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THE METRIC DIMENSION AND PARTITION DIMENSION OF THE AMALGAMATION OF COMPLETE GRAPHS

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ABSTRACT

In this paper, there is a section that identifies the aim of the research and makes it possible to suggest exploring the metric dimension and partition dimension of the amalgamation of complete graphs, which we would denote as $Amal(K_n, v_0)_t$. There are three steps conducted to achieve the research goals in this paper. To begin with, compute the lower bound of the metric dimension and the partition dimension of the graph $Amal(K_n, v_0)_t$. The second step is to find the upper bounds of the metric dimension and partition dimension of the graph $Amal(K_n, v_0)_t$ by demonstrating that the representation of any vertex in $Amal(K_n, v_0)_t$ is distinct. Finally, the exact values of the metric dimension and partition dimension of the graph $Amal(K_n, v_0)_t$ are found if the lower and upper bounds are determined. The exact value of the metric dimension of the graph $Amal(K_n, v_0)_t$ is denoted as $dim(Amal(K_n, v_0)_t)$, while the exact value of the partition dimension is denoted as $pd(Amal(K_n, v_0)_t)$. In this research, it is found that $dim(Amal(K_n, v_0)_t) = (n-2)t$ for $n, t \in \mathbb{N}$ with $n \geq 3$ and $t \geq 2$. It is also found that $pd(Amal(K_n, v_0)_t) = n$ for $2 \leq t \leq C_{n-1}^{n}$.
Keywords: Metric Dimension, Partition Dimension, Amalgamation of Complete Graphs

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PRELIMINARY

Mathematics is a branch of science that plays a significant role in helping to solve various problems. One of the fields that can be developed within mathematics is graph theory. Graph theory is a branch of mathematics that aids in solving problems in a more systematic way. The representation of a graph depicts a discrete object as a vertex and the connections between these objects as edges (Irene et al., 2024). This allows for easier visualization and analysis of relationships between objects.

Graph theory is widely applied in modeling real-world networks, including transportation systems, communication networks, and social structures. In transportation networks, each location can be represented as a vertex, while roads act as edges connecting them. Understanding the structure of these networks is essential for route optimization and network efficiency (Biswal, 2015). One method for analyzing network properties is through metric dimension, which provides a measure of uniqueness in identifying locations within a graph. This study expands on previous research by exploring how the metric and partition dimensions change when complete graphs are amalgamated at a single vertex.

Historically, one of the earliest cases that introduced the use of graphs was the problem of the Königsberg bridges in 1736. The Königsberg bridge problem posed the question of whether it was possible for someone to cross the seven bridges connecting four landmasses exactly once and return to the starting point (Aziz, 2021). In the same year, Leonhard Euler, a Swiss mathematician, successfully provided a solution to this problem with a simple proof by modeling it as a graph.

Metric dimension is a term which appears primarily in the works of graph theory. When determining the metric dimension of a given graph, certain concepts are applied. First is the term of the distance between the two vertices of the graph, and the second is the concept of a resolving set. Harary & Melter (1976) were the first to conduct research on the invention of metric dimension mathematic concepts.

Let $H = (V, E)$ be a connected graph. For every pair of vertices $u, v \in V(H)$ and an ordered set $W \subset V(H)$, with $W = \{w_1, w_2, \dots, w_k\}$, the representation of a vertex v with respect to W, denoted by $r(v|W)$, is a k-vektor

$$r(v|W) = (d(v, w_1), d(v, w_2), \dots, d(v, w_k)).$$

If there exists $w_i \in W$ such that $d(u, w_i)$ is not equal to $d(v, w_i)$, it must follow that $r(u|W)$ is different from $r(v|W)$. If all vertices $u, v \in V(H)$ are represented by W such that there exists $w_i \in W$ for which the representation of u and v with respect to W is not the same, then W is termed a resolving set for the graph H. A resolving set with the minimum number of elements is referred to as a basis of H. The number of elements of the minimum such resolving set is referred to as the metric dimension of the graph H and is denoted $dim(H)$ (Chartrand, Eroh, et al., 2000).

Partition dimension is an extension of metric dimension. A partition is the division of a vertex into several groups or classes. Let H be a connected graph. For any vertices $u, v \in V(H)$ and $S \subseteq V(H)$, suppose $V(H)$ is partitioned into k disjoint subsets $S_1, S_2, \dots, S_k$. Define $\Pi = \{S_1, S_2, \dots, S_k\}$ where $S_i \subseteq V(H)$ for $i = 1, 2, \dots, k$ as the set of k-partitions. The representation of $v \in V(H)$ according to Π is defined as

$$r(v|\Pi) = (d(v, S_1), d(v, S_2), \dots, d(v, S_k)).$$

If, whenever distinct vertices $u, v \in V(H)$, the representations according to $S_i \in \Pi$ are different, this leads us to affirm that Π serves as a resolving partition of the graph H. The number of elements of the minimum such resolving partition is referred to as the partition dimension of the graph H and is denoted $pd(H)$ (Chartrand, Salehi, et al., 2000). An important theorem regarding the partition dimension of graphs can be found in Theorem 1

Theorem 1 (Chartrand, Salehi, et al., 2000). *If G is a nontrivial connected graph, then*

$$pd(G) = \dim(G) + 1.$$

The advancement of knowledge and technology has led to several discoveries related to the determination of metric dimension and partition dimension for various graphs. Among them, Utomo & Dewi (2018) determined the metric dimension of a graph consisting of n complete graphs K_m amalgamated with a complete graph K_n, denoted by the graph $Amal\{nK_m|n \geq 4, m \geq 4\}$. Additionally, Welyyanti et al. (2023) determined the metric dimension of the amalgamation of Theta graphs. Furthermore, Mellany et al. (2023) found the metric dimension of palm graphs. Angraini et al. (2023) determined the metric dimension and partition dimension of the triangular ladder graph TR_n for $n = 2,3$. Putri et al. (2019) determined the metric dimension of the buckminsterfullerene graph. Wijaya (2022) discovered the metric dimension of the graph resulting from identification. Rahmadani & Syafruddin (2015) successfully determined the metric dimension of the barbell graph B2n, where n≥3. Marinda & Syafruddin (2015) found the metric dimension of the dragon graph Tn,m.

Liza (2018) determined the partition dimension of the friendship graph. Daming et al. (2020) determined the partition dimension of the graph resulting from the amalgamation of cycles. Haspika et al. (2023) determined the partition dimension of grid graphs. Haryeni et al. (2017) determined the partition dimension of a disconnected graph. Saifudin (2016) determined the metric dimension and the partition dimension of the family of ladder graphs.

Let G_i be a connected simple graph with $i \in \{1,2,3,\ldots,t\}$ where the vertex set $V(G_i) = \{v_{i,j}|1 \leq j \leq k_i\}$ and $|V(G_i)| = k_i$, for $k_i \geq 2$. Next, consider a finite collection of graphs $(G_1, G_2, \ldots, G_t)$ for $t \geq 2$, where any G_i has a vertex $v_i \in V(G_i)$ referred to as the terminal vertex. The operation of amalgamating these graphs is denoted by $mal(G_i, v)_t$, which results in a graph derived from $G_1, G_2, \ldots, G_t$ that share the terminal vertex v_i, and this vertex becomes a new vertex called v (Iswadi et al., 2010).

This paper will discuss how to determine the metric dimension and partition dimension of the amalgamation of complete graphs. For an integer $t \geq 2$, let $(G_1, G_2, \ldots, G_t)$ be a set of connected, finite, and simple graphs where any G_i has a fixed vertex v_0 (the central vertex). The amalgamation $(G_1, G_2, \ldots, G_t)$, denoted by $Amal(G_i, v_0)_t$, is the graph obtained by identifying the terminal vertices of any graph G_i (Bustan et al., 2023). Next, the graph $Amal(K_n, v_0)_t$ would be presented, where $G \approx K_n$ is the complete graph of order n. The definitions of the vertex set and edge set of $Amal(G_i, v_0)_t$ are as follows.

$$V(Amal(G_i, v_0)_t) = \{v_0\} \cup \{v_{i,j} | i \in [1, t], j \in [1, n-1]\}.$$

$$E(Amal(G_i, v_0)_t) = \{v_0 v_{i,j} | i \in [1, t], j \in [1, n-1]\} \cup \{v_{i,j} v_{i,k} | \ i \in [1, t], \\ j \in [1, n-1], k \in [1, n-1]\}.$$

The graph $Amal(G_i, v_0)_t$ is shown in Figure 1.

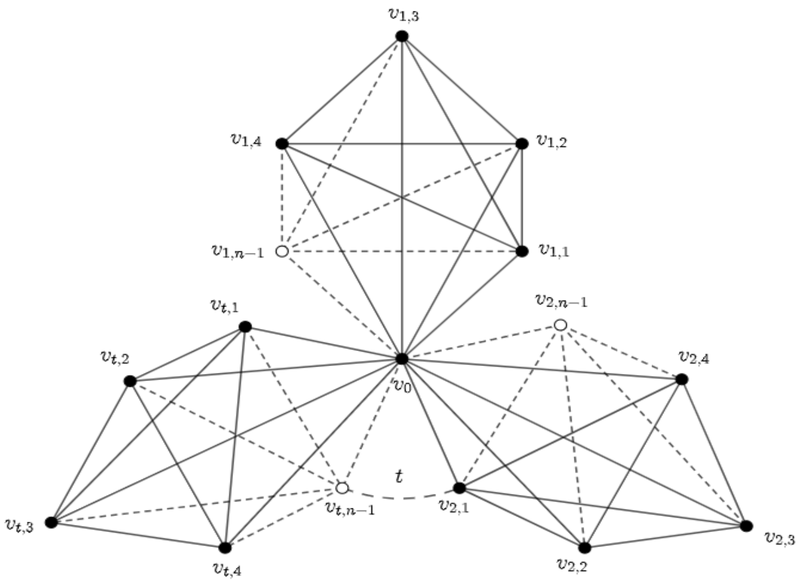

Figure 1. Graph $Amal(K_n, v_0)_t$

Figure 1 will be the object of this paper and will discuss how to determine the metric dimension and partition dimension of the amalgamation of complete graphs, which we would denote as graph $Amal(K_n, v_0)_t$.

METHODS

The steps involved in this research can be summarized as follows:

1. Determining the metric dimension of the graph $Amal(G_i, v_0)_t$:
   - Find the lower bound of the metric dimension of the graph $Amal(G_i, v_0)_t$. If this lower bound does not satisfy the condition for metric dimension, additional resolving sets are added to ensure that the condition for determining the lower bound.

- **Find the upper bound of the metric dimension of the graph** $Amal(G_i, v_0)_t$ by demonstrating that the representation of any vertex in the graph G must be different.
- The exact value of the metric dimension of the graph $Amal(G_i, v_0)_t$ is determined if the lower bound of the metric dimension is $dim(Amal(G_i, v_0)_t) \geq a$ and the upper bound is $dim(Amal(G_i, v_0)_t) \leq a$. In this case, the exact metric dimension of $Amal(G_i, v_0)_t$ is $dim(Amal(G_i, v_0)_t) = a$.

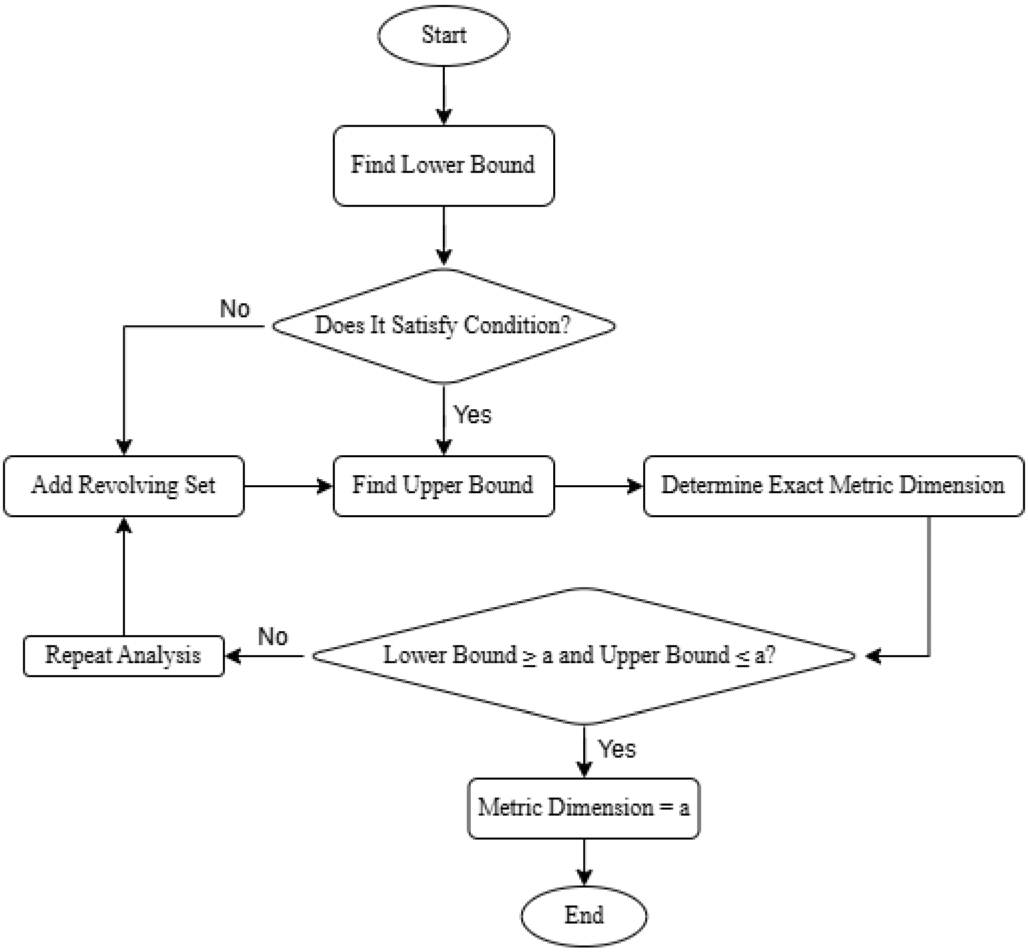

Figure 2. Flowchart of metric dimension

2. Determining the partition dimension of the graph $Amal(G_i, v_0)_t$:
   - Find the lower bound of the partition dimension of the graph $mal(G_i, v_0)_t$**.** If this lower bound does not satisfy the condition for partition dimension, additional resolving partitions are added to ensure that the condition for determining the lower bound.

- Find the upper bound of the partition dimension of the graph $Amal(G_i, v_0)_t$ by demonstrating that the representation of any vertex in the graph G must be different.
- **The exact value of the partition dimension of the graph** $Amal(G_i, v_0)_t$ is determined if the lower bound of the partition dimension is $pd(Amal(G_i, v_0)_t) \geq a$ and the upper bound is $pd(Amal(G_i, v_0)_t) \leq a$. In this case, the exact partition dimension of $Amal(G_i, v_0)_t$ is $pd(Amal(G_i, v_0)_t) = a$.

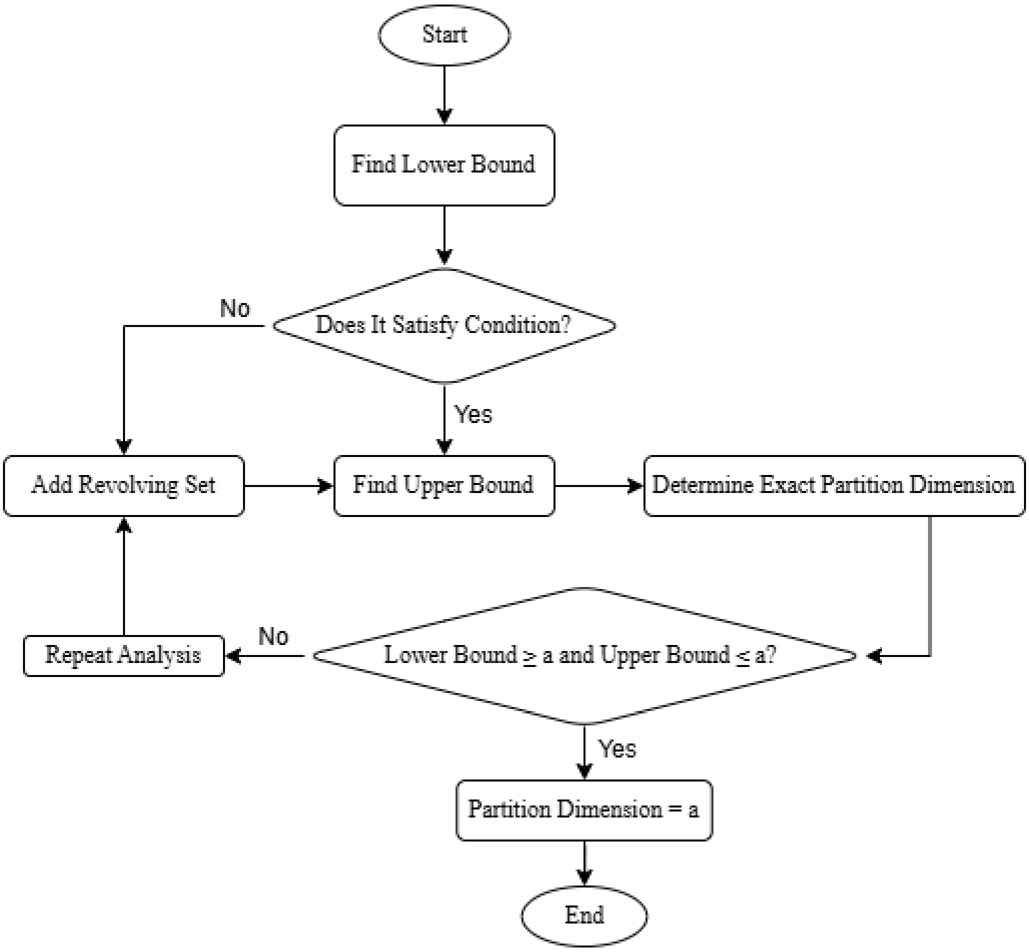

Figure 3. Flowchart of partition dimension

RESULT AND DISCUSSION

A. Metric Dimension of Amalgamation of Complete Graphs

Theorem 2 would discuss the metric dimension of the graph $Amal(G_i, v_0)_t$ for $n, t \in \mathbb{N}$ with $n \geq 3$ and $t \geq 2$.

Theorem 2. *Let* $Amal(G_i, v_0)_t$ *be the amalgamation of complete graphs* K_n, *for* $n, t \in \mathbb{N}$ *with* $n \geq 3$ and $t \geq 2$. *Then*

$$dim(Amal(G_i, v_0)_t) = (n-2)t.$$

Proof.

Let G be $Amal(G_i, v_0)_t$ with the vertex set and edge set given in the preliminary. Assume that $dim(G) = (n-2)t - 1$. Consider $W = \{v_{1,k} | k \in [3, n-1]\} \cup \{v_{i,j} | i \in [2, t], j \in [2, n-1]\}$. It follows that there would be two vertices in the first K_n that are not element of the resolving set, specifically the vertices $v_{1,1}$ and $v_{1,2}$. Consequently, these two vertices would not provide different representations because they have the same distance to W. Therefore, at least $n-2$ vertices in any K_n of the graph G must be included in the resolving set. Hence, $dim(G) \geq (n-2)t$.

Let $W = \{v_{i,j} | i \in [1, t], j \in [2, n-1]\}$, where $v_{i,j} \in G$ with $n \geq 3$ and $t \geq 2$. Consider all the vertex representations of $V(G)$ with respect to W as follows.

To analyze the uniqueness of vertex identification in $Amal(G_i, v_0)_t$, we present the vertex representations in Table 1. This table illustrates how each vertex is distinguished based on its distance to a selected resolving set, which is crucial in determining the metric dimension of the graph.

Table 1. Vertex Representations of $\boldsymbol{Amal(G_i, v_0)_t}$

Vertex Representations
$r(v_0 \| W) = (\underbrace{1,1,\dots,1,1}_{(n-2)t}),$
$r(v_{1,1} \| W) = (\underbrace{1,1,\dots,1}_{n-2}, \underbrace{2,2,\dots,2}_{(t-1)(n-2)}),$
$r(v_{1,2} \| W) = (0, \underbrace{1,1,\dots,1}_{n-3}, \underbrace{2,2,\dots,2}_{(t-1)(n-2)}),$
$r(v_{1,3} \| W) = (1,0, \underbrace{1,1,\dots,1}_{n-4}, \underbrace{2,2,\dots,2}_{(t-1)(n-2)}),$
$r(v_{1,4} \| W) = (1,1,0, \underbrace{1,1,\dots,1}_{n-5}, \underbrace{2,2,\dots,2}_{(t-1)(n-2)}),$
$r(v_{1,j} \| W) = (\underbrace{1,1,\dots,1}_{j-2}, 0, \underbrace{1,1,\dots,1}_{n-j-1}, \underbrace{2,2,\dots,2}_{(t-1)(n-2)}),$
$r(v_{1,n-1} \| W) = (\underbrace{1,1,\dots,1}_{n-3}, 0, \underbrace{2,2,\dots,2}_{(t-1)(n-2)}),$
$r(v_{2,1} \| W) = (\underbrace{2,2,\dots,2}_{n-2}, \underbrace{1,1,\dots,1}_{n-2}, \underbrace{2,2,\dots,2}_{(t-2)(n-2)}),$
$r(v_{2,2} \| W) = (\underbrace{2,2,\dots,2}_{n-2}, 0, \underbrace{1,1,\dots,1}_{n-3}, \underbrace{2,2,\dots,2}_{(t-2)(n-2)}),$
$r(v_{2,3} \| W) = (\underbrace{2,2,\dots,2}_{n-2}, 1,0, \underbrace{1,1,\dots,1}_{n-4}, \underbrace{2,2,\dots,2}_{(t-2)(n-2)}),$
$r(v_{2,4} \| W) = (\underbrace{2,2,\dots,2}_{n-2}, 1,1,0, \underbrace{1,1,\dots,1}_{n-5}, \underbrace{2,2,\dots,2}_{(t-2)(n-2)}),$
$r(v_{2,j} \| W) = (\underbrace{2,2,\dots,2}_{n-2}, \underbrace{1,1,\dots,1}_{j-2}, 0, \underbrace{1,1,\dots,1}_{n-j-1}, \underbrace{2,2,\dots,2}_{(t-2)(n-2)}),$

$$r(v_{2,n-1}|W) = (\underbrace{2,2,\dots,2}_{n-2}, \underbrace{1,1,\dots,1}_{n-3}, 0, \underbrace{2,2,\dots,2}_{(t-2)(n-2)}),$$
$$r(v_{t,1}|W) = (\underbrace{2,2,\dots,2}_{(t-1)(n-2)}, \underbrace{1,1,\dots,1}_{n-2}),$$
$$r(v_{t,2}|W) = (\underbrace{2,2,\dots,2}_{(t-1)(n-2)}, 0, \underbrace{1,1,\dots,1}_{n-3}),$$
$$r(v_{t,3}|W) = (\underbrace{2,2,\dots,2}_{(t-1)(n-2)}, 1,0, \underbrace{1,1,\dots,1}_{n-4}),$$
$$r(v_{t,4}|W) = (\underbrace{2,2,\dots,2}_{(t-1)(n-2)}, 1,1,0, \underbrace{1,1,\dots,1}_{n-5}),$$
$$r(v_{t,j}|W) = (\underbrace{2,2,\dots,2}_{(t-1)(n-2)}, \underbrace{1,1,\dots,1}_{j-2}, 0, \underbrace{1,1,\dots,1}_{n-j-1}),$$
$$r(v_{t,n-1}|W) = (\underbrace{2,2,\dots,2}_{(t-1)(n-2)}, \underbrace{1,1,\dots,1}_{n-3}, 0).$$

Table 1 presents the vertex representations in the graph $Amal(G_i, v_0)_t$. The distance of the vertex v_0 with respect to elements of W is consistently 1. For each vertex $v_{i,j}$ with $i = 1,2,\dots,t$ and $j = 1,2,\dots,(n-1)$, its representation with respect to W is 0 for itself, 1 for vertices within the same K_n, and 2 otherwise. Consequently, each vertex $v_{i,j}$ is uniquely distinguished by the placement of 0, 1, and 2 values.

Since no two vertices share the same representation with respect to W, it follows that W constitutes a resolving set. Therefore, the metric dimension satisfies $dim(G) \leq (n-2)t$, leading to the conclusion that the metric dimension of $Amal(G_i, v_0)_t$ is $dim(Amal(G_i, v_0)_t) = (n-2)t$. These findings confirm the validity of the chosen resolving set and provide a precise characterization of vertex distinguishability within the graph structure.

■

B. Partition Dimension of Amalgamation of Complete Graphs

Theorem 3 would discuss the partition dimension of the graph $Amal(G_i, v_0)_t$ for $n, t \in \mathbb{N}$ with $n \geq 3$ and $t \geq 2$.

Theorem 3. *Let $Amal(G_i, v_0)_t$ be the amalgamation of complete graphs K_n, for $n, t \in \mathbb{N}$ with $n \geq 3$ and $t \geq 2$. Then*

$$pd(Amal(G_i, v_0)_t) = n, \ for \ 2 \leq t \leq C_{n-1}^{n}.$$

Proof.

Let G be $Amal(G_i, v_0)_t$ with the vertex set and edge set given in the preliminary. We would determine that $pd(G) \geq n$ for $2 \leq t \leq C_{n-1}^{n}$. According to Theorem 1 in the preliminary, which states that $pd(H) \leq dim(H) + 1$, and based on the previously obtained metric dimension results for the graph $Amal(G_i, v_0)_t$, we have $pd(G) \geq n$ for $2 \leq t \leq$

C_{n-1}^{n}. Next, we would show that $pd(G) \le n$ for $2 \le t \le C_{n-1}^{n}$. Let $\Pi = \{S_1, S_2, \dots, S_n\}$ be a resolving partition of $mal(G_i, v_0)_t$. The partition classes of the graph $Amal(G_i, v_0)_t$ are shown in Figure 2.

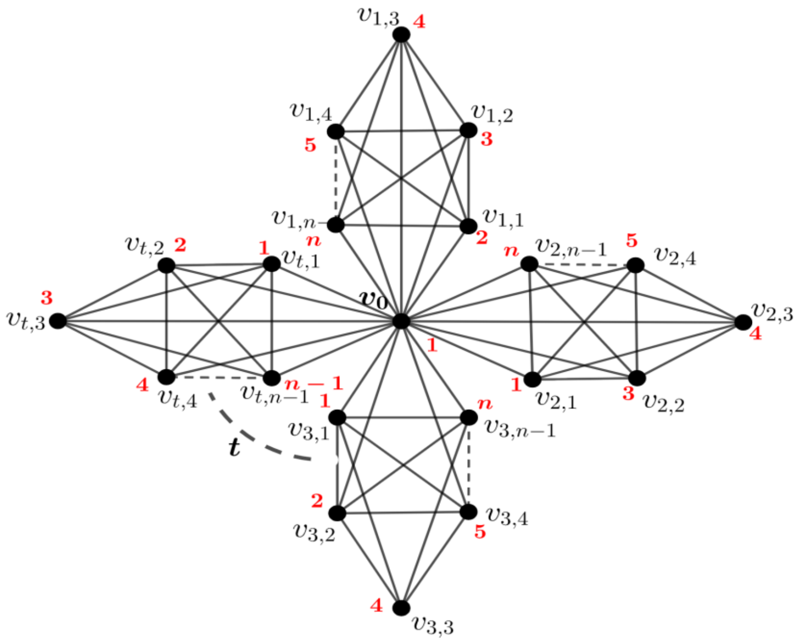

Figure 4. The partition classes of the graph $Amal(G_i, v_0)_t$

As illustrated in Figure 4, the vertex representations in the graph $Amal(G_i, v_0)_t$ are determined based on their distances to elements of the resolving set Π. Each row in Table 2 corresponds to a vertex, while the columns indicate the respective distance values. These distances uniquely distinguish each vertex, ensuring that no two vertices share the same representation. The placement of values 0, 1, and 2 reflects the relative proximity of each vertex within the graph structure, which plays a crucial role in accurately determining its partition dimension.

Table 2. Vertex Representations of $Amal(G_i, v_0)_t$

Vertex Representations	
$r(v_0\|\Pi) = (0, \underbrace{1,1,\dots,1}_{n-1}),$	$r(v_{3,1}\|\Pi) = (0,1,2, \underbrace{1,1,\dots,1}_{n-3}),$
$r(v_{1,1}\|\Pi) = (1,0, \underbrace{1,1,\dots,1}_{n-2}),$	$r(v_{3,2}\|\Pi) = (1,0,2, \underbrace{1,1,\dots,1}_{n-3}),$
$r(v_{1,2}\|\Pi) = (1,1,0, \underbrace{1,1,\dots,1}_{n-3}),$	$r(v_{3,3}\|\Pi) = (1,1,2,0, \underbrace{1,1,\dots,1}_{n-4}),$
$r(v_{1,3}\|\Pi) = (1,1,1,0, \underbrace{1,1,\dots,1}_{n-4}),$	$r(v_{3,4}\|\Pi) = (1,1,2,1,0, \underbrace{1,1,\dots,1}_{n-5}),$
$r(v_{1,4}\|\Pi) = (1,1,1,1,0, \underbrace{1,1,\dots,1}_{n-5}),$	$r(v_{3,j}\|\Pi) = (1,1,2, \underbrace{1,1,\dots,1}_{j-3}, 0, \underbrace{1,1,\dots,1}_{n-j-1}),$
$r(v_{1,j}\|\Pi) = (\underbrace{1,1,\dots,1}_{j}, 0, \underbrace{1,1,\dots,1}_{n-j-1}),$	$r(v_{3,n-1}\|\Pi) = (1,1,2, \underbrace{1,1,\dots,1}_{n-3}),$

$r(v_{1,n-1}\|\Pi) = (\underbrace{1,1,\dots,1}_{n-1},0)$,	$r(v_{t,1}\|\Pi) = (0,\underbrace{1,1,\dots,1}_{n-2},2)$,
$r(v_{2,1}\|\Pi) = (0,2,\underbrace{1,1,\dots,1}_{n-2})$,	$r(v_{t,2}\|\Pi) = (1,0,\underbrace{1,1,\dots,1}_{n-3},2)$,
$r(v_{2,2}\|\Pi) = (1,2,0,\underbrace{1,1,\dots,1}_{n-3})$,	$r(v_{t,3}\|\Pi) = (1,1,0,\underbrace{1,1,\dots,1}_{n-4},2)$,
$r(v_{2,3}\|\Pi) = (1,2,1,0,\underbrace{1,1,\dots,1}_{n-4})$,	$r(v_{t,4}\|\Pi) = (1,1,1,0,\underbrace{1,1,\dots,1}_{n-5},2)$,
$r(v_{2,4}\|\Pi) = (1,2,1,1,0,\underbrace{1,1,\dots,1}_{n-5})$,	$r(v_{t,j}\|\Pi) = (\underbrace{1,1,\dots,1}_{j-1},0,\underbrace{1,1,\dots,1}_{n-j-1},2)$,
$r(v_{2,j}\|\Pi) = (1,2,\underbrace{1,1,\dots,1}_{j-2},0,\underbrace{1,1,\dots,1}_{n-j-1})$,	$r(v_{t,n-1}\|\Pi) = (\underbrace{1,1,\dots,1}_{n-2},0,2)$,
$r(v_{2,n-1}\|\Pi) = (1,2,\underbrace{1,1,\dots,1}_{n-2})$.	

With the partition classes defined as shown in Table 2, the set Π provides a unique representation for each vertex, thereby satisfying the condition for a resolving partition. Consequently, the partition dimension of the graph satisfies $pd(G) \leq n$ for $2 \leq t \leq C^n_{n-1}$. It follows that the partition dimension of $Amal(G_i, v_0)_t$ is precisely $pd(Amal(G_i, v_0)_t) = n$, confirming the validity of the selected resolving partition and ensuring the distinct identification of all vertices within the graph structure.

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CONCLUSION

This paper has examined the metric and partition dimensions of the graph $Amal(G_i, v_0)_t$. The findings indicate that $dim(Amal(G_i, v_0)_t) = (n-2)t$ and $pd(Amal(G_i, v_0)_t) = n$ for $2 \leq t \leq C^n_{n-1}$. These results contribute to the broader understanding of vertex identification in graph structures, with potential applications in network optimization and fault-tolerant systems. Future research could extend this analysis to other graph families or explore different amalgamation models to further investigate their impact on graph dimensions.

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