to be used in the equation.
   a. Growth rate.
   b. b. Carrying capacity.
2. Provide the Verhulst equation:

$$\frac{dN}{dt} = k\left(1 - \frac{N}{K}\right)N = f(t, N)$$

3. Calculate the four initial solution y_0, y_1, y_2 and y_3 using the fourth-order Runge-Kutta method.

$$y_{r+1} = y_r + \left(\frac{k_1 + 4k_2 + k_3}{6}\right)$$

4. Determine the values $f_r, f_{r-1}, f_{r-2}, f_{r-3}$ with $r = 3,4,\ldots,n$ pada persamaan.

$$\frac{dP}{dt} = k\left(1 - \frac{N}{K}\right)N = f(t, N)$$

5. Determine the numerical solution using the Adams-Bashforth method with the predictor equation.

$$y_{n+1}^{(0)} = y_n + \frac{h}{24}(55f_n - 59f_{n-1} + 37f_{n-2} - 9f_{n-3})$$

6. Determine the numerical solution using the Adams-Moulton method with the corrector equation.

$$y_{n+1}^{(k)} = y_n + \frac{h}{24}(9f_{n+1} + 19f_n - 5f_{n-1} + f_{n-2})$$

7. Adams-Moulton correction is iterated for r until it meets the condition

$$\frac{\left|y_{r+1}^{(1)} - y_{r+1}^{(0)}\right|}{\left|y_{r+1}^{(1)}\right|} < \varepsilon$$

For $\varepsilon = 5 \times 10^{-7}$.

8. If the stopping criteria are not met, then a step size analysis is performed *h*.

a. If $\frac{19}{270}.\frac{\left|y_{r+1}^{(1)}-y_{r+1}^{(0)}\right|}{\left|y_{r+1}^{(1)}\right|} > 10^{-7}$, then h is replaced with $\frac{h}{2}$.

b. If $\frac{19}{270}.\frac{\left|y_{r+1}^{(1)}-y_{r+1}^{(0)}\right|}{\left|y_{r+1}^{(1)}\right|} < 10^{-7}$, then h is replaced with $2h$.

9. If the stopping criteria are met, proceed by returning to step (5).

RESULT AND DISCUSSION

3.1 Research resul

According to the research procedure, there are several steps that can be taken to obtain approximation results on using the Adam Bashforth Moulton (ABM) method as follows:

Step 1 Determine the data used, growth rate and capacity.

The data used in this study is the amount of coffee production starting from 1999 to 2021 obtained from the BPS Aceh Province website.

a. Growth rate

To determine the growth rate:

$$k = \frac{1}{t} ln\left(\frac{N(t)}{N_0}\right)$$

$$k = \frac{1}{1} ln\left(\frac{41535}{39922}\right) = 0{,}04$$

Based on the value of k = 0.04, the growth rate of coffee production is 4% and the population will increase exponentially.

b. *Carrying capacity*

The value of the carrying capacity (K) can be obtained by trial and error, namely substituting the estimated value of K into the Verhults model (Side, 2019). In the interval [0.25] with many iterations $n = 25$,

$$h = \frac{b-a}{n} = \frac{25-0}{25} = 1$$

And step size $h = 1$.

Step 2 Determine the Verhulst model as a nonlinear differential equation.

$$\frac{dN}{dt} = k\left(1-\frac{N}{K}\right)N, \text{ with initial value } N(t_0) = N_0$$

$$= 0{,}04\left(1-\frac{N}{150000}\right)N$$

The Verhults model equation is solved to find a numerical solution, so the solution is obtained using the Runge-Kutta method and the Adams Basforth Moulton method.

Step 3 Define four initial solutions N_0, N_1, N_2 and N_3 using the Runge Kutta method using equations (11), (12), (13) and (14).

Is known $\frac{dN}{dt} = 0{,}04\left(1 - \frac{N}{150000}\right)N$ with $N_0 = 39922$ and $h = 1$.

Table 3. Initial solution using the Runge Kutta method

r	t_r	$h = 1$	
		N	$N' = f_r(t_r, N) = 0{,}04\left(1 - \frac{N}{150000}\right)N$
0	1	39922,000000000	1171,875710880
1	2	41104,781905311	1193,630450699
2	3	42309,137614383	1215,015337779
3	4	43534,673231173	1235,982189644

In table 3 it can be seen that the value of the initial solution using the Runge Kutta method will be applied to the Verhults model.

Step 4 Determine the numerical solution using the Adams Bashforth method

$$N_{r+1}^{(0)} = N_r + \frac{h}{24}(55f_r - 59f_{r-1} + 37f_{r-2} - 9f_{r-3})$$

With,

$N_r =$ The value of the initial solution to the Runge Kutta equation

$f_r =$ The value of the initial solution using the Verhults equation

For $t_{r+1} = 4$

$N_r = 43534{,}673231173$

$$N_4^{(0)} = 43534{,}673231173 + \frac{1}{24}(55(1235{,}982189644) - 59(1215{,}015337779) + 37(1193{,}630450699) - 9(1171{,}875710880))$$

$N_4^{(0)} = 44780{,}946597265$

Step 5 Determine the numerical solution using the Adams Moulton method

$$N_{r+1}^{(k)} = N_r + \frac{h}{24}\left(9f_{r+1}^{(0)} + 19f_r - 5f_{r-1} + f_{r-2}\right)$$

$$f_{r+1}^{(0)} = f(f_{r+1}, N_{r+1}^{(0)})$$

For $t_{r+1} = 4$

$N_r = 43534{,}673231173$

$f_4^{(0)} = f_4(4; 44780{,}946597265)$

$= 0{,}04\left(1 - \frac{44780{,}946597265}{150000}\right)44780{,}946597265$

$= 1256{,}4823496854$

$$N_{3+1}^{(1)} = 43534{,}673231173 + \frac{1}{24}(9(1256{,}4823496854) + 19(1235{,}982189644) - 5(1215{,}015337779) + (1193{,}630450699)$$

$N_4^{(1)} = 44780{,}946419242$

Table 4. Approximation Results Using the Adams Basforth Moulton Method

Years	t_r	$h = 1$		Relative Error
		$N_r^{(0)}$	N_r	
1999	0		39922,000000000	
2000	1		41104,781905311	
2001	2		42309,137614383	
2002	3		43534,673231173	
2003	4	44780,946597265	44780,946419242	4×10^{-9}
2004	5	46047,466141088	46047,466038366	2×10^{-9}
2005	6	47333,691979217	47333,691955913	5×10^{-10}
2006	7	48639,035042857	48639,035103006	1×10^{-9}
2007	8	49962,857604455	49962,857751355	3×10^{-9}
2008	9	51304,473789854	51304,474026263	5×10^{-9}
2009	10	52663,150334244	52663,150662268	6×10^{-9}
2010	11	54038,107584936	54038,108005967	8×10^{-9}
Years	t_r	$h = 1$		**Relative Error**
		$N_r^{(0)}$	N_r	
2011	12	55428,520753944	55428,521268613	9×10^{-9}
2012	13	56833,521420824	56833,522028956	1×10^{-8}
2013	14	58252,199284017	58252,199984598	1×10^{-8}
2014	15	59683,604156703	59683,604947856	1×10^{-8}
2015	16	61126,748200830	61126,749079801	1×10^{-8}
2016	17	62580,608390631	62580,609353793	2×10^{-8}
2017	18	64044,129194659	64044,130237523	2×10^{-8}
2018	19	65516,225463073	65516,226580312	2×10^{-8}
2019	20	66995,785504742	66995,786690233	2×10^{-8}
2020	21	68481,674336701	68481,675583573	2×10^{-9}
2021	22	69972,737086572	69972,738387271	2×10^{-8}
2022	23	71467,802526874	71467,803873241	2×10^{-8}
2023	24	72965,686718650	72965,688102003	2×10^{-8}
2024	25	74465,196740599	74465,198151828	2×10^{-8}

In table 4 it can be seen that the error is relatively smaller than the termination criteria, so the iteration is continued until the 25th iteration. The approximation results using the Adams Basforth Moulton method on the Verhults model in Table 4 have met the termination criteria $\varepsilon = 5 \times 10^{-7}$. Score N_r for r = 4 to 25 in Table 4 is the Adam-Moulton corrector.

Step 6. Determine the MAPE value on the approximation results of the amount of coffee production.

$$\text{MAPE} = \frac{1}{n}\sum_{i=1}^{n}\frac{|Actual - Approximation|}{|Approximation|} \times 100\%$$

$$\frac{\left|\left(\left(\frac{39922-39922}{39922}\right)+\left(\frac{41535-41104,781905311}{41535}\right)+\cdots+\left(\frac{73674-69972,738387271}{73674}\right)\right)\right| \times 100}{23}$$

$$= 0{,}1285475584930\% \approx 0{,}129\%$$

The MAPE value obtained is close to 0.129% which means that the approximation criterion is very good.

3.2 Discussion

This study discusses the application of the Adams Bashforth Moulton method to the Verhults equation in approximation. The results of the analysis of the application of the Adams Bashforth Moulton obtained is the approximation by using the following equation $\frac{dN}{dt} = k\left(1 - \frac{N}{K}\right)N$ with a value of k = 0.04 and K = 150,000 tons. The equations in the Verhults model are first solved to get the initial solution f_0 =1.171,87571088 , f_1 =1.193,630450699, f_2 =1.215,015337779 and f_3 =1.235,982189644. The initial solution is substituted for the Adam Basforth Moulton method to obtain approximation results using r = 3,4,...,n then the approximation results are corrected using the Adam Moulton method. For a comparison of the actual data on the amount of coffee production and the approximation results for the coffee production, found that the approximation results for the amount of coffee production are close to the actual data from the BPS Aceh Province and the approximation results in 2022-2024 are in the 23rd to 25th iterations with the approximation number can be seen in table 5.

Table 5. The results of the approximation of the amount of coffee production

Years	Production Coffee (Ton)
2022	71.468
2023	72.966
2024	74.465

Table 5 shows that the results of the total coffee production in Aceh Province in 2022-2024 are increasing every year. The accuracy of the approximation results is known by calculating the MAPE value, namely by using the actual value and the approximation value. The MAPE value obtained is close to 0.129% indicates that the current approach is quite good at predicting coffee production in Aceh. However, it should be noted that using ABM can provide better results depending on the available historical data and the accuracy in model adjustments.

The ABM method may have limitations in cases where historical data is incomplete or there is uncertainty in production trends. Additionally, the success of this method also highly depends on the quality and quantity of the data used. In practical implementation, ABM may require more complex calculations and more computation compared to other simpler approaches. By adopting the Adams-Bashforth-Moulton method, future research is

expected to produce more accurate and reliable predictions for coffee production in Aceh Province, as well as provide deeper insights into the factors affecting production.

CONCLUSION

Based on the problem formulation and results obtained from this research, it can be concluded that the results of the analysis of the application of the Adams Bashforth Moulton Verhults equation method in approaching the amount of coffee production show that every year the amount of production increases. The results of the total coffee production in Aceh Province in 2022 will reach 71,468 tons and in 2023 it will reach 72,966 tons and in 2024 it will reach 74,465 tons.

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